Decentralized Observer-Based Robust Model Predictive Control for a Class of Distributed Networked Systems

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Abstract: This paper presents a decentralized observer-based robust model predictive control strategy for a class of distributed networked systems. The overall system is composed of a number of interconnected nonlinear subsystems with time-varying transmission delays. Time delays are appeared in distributed subsystems due to the information transmission in the communication networks. A decentralized control strategy is implemented for each subsystem based on model predictive controllers. Each MPC problem is formulated with a memory-less observer-based feedback control law to minimize the upper bound of the infinite horizon cost that satisfies the sufficient stability conditions. Novel delay dependent sufficient conditions are proposed for the existence of such controllers in the form of a linear matrix inequality optimization problem. The simulation results for a chemical reactor plant are exploited to illustrate the applicability of the proposed method.

Key-Words: Model Predictive Control- Decentralized- transmission delays- observer- linear matrix inequality

1 Introduction
Processes in modern industries are physically distributed and generally composed of different subsystems, which are interconnected and characterized by significant interactions; examples of such systems are presented in [1]-[3]. Besides, the implementation of a control strategy can be economically achieved via a communication networks. In these networked systems, the information exchange through the communication networks unavoidably introduces time delays. Therefore, it is very important to investigate the control problem for distributed networked systems with transmission delays.

In real applications, it could be a challenge to address the control problem in interconnected systems using a centralised architecture because of the constraints on the computational capabilities and the communication bandwidth. Consequently, the area of decentralised control has attracted broad attention ([4], [5]). In other hand, some difficulties are appeared due to the fact that each local controller has only local information. Thus we need to consider disturbances and/or neglected dynamics which, if ignored, can harm the control design.

In recent years, model predictive control (MPC) has become an important research interest in control community within industry and academia. This is illustrated by its applications to industrial and practical implementations and by many valuable works on this topic. See for example the references [6] and [7]. MPC algorithms which explicitly take into account uncertainties within their formulation are called Robust Model Predictive Control (RMPC) schemes. It is well known that time-delays as well as uncertainties are most often the main cause of deterioration of system performance and instability of the systems. In [8]-[10], Wuhua et al., Zhang et al., and Mei and Huihe have proposed robust model predictive control algorithms for discrete linear systems with both state and input delays. In the actual control problems, the states may not be available directly. Thus, some observer-based control strategies are required. Vesely and Rosinov have considered the problem of designing a robust output-feedback model predictive control for linear polytopic systems [11]. The problem of observer-based robust predictive control for the linear singular systems with norm-bounded uncertainties and just state delay is studied by Xiaohua and Yuanhua [12].
It is worth mentioning that most of the works consider the linear system formulation to develop their robust model predictive control (see [8]-[12]). Since many real-world systems exhibit nonlinear dynamic behaviour, the control of uncertain nonlinear systems is of practical importance. In the references [13] and [14], we proposed a robust predictive control approach for additive discrete time uncertain nonlinear systems. In [15], we presented a linear matrix inequality (LMI) framework to design robust MPC for a class of continuous-time nonlinear uncertain systems. The controller design is characterized as an optimization problem of the “worst-case” objective function over infinite moving horizon. A sufficient state feedback synthesis condition is provided in the form of LMI optimization and will be solved online at each time step. Our aim in this paper is to extend these results to continuous-time nonlinear networked systems with transmission delays.

Motivated by the above discussions, this paper presents decentralized observer-based robust model predictive controllers for a class of distributed networked systems. The overall system is composed of a number of interconnected nonlinear subsystems with time-varying transmission delays. Time delays are appeared in distributed subsystems due to the information transmission in the communication network. A decentralized control strategy is implemented for each subsystem with model predictive controllers. Each MPC problem is formulated with a memory-less observer-based feedback control law to minimize the upper bound of the infinite horizon cost that satisfies the sufficient stability conditions. Novel delay dependent sufficient conditions are proposed for the existence of such controllers in the form of a linear matrix inequality (LMI) optimization problem. The simulation results for a chemical reactor plant are exploited to illustrate the applicability of the proposed method.

The rest of this article is organized as follows. The problem description is illustrated in Section 2. In Section 3, the main results are presented. In order to demonstrate the validity of the approach the simulation results are illustrated in Section 4. Section 5 provides the concluding remarks.

2 Problem Description

Consider a plant that composed of \( N \) different sub-plants with interactions, and the \( i \)th sub-plant is described by

\[
\dot{x}_i = A_i x_i + A_{ii} x_i (t-d_{ii}(t)) + f_i(x_i(t),u_i(t-d_{ii}(t))) + B_i u_i(t-d_{ii}(t)) \\
y_i = C_i x_i(t),
\]

where \( x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}, y_i \in \mathbb{R}^{p_i} \), and \( x \) is the \( i \)th sub-plant state, input, output, and the plant state vectors. \( A_i, A_{ii}, B_i \) are the \( i \)th sub-plant constant matrices and \( f_i(\cdot) \) is the nonlinear function represent how other sub-plants interact with \( i \)th sub-plant. The time delays \( d_{ii}(t), d_{i}(t) \) encountered in the system due to information exchange in the \( i \)th sub-plant and between distributed sub-plants, between \( i \)th controller and sub-plant, and between \( i \)th sub-plant and controller via the communication network respectively.

**Remark 1.** It can be considered that \( d_{ii}(t) = \max \{d_{in}(t), d_{i}(t)\} \), where \( d_{in}(t) \) and \( d_{i}(t) \) are the time delays related to information exchange in the \( i \)th sub-plant and between \( i \)th and \( j \)th sub-plants.

**Assumption 1.** \( d_{i}(t), d_{i}(t) \) and \( d_{i}(t) \) are time-varying bounded delays satisfying the following inequalities

\[
0 \leq d_{ii}(t) \leq d_{ii}^*, d_{i}(t) \leq d_{i}^* \quad \forall w = 1,2,3
\]

**Assumption 2.** \( f_i(x, x(t-d_{ii}(t))) \) is a Lipschitz bounded nonlinear term that satisfies the following condition

\[
\|f_i(x, x(t-d_{ii}(t)))\| \leq \alpha \|x\| + \gamma \|x(t-d_{ii}(t))\|
\]

where \( \alpha \) and \( \gamma \) are positive real constant known numbers.

Now one can consider the nonlinear term as the model uncertainty which is unknown. This idea can be used to reformulate the system to an uncertain one and motivate the robust model predictive control (RMPC) approach.
Assumption 3. Suppose that the matrix $C_i$ has full row rank (i.e., $\text{rank}(C_i) = p$). Then the singular value decomposition of $C_i$ is
\[
C_i = U_i \begin{bmatrix} \bar{S}_i & \mathbf{0} \end{bmatrix} V_i^T
\]
(4)
where $\bar{S}_i \in \mathbb{R}^{n \times n}$ is a diagonal matrix with positive elements in a descending order, $0 \in \mathbb{R}^{n \times (n-p)}$ is a zero matrix, and $U_i, V_i \in \mathbb{R}^{n \times n}$ are unitary matrices.

Let $x_i(kT+l,kT)$ and $u_i(kT+l,kT)$ denote the predicted state and control actions of the $i$th sub-plant at time $kT$, where $T$ is the sampling time. Then a quadratic cost function may be given as:
\[
J_{ik} = \int_{kT}^{(k+1)T} \left( x_i(kT+l,kT)^T Q x_i(kT+l,kT) + u_i(kT+l,kT)^T R_i u_i(kT+l,kT) \right) dt
\]
(5)
where $Q_i > 0$ and $R_i > 0$ are symmetric weighting matrices.

Consider the following observer-based control for system (1):
\[
\begin{align*}
\dot{x}_i &= A_i x_i + A_i \hat{x}_i (t - d_1(t)) + B_i u_i(t - d_2(t)) \\
\dot{\hat{x}}_i &= C_i x_i (t) + d_i(t) + L_i e_i(t), \\
u_i(kT+l,kT) &= K_i \hat{x}_i(kT+l,kT),
\end{align*}
\]
(6)

where $\hat{x}_i(t) \in \mathbb{R}^{n}$ is the state estimation vector of the $i$th sub-plant, $K_i \in \mathbb{R}^{n \times n}$ and $L_i \in \mathbb{R}^{n \times n}$ are $i$th controller and observer gain respectively.

Suppose $e_i = x_i - \hat{x}_i$ is the estimated error of the $i$th sub-plant, then the dynamic error equation and closed-loop system are respectively expressed as
\[
\begin{align*}
\dot{e}_i &= A e_i + A \xi_i (t - d_i(t)) + f_i(x_i(x(t - d_i(t))) - L_i e_i(t - d_2(t)), \\
\dot{\xi}_i &= A \xi_i + f_i(x_i(x(t - d_1(t))) \\
&= B_i x_i(t - d_2(t)) + L_i e_i(t - d_3(t)).
\end{align*}
\]
(7)

The objective is to design a robust observer-based predictive controller (RMPC) for each sub-plant (1) at each sampling time $kT$ to obtain optimal control sequence $u_i(kT+l,kT), l \in [0, \infty)$ to minimize $J_{ik}$ subject to model uncertainty. The proposed Distributed Networked Control System (DNCS) structure is presented in Fig. 1.

**3 Main Results**

In this section, we propose a new RMPC technique to design observer-based controller for the system (1) using Lyapunov-Krasovskii function. For simplicity of notation we ignore the subscript $i$ in this section.

Consider the quadratic Lyapunov-Krasovskii function $V(x(t), e(t))$ for the system in (7).
\[
V = x^T P x + e^T P e + \sum_{\nu = 1}^{n} \int_{\nu - 1}^{\nu} x^T (\sigma) S_{\nu} x (\sigma) d\sigma + \int_{-\infty}^{-1} e^T (\sigma) S_{-1} e (\sigma) d\sigma
\]
where $P > 0$, $S_\nu > 0$, and $\bar{S}_\nu > 0$. According to the Lyapunov stability theory, the state $x(t)$ and $e(t)$ asymptotically converges to zero if the function $V(x(t), e(t))$ is positive definite and $\dot{V}(x(t), e(t))$ is negative definite for all $x(t)$ and $e(t)$. These conditions establish the stability constraints for our problem at each time instant $kT$. At each sampling time $kT$, suppose that $V(x(t), e(t))$ satisfies the following robust stability constraint
\[
\frac{d}{dt} V(x(kT+l,kT), e(kT+l,kT)) \leq -x(kT+l,kT)^T \times
\]
\[
Q x(kT+l,kT) - u(kT+l,kT)^T Ru(kT+l,kT)
\]
(9)

In order to make $J_{ik}$ finite, we must have $x(\infty,kT) = 0$, $V(x(\infty,kT), e(\infty,kT)) = 0$ under the control law (6). Integrating both sides of the inequality (9) from $l = 0$ to $T$, we obtain
\[
\int_{l=0}^{T} x^T (kT+l,kT) Q x(kT+l,kT) + u^T (kT+l,kT) R u(kT+l,kT) dt \geq \int_{l=0}^{T} x^T (kT+l,kT) V x (kT) dt + u^T (kT+l,kT) R u(kT+l,kT) dt +
\]
\[ e^T (kT + T) Pe(kT + T) - e^T (kT) Pe(kT) + \sum_{\tau = 0}^{\infty} \int_{t - kT}^{t} \int_{t - kT}^{t} e^T (\tau) S_x \phi(\tau) \phi(\tau) e^T (\tau) d\tau + \int_{t - kT}^{t} e^T (\tau) S_x e(\tau) d\tau \]

As \( T \to \infty \), we can obtain:

\[ J_k < V_i(kT) \quad (11) \]

where

\[ V_i(kT) = x^T(kT) P x(kT) + e^T(kT) Pe(kT) + \sum_{\tau = 0}^{\infty} \int_{t - kT}^{t} \int_{t - kT}^{t} e^T (\tau) S_x \phi(\tau) \phi(\tau) e^T (\tau) d\tau. \]

Therefore, the RMPC problem at time \( kT \) can be solved by minimizing \( V_i(kT) \) instead of minimizing \( J_k \) subject to model uncertainty.

**Theorem 1.** For the system (1), \( \hat{x}(kT) \) is the estimated state from the observer system (6) at each sampling time \( kT \). Then the observer-based feedback matrix \( K \) in the control law \( u(kT + 1, kT) = K \hat{x}(kT + 1, kT) \) and observer gain matrix \( L \), which minimize the upper bound on the predictive control objective function \( (V_i(kT)), \) are given by \( K = X_1 \tilde{P}^{-1} \) and \( L = \eta^{-1} D \tilde{P}^{-1} \), where \( X_1, P, D, \eta \) and \( \tilde{P} \) are obtained from the solution of the following LMI problem:

\[ \min_{\tilde{K}, \tilde{S}_d, \tilde{X}_1, \tilde{S}_x, \tilde{Z}_x} \eta + \sum_{\tau = 1}^{\infty} \text{trace}(\tilde{Z}_x) + \text{trace}(\tilde{Z}_x) \quad (12) \]

subject to

\[ \begin{bmatrix} I & x^T(kT) & e^T(kT) \\ x(kT) & \tilde{P} & 0 \\ e(kT) & 0 & \tilde{P} \end{bmatrix} \geq 0 \quad (13) \]

\[ \begin{bmatrix} -\tilde{Z}_x & q_s^- \\ q_s^- & -\tilde{U}_s \end{bmatrix} \leq 0, \quad (14) \]

\[ \begin{bmatrix} -\tilde{Z}_x & q_s^- \\ q_s^- & -\tilde{U}_s \end{bmatrix} \leq 0 \quad (15) \]

where the symbol * depicts a symmetric structure, \( I \) is an appropriately dimensioned identity matrix,

\[ \tilde{A}_n = A \tilde{P} + A \tilde{P}^T + \sum_{\tau = 0}^{\infty} \int_{t - kT}^{t} \int_{t - kT}^{t} \phi^T (\tau) S_x \phi(\tau) d\tau + \sum_{\tau = 0}^{\infty} \int_{t - kT}^{t} \int_{t - kT}^{t} \phi^T (\tau) S_x e(\tau) d\tau \]

and \( \tilde{P} = G_{\theta} H_{\theta}^{-1} G_{\theta}^T \) obtains from \( \tilde{P}, \eta \) and \( C \) based on the following equalities:

\[ \eta^{-1} \tilde{P} = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \tilde{P}^T \quad , \quad C = \begin{bmatrix} G_{\theta}^T & 0 \\ G_{\theta}^T \end{bmatrix} \tilde{P}^T \quad . \]

If the minimizing problem (12) has a feasible solution at the initial time, the closed-loop system (7) with feedback matrix \( K \) is asymptotically stable.

**Proof.** According to \( \int_{-\infty}^{0} \phi(\tau) \phi(\tau) d\tau = q_s^- q_s^- \), we can obtain

\[ \int_{-\infty}^{0} e^T (\tau) S_x e(\tau) d\tau = \int_{-\infty}^{0} \text{trace}(e^T (\tau) S_x e(\tau) d\tau) = \text{trace}(q_s^- q_s^- U_w) \quad , \quad w = 1, 2, 3 \quad (17) \]

\[ \int_{-\infty}^{0} e^T (\tau) S_x e(\tau) d\tau = \int_{-\infty}^{0} \text{trace}(e^T (\tau) U_w) = \text{trace}(q_s^- q_s^- U_w) \quad , \quad w = 1, 2, 3 \quad (18) \]

where \( S_w = U_w^{-1} \) and \( S_w = U_w^{-1} \). Assume there exist matrices \( Z_x \) and \( Z_x \) such that \( \text{trace}(q_s^- U_w q_s^-) < Z_x \) and \( \text{trace}(q_s^- U_w q_s^-) < Z_x \), then (14) and (15) hold by the Schur complement ([16]). If there exist a scalar
\( \eta \) satisfying \( x^T(kT)P_S(kT) + \epsilon^T(kT)Pe^T(kT) \leq \eta \), then \( x^T(kT)P_S(kT) + \epsilon^T(kT)Pe^T(kT) \leq \eta \) is equivalent to (13) by the Schur complement, where \( P = \eta P^{-1} \). So, minimization of \( J_k \) subject to model uncertainty can be converted into the minimization of \( \eta + \sum_{i=1}^3 (\text{trace}(Z_\eta) + \text{trace}(Z_{\eta})) \).

According to inequality (3) and Young's inequality ([17]), we have:

\[
I \leq \eta^T P x(t) x(t) + \epsilon^T P \epsilon(t) + \sum_{i=1}^3 (\alpha_i + \gamma_i) \eta_i^T \eta_i + \eta^T P_x \eta(t) + \epsilon^T P_{\epsilon} \epsilon(t) + \sum_{i=1}^3 (\alpha_i + \gamma_i) \eta_i^T \eta_i \\
\leq \frac{\eta^T P_x \eta(t) + \epsilon^T P_{\epsilon} \epsilon(t) + \sum_{i=1}^3 (\alpha_i + \gamma_i) \eta_i^T \eta_i}{2} \leq \frac{\eta^T P_x \eta(t) + \epsilon^T P_{\epsilon} \epsilon(t) + \sum_{i=1}^3 (\alpha_i + \gamma_i) \eta_i^T \eta_i}{2}
\]

(19)

Without loss of generality, we assume that \( \eta \leq \zeta \).

Now consider the time differentiation of \( V(x(t), e(t)) \) and substituting inequalities (3) and (19), we get the following inequality

\[
\dot{V} \leq \epsilon^T (P + \sum_{i=1}^3 \gamma_i I) \epsilon + \eta^T (P + \sum_{i=1}^3 \alpha_i I) \eta + \epsilon^T \\alpha^T \eta + \epsilon^T \gamma^T \eta + \sum_{i=1}^3 \gamma_i \eta_i + \sum_{i=1}^3 \alpha_i \eta_i
\]

By some algebraic operations, we can reach

\[
\dot{V} \leq \epsilon^T \epsilon \leq 0
\]

where

\[ \epsilon^T = [\epsilon' \epsilon' x(t-d_i) \cdots x(t-d_i) x'(t-d_i) \cdots x'(t-d_i)] \]

Based on Lyapunov-stability theory and Barbalat’s lemma ([18]), the proposed controller will guarantee that the closed-loop system (7) is asymptotically stable.

**Lemma 1** [19]. For a given \( C \in \mathbb{R}^{m \times n} \) with rank\( (C) = p \), assume that \( Q \in \mathbb{R}^{m \times m} \) is a symmetric matrix; then there exists a matrix \( \bar{P} \in \mathbb{R}^{m \times m} \) such that \( C^T \bar{P} = \bar{P} C \) if and only if

\[
\bar{Q} = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}
\]

where \( H_1 \in \mathbb{R}^{p \times p} \), \( H_2 \in \mathbb{R}^{(m-p) \times (m-p)} \), and \( \bar{V}_i \in \mathbb{R}^{m \times m} \).

According to Assumption 3, the conditions \( \epsilon^T \epsilon \geq \xi \), \( \epsilon^T P \epsilon \leq \eta \), and \( P = \eta P^{-1} \), Lemma 1 implies that \( C^T \bar{P} = \bar{P} C \).

Pre-multiplying and post-multiplying \( E \leq 0 \) by the matrix \( T = \text{diag}(\eta^2, \eta^2, \ldots, \eta^2, P^{-1}) \) and setting \( \eta P^{-1} = \bar{P}, \eta P^{-1} S_S P^{-1} = S_i, \eta P^{-1} S_h P^{-1} = \eta^2 \), \( \eta P^{-1} Q P^{-1} = Q \) and \( D = \eta P \), we derive \( T^T \dot{E} = \bar{E} \leq 0 \). Applying Schur complement lemma to the \( \bar{E} \leq W_{l-1} W_{l-1}^T \leq 0 \), one can get \( H = \begin{bmatrix} \bar{E} & W_l \\ W_l^T & W_{l-1} \end{bmatrix} \leq 0 \), where \( W_l = [0 0 0 0 0 0 0 X_l] \) and \( W_{l-1} = \eta^T R \).

This completes the proof of theorem 1.

**4 Simulation Results**

In this section, a distributed networked chemical reactor plant with transmission delays is considered [5]. This system consists of two sub-plants, either of which has three reactors. The system structure is depicted in Fig. 2. The compositions of the produce stream from the reactor \( C_{11}, C_{12}, C_{13}, C_{21}, C_{22}, C_{23}, C_{31}, C_{32}, C_{33} \) are the system state to be controlled. The model of this networked chemical reactor plant is described by

\[
\begin{align*}
C_{i1}(t) &= -k_{i1} C_{i1}(t) - \frac{R_{1i}}{V_{1i}} C_{i1}(t) - \frac{1}{V_{1i}} u_i(t) \quad (i = 1, 2, 3) \\
C_{i2}(t) &= -k_{i2} C_{i2}(t) - \frac{R_{2i}}{V_{2i}} C_{i2}(t) - \frac{1}{V_{2i}} u_i(t) \quad (i = 1, 2, 3) \\
C_{i3}(t) &= -k_{i3} C_{i3}(t) - \frac{R_{3i}}{V_{3i}} C_{i3}(t) - \frac{1}{V_{3i}} u_i(t) \quad (i = 1, 2, 3)
\end{align*}
\]

where \( V_1 = [C_{11} C_{12} C_{13}] \), \( C_{14} = [C_{11} C_{12} C_{13}] \), \( u_i(t) \) and \( u_j(t) \) are the first sub-plant and second sub-plant states and control input respectively. \( d_{i1}(t) \) and \( d_{i2}(t) \) are the time delays related to information exchange in piping communication network within the \( i \) th sub-plant and between \( i \)th and \( j \)th sub-plants.
Recent Advances in Electrical Engineering

Other parameters of this chemical reactor plant for \( i = 1,2 \) are described in Table 1.

**Table 1: Parameters of chemical reactor plant for \( i = 1,2 \)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_a, R_a, R_c )</td>
<td>Recycle flow rates</td>
<td>( k_a = k_B = k_C = 0.5 )</td>
</tr>
<tr>
<td>( \theta_a, \theta_B, \theta_C )</td>
<td>Reactor residence time</td>
<td>( R_a = R_B = R_C = 0.5 )</td>
</tr>
<tr>
<td>( k_a, k_B, k_C )</td>
<td>Reaction constants</td>
<td>( V_a = V_B = V_C = 0.5 ) and ( F_i = 0.5 ), the equilibrium points will be ( C^<em>_a = 3/32 ), ( C^</em>_B = 3/16 ) and ( C^<em>_C = 3/8 ). Let us define ( x_i(t) = C^</em>_a (t) - C^<em><em>a ), ( x</em>{i2}(t) = C^</em>_B (t) - C^<em><em>a ) and ( x</em>{i3}(t) = C^</em>_C (t) - C^*_a ) for ( i = 1,2 ). Now we have</td>
</tr>
<tr>
<td>( F_i )</td>
<td>Feed rates</td>
<td>( x_{i1}(t) = -0.5x_{i1}(t) - 0.5(x_{i1}(t-d_{i1}) + x_{i2}(t-d_{i1})) + x_{i2}(t) )</td>
</tr>
<tr>
<td>( V_a, V_B, V_C )</td>
<td>Volume of the reactors</td>
<td>( x_{i1}(t) = -0.5x_{i1}(t) - 0.5(x_{i1}(t-d_{i1}) + x_{i2}(t-d_{i1})) + x_{i2}(t) )</td>
</tr>
<tr>
<td>( H_a, H_B, H_C )</td>
<td>Nonlinear function represents the complex behavior of the systems</td>
<td>( x_{i1}(t) = -0.5x_{i1}(t) - 0.5(x_{i1}(t-d_{i1}) + x_{i2}(t-d_{i1})) + x_{i2}(t) )</td>
</tr>
<tr>
<td>( f_i(\cdot) )</td>
<td>Nonlinear functions for describing the systems uncertainties and external disturbance</td>
<td>( x_{i1}(t) = -0.5x_{i1}(t) - 0.5(x_{i1}(t-d_{i1}) + x_{i2}(t-d_{i1})) + x_{i2}(t) )</td>
</tr>
</tbody>
</table>

If one chooses \( \theta_{i1} = \theta_{i2} = \theta_{i3} = 2 \), \( k_a = k_B = k_C = 0.5 \), \( R_a = R_B = R_C = 0.5 \), \( V_a = V_B = V_C = 0.5 \) and \( F_i = 0.5 \), the equilibrium points will be \( C^*_a = 3/32 \), \( C^*_B = 3/16 \) and \( C^*_C = 3/8 \). Let us define \( x_{i1}(t) = C^*_a (t) - C^*_a \), \( x_{i2}(t) = C^*_B (t) - C^*_a \) and \( x_{i3}(t) = C^*_C (t) - C^*_a \) for \( i = 1,2 \). Now we have

\[
\begin{align*}
x_{i1}(t) &= -0.5x_{i1}(t) - 0.5(x_{i1}(t-d_{i1}) + x_{i2}(t-d_{i1})) + x_{i2}(t) \\
x_{i2}(t) &= -0.5x_{i2}(t) - 0.5(x_{i1}(t-d_{i2}) + x_{i2}(t-d_{i2})) + x_{i3}(t) \\
x_{i3}(t) &= -0.5x_{i3}(t) - 0.5(x_{i1}(t-d_{i3}) + x_{i2}(t-d_{i3})) + x_{i3}(t) \\
y_i(t) &= x_{i1}(t-d_{i1}) \\
\end{align*}
\]

where \( d_{i1}, d_{i2}, d_{i3} \) are the time delays related to information exchange between \( i th \) controller and sub-plant, and between \( i th \) sub-plant and controller via the communication network respectively. \( y_i(t) \) is the \( i th \) sub-plant output for \( i = 1,2 \) and

\[
\begin{align*}
f_1(x_i(t), x(t-d_{i1})) &= e^{x_i(t)}x_i(t-d_{i1}) + e^{x_i(t)}x_i(t-d_{i1}) + x_i(t-d_{i1}) \\
f_2(x_i(t), x(t-d_{i2})) &= e^{x_i(t)}x_i(t-d_{i2}) + e^{x_i(t)}x_i(t-d_{i2}) + x_i(t-d_{i2}) \\
\end{align*}
\]

This system can be reformulated in the form of the system in (1) as

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= x_1(t) \\
\end{align*}
\]

When \( \alpha = 0.5 \) and \( \gamma = 0.2 \) the Lipschitz nonlinearity is satisfied. In order to study the behavior of this system in numerical simulation, integration increment in simulation environment is set to \( T_i = 0.01 \). Cost function parameters are chosen as \( Q = I \) and \( R = 10^6 I \), and the time delay upper bounds are \( d_{i1}^* = 0.2 \), \( d_{i2}^* = 0.1 \) and \( d_{i3}^* = 0.15 \) second respectively. Our LMI based method is applied to this problem, this system is initialized from
\[ x_1(0) = [1 \ 0 \ 1]^T, \ x_2(0) = [0.8 \ 0.8]^T \] and
\[ \hat{x}_i(0) = [0.5 \ 0.5 \ 0.5]^T \] for \( i = 1, 2 \).

The states and their estimation response and the applied control efforts are shown in Fig. 3- Fig. 5 respectively. As one sees, the system states and their estimation are converged together and the compositions of the produce stream from the reactor (\( C_{1A}, C_{1B}, C_{1C}, C_{2A}, C_{2B}, \) and \( C_{2C} \)) are controlled and converged exponentially to their equilibrium points with reasonable control efforts.

### 5 Conclusion

This paper presented a decentralized observer-based robust model predictive control strategy for a class of distributed networked systems. The overall system is composed of a number of interconnected nonlinear subsystems with time-varying transmission delays. Time delays are appeared in distributed subsystems due to the information transmission in the communication network. A decentralized control strategy implemented for each subsystem with model predictive controllers. Each MPC problem was formulated with a memory-less observer-based feedback control law to minimize the upper bound of the infinite horizon cost that satisfies the sufficient stability conditions. The Novel delay dependent sufficient conditions were proposed for the existence of such controllers in the form of a linear matrix inequality (LMI) optimization problem. The simulation results for a chemical reactor plant were exploited to illustrate the applicability of the proposed method.

### References:


