Optimal Constrained Control Allocation for Underwater Robotic Vehicle - Comparison of Algorithms

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Abstract: - The paper describes methods of thrusts allocation in a propulsion system of an underwater robotic vehicle. The methods are directed towards minimization of energy expenditures necessary to obtain required control. A power/thrust relation is mapped by quadratic and linear functions. Such approach allows to compare efficiency of optimization-based control allocation strategies to finding constrained thrust allocations for generalized forces and moments. Selected results of computer simulations are inserted.

Key-Words: - Underwater robotic, control, thrust allocation, optimization problem

1 Introduction

Nowadays, it is common to use an underwater robotic vehicle (URV) to accomplish such missions as inspection of coastal and off-shore structures, cable maintenance, as well as hydrographical and biological surveys. In a military area it is employed in such tasks as surveillance, intelligence gathering, torpedo recovery and mine counter measures.

Its motion of six degrees of freedom describes the following vectors [1, 2]:

$$\begin{aligned}\boldsymbol{\eta} &= [x, y, z, \phi, \theta, \psi]^T \\ \mathbf{v} &= [u, v, w, p, q, r]^T \\ \boldsymbol{\tau} &= [X, Y, Z, K, M, N]^T\end{aligned} \tag{1}$$

where:

- $\boldsymbol{\eta}$ – vector of position and attitude in the inertial frame;
- x, y, z – coordinates of position;
- ϕ, θ, ψ – coordinates of attitude (Euler angles);
- $\mathbf{v}$ – vector of linear and angular velocities in the body-fixed frame;
- u, v, w – linear velocities in longitudinal, transversal and normal axes;
- p, q, r – angular velocities about longitudinal, transversal and normal axes;
- $\boldsymbol{\tau}$ – vector of generalized forces and moments acting on the robot in the body-fixed frame;
- X, Y, Z – forces in longitudinal, transversal and normal axes;
- K, M, N – moments about longitudinal, transversal and normal axes.

Nonlinear dynamic equations of motion can be written in form [2, 4, 8]:

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \tag{2}$$

where:

- $\mathbf{M}$ – inertia matrix (including added mass);
- $\mathbf{C}(\mathbf{v})$ – matrix of Coriolis and centripetal terms (including added mass);
- $\mathbf{D}(\mathbf{v})$ – hydrodynamic damping and lift matrix;
- $\mathbf{g}(\boldsymbol{\eta})$ – vector of gravitational forces and moments.

The contemporary URV is equipped with an automatic control system to execute complex manoeuvres without constant human intervention. Basic modules of the control system are depicted in Fig. 1. The autopilot computes demanded generalized forces and moments by comparing desired position and attitude of the robot with their current estimates. Thrusts, which should be developed by thrusters to produce desired forces and moments, are calculated in the thrust distribution module and transmitted as control inputs into the propulsion system.

The underwater robot has no other actuators except thrusters and both movement and positioning are realised only by change of thrusts.

2 Robot's propulsion system

The conventional URV operates in a crab-wise manner with small roll and pitch angles that can be neglected during normal operations. Therefore, its basic motion is movement in a horizontal plane with some variation due to diving, i.e. motion in four degrees of freedom. Thus, it is purposeful to regard its spatial motion as superposition of two displacements: motion in a vertical plane and motion in a horizontal plane. It allows to divide the robot's propulsion system into two independent subsystems responsible for movement in these planes, respectively. The most often applied configuration of thrusters in the propulsion system shows Fig. 2.

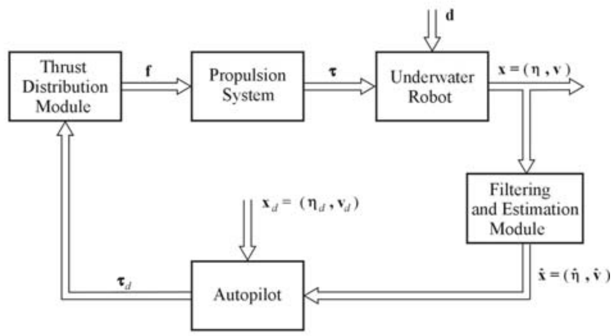

Fig. 1. A block diagram of the control system (**d** – environmental disturbances).

The subsystem realizing vertical motion, i.e. heave, consists of 1 or 2 thrusters generating thrusts in the vertical axis. The allocation control is realized in such a way that a thrust or sum of the thrusts of thrusters should be equal to the demanded force *Z*.

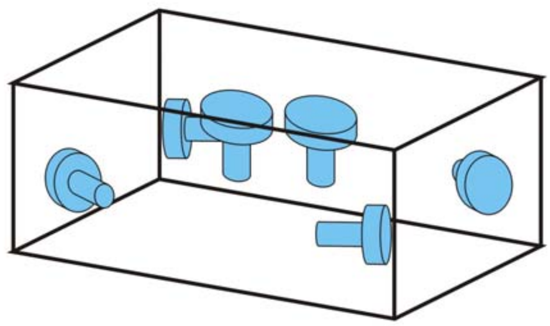

Fig. 2. Layout of thrusters in the propulsion system.

The subsystem assuring motion in the horizontal plane, i.e. surge, sway and yaw, is usually composed of four thrusters mounted askew in relation to main robot's symmetry axes (see Fig. 3). Forces *X* and *Y* acting in the longitudinal and transversal axes and the moment *M* acting about the vertical axis are a combination of thrusts produced by four thrusters of the subsystem. It is the overactuated control problem since a number of thrusters is greater than the number degrees of freedom. Hence, the control system should include a procedure of allocation control assuring that the produced generalized forces are equal to the demanded ones.

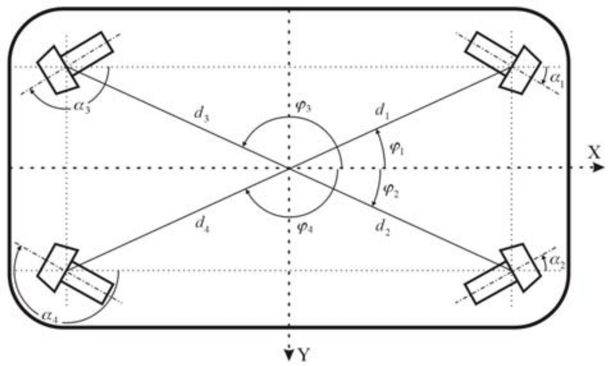

Fig. 3. A configuration of thrusters in subsystem responsible for horizontal motion (d_i – distance of i^{th} actuator from centre of gravity, α_i – angle between longitudinal axis and symmetry axis of i^{th} actuator, φ_i – angle between longitudinal axis and line connecting centre of gravity with centre of symmetry of i^{th} actuator).

A relationship between the desired propelling forces and moments $\boldsymbol{\tau}_d$ and corresponding them thrusts **f** produced by the propulsion system is a complicated function depending on a density of water, robot's velocity **v**, actuators diameters and revolutions, etc. A detailed analysis of thruster's dynamics can be found in [5, 6, 7].

In practical applications, the vector $\boldsymbol{\tau}_d$ acting on the vehicle in the horizontal plane is described as a function of the thrust vector **f** by the following expression [1, 3]:

$$\boldsymbol{\tau}_d = \mathbf{Tf} \tag{3}$$

where:

$\boldsymbol{\tau}_d = [\tau_{dX}, \tau_{dY}, \tau_{dN}]^T$,

τ_{dX} – force in the longitudinal axis,

τ_{dY} – force in the transversal axis,

τ_{dN} – moment about the vertical axis,

$\mathbf{f} = [f_1, f_2, f_3, f_4]^T$,

f_i – thrust of the i^{th} thruster,

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T – thruster configuration matrix:

$$\mathbf{T} = \begin{bmatrix} \cos\alpha_1 & \dots & \cos\alpha_4 \\ \sin\alpha_1 & \dots & \sin\alpha_4 \\ d_1 \sin(\alpha_1 - \varphi_1) & \dots & d_4 \sin(\alpha_4 - \varphi_4) \end{bmatrix},$$

The solution of the above problem is shown in [2] as:

$$\mathbf{f} = \mathbf{T}^* \boldsymbol{\tau}_d \tag{4}$$

where:

$$\mathbf{T}^* = \mathbf{T}^T \left(\mathbf{T}\mathbf{T}^T\right)^{-1} \tag{5}$$

The above approach is an effective method of finding the optimal allocation for the multi-thrusters propulsion system under assumption that the vector $\boldsymbol{\tau}_d$ is bounded, in such a way that the calculated vector **f** never exceed the lower and upper boundary vectors $\mathbf{f}_{\min} < 0$ and $\mathbf{f}_{\max} > 0$. Whether that this constraint is not taken into account it can give unsatisfied solution, i.e. compute values f_i that cannot be developed by thrusters. I such case, the desired vector $\boldsymbol{\tau}_d$ may not be produced due to work of one or more thrusters in saturation. It can contribute that a behavior of the robot may differ significantly from the required one.

Therefore, in order to overcome this difficulties, two constrained trust allocation methods are regarded.

3 Constrained control allocation

3.1 Control allocation as QP optimization problem

Literature study shows that due to quadratic mapping of the power/thrust relation a main method to solve the constrained control is to use the quadratic programming (QP) technique written as:

$$J = \min_{\mathbf{f}} \frac{1}{2} \mathbf{f}^T \mathbf{H} \mathbf{f} \tag{6}$$

subject to:

$$\begin{aligned} &\boldsymbol{\tau}_d - \mathbf{T}\mathbf{f} = \mathbf{0} \\ &\mathbf{f}_{\min} \le \mathbf{f} \le \mathbf{f}_{\max} \end{aligned} \tag{7}$$

where:

H – a diagonal weighting matrix,

$$\begin{aligned} \mathbf{f}_{\min} &= \left[f_1^{\min}, f_2^{\min}, f_3^{\min}, f_4^{\min}\right]^T \\ \mathbf{f}_{\max} &= \left[f_1^{\max}, f_2^{\max}, f_3^{\max}, f_4^{\max}\right]^T \end{aligned} \tag{8}$$

This approach ensures that the constraints $f_i^{\min} \le f_i \le f_i^{\max}$ are satisfied. A solution of the above optimization problem can be obtained by using any of the well-known QP algorithms or dedicated QP software. A basic disadvantage of the presented above method is its time-consuming due to computational complexity. As our experiences shown, its practical implementation is restricted by power of a board computer. Therefore, an alternative solution requiring a much less amount of computation is wanted.

3.2 Control allocation as LP optimization problem

Linear programing (LP) seems to be an attractive alternative to QP approach. However, linear approximation to the thrust allocation problem requires to take into account that criterion to be minimized has to be formulated as:

$$J = \min_{\mathbf{f}} \mathbf{c}^T |\mathbf{f}| \tag{9}$$

subject to:

$$\begin{aligned} &\boldsymbol{\tau}_d - \mathbf{T}\mathbf{f} = \mathbf{0} \\ &-\mathbf{f} \le -\mathbf{f}_{\min} \\ &\mathbf{f} \le \mathbf{f}_{\max} \end{aligned} \tag{10}$$

where **c** is positive definite vector.

Due to absolute values in the cost function (9) it cannot be solved by a linear optimization procedure. To make the problem compatible with a standard LP form it is reformulated as follows:

$$J = \min_{\mathbf{u}} \mathbf{c}^T \mathbf{u} \tag{11}$$

subject to:

$$\begin{aligned} &\boldsymbol{\tau}_d - \mathbf{T}\mathbf{f} = \mathbf{0} \\ &-\mathbf{f} \le \mathbf{u} \\ &\mathbf{f} \le \mathbf{u} \\ &-\mathbf{f} \le -\mathbf{f}_{\min} \\ &\mathbf{f} \le \mathbf{f}_{\max} \end{aligned} \tag{12}$$

A solution of the above optimization problem can be easy find by using the Simlex algorithm, which computational complexity with comparison to QP algorithms is greatly reduced, or LP software.

4 Simulation study

Computer experiments have been carried out to compare considered control allocation methods for the underwater robot shown in Figure 4. It is the open-frame submersible physically connected to a surface craft by an umbilical cable providing power and communications. Its propulsion system consists of six tunnel thrusters, two to motion in the vertical plane and four to motion in the horizontal one. The robot is also equipped with a manipulator arm to be able to perform some underwater tasks.

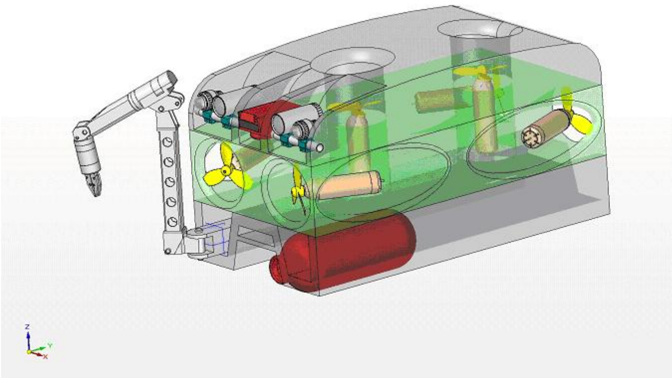

Fig. 4. A virtual view of the URV.

A displacement in the horizontal plane is realized by four thrusters assuring, developing thrust up to ±1000 N each. Its thruster configuration matrix, depending on location of the actuators, has the following form:

$$\mathbf{T} = \begin{bmatrix} 0.875 & 0.875 & -0.875 & -0.875 \\ 0.485 & -0.485 & 0.485 & -0.485 \\ 0.332 & -0.332 & -0.332 & 0.332 \end{bmatrix} \qquad (13)$$

Computer experiments have been made in MATLAB/Simulink environment using m-file function *quadprog.m* for QP problem and *linprog.m* for LP problem. Some results of simulations are depicted in Fig. 5 and Fig. 6.

The Figure 6 shows values of the thrust vector $\mathbf{f} = [f_1, f_2, f_3, f_4]^T$ computed by means of *quadprog.m* and *linprog.m* for given generalized forces $\boldsymbol{\tau}_d = [\tau_{dX}, \tau_{dY}, \tau_{dN}]^T$ depicted in Fig. 5. It can be noticed that in both cases the calculated values of the thrust vector **f** are very similar.

In order to compare a power consumption the following formulea was applied to both methods:

$$E = \sum_i \left(|f_{1i}| + |f_{2i}| + |f_{31i}| + |f_{4i}| \right) \qquad (14)$$

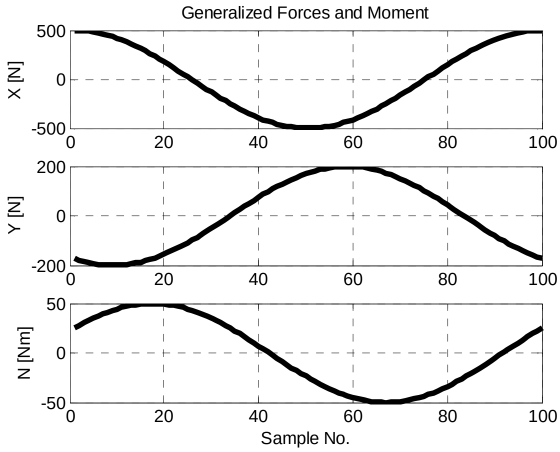

Fig. 5. Courses of given forces *X* and *Y* and moment *N*.

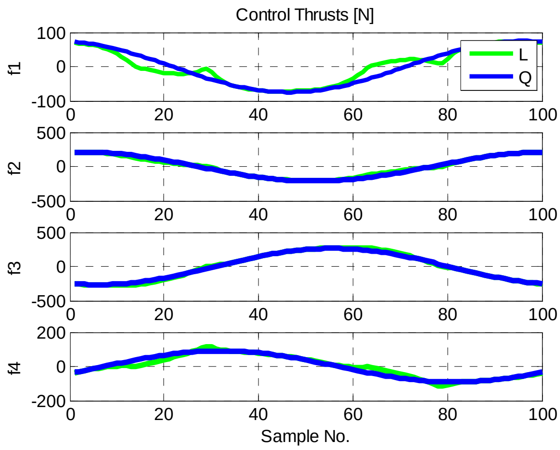

Fig. 6. Thrusts developed by thrusters (L – solution for LP problem, Q – solution for QP problem).

Analysis of obtained results showed that the power consumption *E* for LP problem is about 3÷5% less than for QP problem.

Conducted researches confirm that using LP algorithm can be attractive alternative method to solve the constrained thrust allocation problem, especially for small, but very popular currently, Low Cost Underwater Vehicle having limited computational possibilities.

5 Conclusions

The paper presents two methods of thrust allocation in the propulsion system of the underwater robotic vehicle.

The control allocation problem was regarded as the constrained optimization problem. The problem was solved using the quadrating programing and the linear programing. The optimization was directed towards minimization of energy expenditures necessary to obtain required control.

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The considered methods of power distribution are of a general character and can be successfully applied to different types of the underwater robotic vehicles.

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