The Use of MEMS Accelerometers for Control of a Small Unmanned
Underwater Vehicle

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Abstract: - In this paper the attempt to make an analysis of short duration Micro ROV distance measurement using low-cost MEMS accelerometers was presented. The main emphasis was placed on the gravity compensation and state estimation with Kalman filter method. The MEMS accelerometers error characteristics and the process model for measuring displacement of Micro ROV using accelerometers were presented. Finally, the examples of the verification results were performed.

Key-Words: - Underwater vehicles, Kalman filter, MEMS accelerometers, distance measurement

1 Introduction
Unmanned Underwater Vehicles (UUVs) are powerful tools for exploring and investigating underwater resources. Nowadays, the surveillance and inspection of underwater objects is carried out by trained operators, who control a Remotely Operated Vehicle (ROV) with cameras mounted over it. This is a tedious, time-consuming and expensive task. For this purpose, development of a vision system for measuring distance to an inspected object should be developed.

Depth perception is one of the most important tasks of a computer vision. The 3D shape reconstruction from a single image is an ill-posed problem. To solve this problem, it is possible to use at least two images captured from different viewpoints. If the location of an object on both images is known, it is feasible to perform triangulation technique to estimate the 3D position of the object. Calculating the distance of various points in a scene, relative to the position of a camera, allows the performance of complex tasks, such as depth measurements and environmental reconstruction [10].

The special visual features that artificial underwater objects possess (edges, texture, colour, brightness) allow distinguishing them from the rest of space presented in a natural scenario even in noisy images. For the purpose of distinguish these features, digital image processing is required. Next, to every feature, the distance using triangulation method is calculated. After that, the information to control system is send. It is expected, that latency of computer system or lack of distinguishable features may cause break in control system operation. The assumption that break time will not exceed few second allow us to suppose, that accelerometers may be usable for measure underwater vehicle distance travelled when vision system is unavailable.

2 Accelerometers
The accelerometers are intended for the positioning systems of a mobile robots or underwater vehicles. The covered distance is obtained by double integration of the sensor signal with time. Therefore, bias offset drift exhibited in the acceleration signal is accumulated and the accuracy of the distance measurement lower with time due to the integration. This problem may be fixed by periodic recalibration using external measurements on position, velocity and attitude. These external signals can be calculated by GPS, acoustic altimeter, pressure or Doppler sensors. For providing optimum estimate of the system error, Kalman filter may be implemented [1].

The accelerometers are broadly classified as mechanical, solid state or MEMS devices. A mechanical accelerometer consists of a mass suspended by springs. The displacement of the mass is measured using a displacement pick-off, giving a signal that is proportional to the force $F$ acting on the mass in the direction of the input axis. Newton’s second law $F = ma$ is then used to calculate the acceleration acting on the device.
Solid-state accelerometers can be broken into various sub-groups, including surface acoustic wave, vibratory, silicon and quartz devices. Solid state accelerometers are small, reliable and rugged.

Micro-machined silicon accelerometers use the same principles as mechanical and solid state sensors. There are two main classes of MEMS accelerometer. The first class consists of mechanical accelerometers manufactured using MEMS techniques. The second class consists of devices which measure the change in frequency of a vibrating element caused by a change of tension. MEMS accelerometers are small, light and have low power consumption and start-up times. Their main disadvantage is that they are not currently as accurate as accelerometers manufactured using traditional techniques, although the performance of MEMS devices is improving rapidly [2].

3 MEMS Accelerometer Error Characteristics

All types of accelerometers exhibit biases, scale factor, and cross-coupling errors, random noise to a certain extent, the nonlinearity and composite error.

3.1 Bias
The bias is a constant error exhibited by all accelerometers. It is independent of the underlying specific force and angular rate. It is convenient to split the accelerometer bias \( b_s \) into static \( b_{as} \) and dynamic \( b_{ad} \) components, where:

\[
b_s = b_{as} + b_{ad}
\]  
(1)

The static component contains the run-to-run variation bias and the residual fixed bias remaining after sensor calibration. It is constant throughout an operating period, but varies from run to run. The dynamic component varies over periods of order a minute and also incorporates the residual temperature-dependent bias. The dynamic bias is typically about 10 percent of the static bias [3].

3.2 Scale Factor and Cross-Coupling Errors
The scale factor error is dependent on the input-output gradient of the instrument from unity following unit conversion. The accelerometer output error due to the scale factor error is proportional to the true specific force along the sensitive axis and is denoted by vector:

\[
s_a = \left( s_{ax}, s_{ay}, s_{az} \right)
\]  
(2)

where: \( s_{a,\alpha} \) – scale factor error in \( \alpha \) axis.

Cross-coupling errors arise from the misalignment of the sensitive axes of the inertial sensors with respect to the orthogonal axes of the body frame. It is responsible for making each accelerometer sensitive to the specific force along the axes orthogonal to its sensitive axis.

Finally, the scale factor and cross-coupling errors for a nominally orthogonal accelerometer may be expressed by matrix:

\[
M_a = \begin{bmatrix}
    s_{a,x} & m_{a,xy} & m_{a,xz} \\
    m_{a,yx} & s_{a,y} & m_{a,yz} \\
    m_{a,zx} & m_{a,zy} & s_{a,z}
\end{bmatrix}
\]  
(3)

where: \( m_{a,\alpha\beta} \) – cross-coupling error between \( \alpha \) and \( \beta \) axes.

3.3 Random Noise
All accelerometers exhibit random noise from a number of sources. Electrical noise limits the resolution of sensors, particularly MEMS sensors, where the signal is very weak. It is widely assumed that the spectrum of accelerometer noise for frequencies below 1 Hz is approximately white, so the standard deviation of the average specific force and angular rate noise varies in inverse proportion to the square root of the averaging time. The accelerometer random noise is often described as random walk. The standard deviation of a random walk process is proportional to the square root of the integration time. [3].

3.4 Nonlinearity and Composite Error
The scale factor may not be exactly linear but may have second or higher order terms relating signal to input. To see these, it usual to run a test in which the acceleration rate is varied in steps, and the sensor output is measured. The data are fitted to a straight line by the least squares method, and the residuals are plotted. The standard deviation of residuals, called the standard error, can be used to specify sensor quality. The composite error is the ratio of the largest residual to the full scale range, and it combines errors due to hysteresis and resolution, among others [9].

4 Acceleration
Let the acceleration measurement of underwater vehicle be denoted by $f_{acc}$ representing specific force. The linear acceleration $a_{cc}$ is related to specific force as [5][6]:
\[
f_{acc} = a_{cc} + g^b
\]
where: $g^b$ – acceleration of gravity decomposed in the $b$-frame (body-fixed reference frame).

For denoted the $b$-frame linear acceleration of the moving body as $a^b$, the quantities related to the measurements may be described as:
\[
a^b = f_{acc} + b_{acc} + g^b + w
\]
where: $b_{acc}$ – accelerometer bias, $w$ – zero mean white noise.

Assuming that the NED reference frame is the inertial system implies:
\[
\dot{v}^n = a^n = R^b_n(\Theta) a^b
\]
where: $\Theta$ – vector of Euler angles $[\phi, \theta, \psi]$, $R^b_n$ – rotation matrix between the NED and body-fixed reference frames.

Substitution of (5) into (6), yields:
\[
\dot{v}^n \approx R^b_n(\Theta)(f_{acc} + b_{acc} + w) + g^n
\]
where: $g^n$ – acceleration of gravity in NED reference frame, $R^b_n$ – rotation matrix between the body-fixed and NED reference frames.

Rotation matrix between the body-fixed and NED reference frame can be expressed as:
\[
R^b_n(\Theta) = \begin{bmatrix}
\cos \psi & \cos \psi \cos \theta & -\sin \psi \\
\sin \psi & \cos \psi \cos \phi + \sin \psi \sin \theta \cos \phi & \sin \psi \sin \phi \\
-\sin \theta & -\cos \theta \sin \phi & \cos \theta \cos \phi \\
\end{bmatrix}
\]
where: $\phi$ – rotation about the $x$ axis (roll, heel), $\theta$ – rotation about the $y$ axis (pitch, trim), $\psi$ – rotation about the $z$ axis (yaw).

The accelerometer force measurements can be transformed to roll and pitch angles. It can be shown that three orthogonal accelerometers at rest produce:
\[
\begin{bmatrix}
f_x \\
f_y \\
f_z
\end{bmatrix} = R^b_n(\Theta) \begin{bmatrix}
0 \\
0 \\
g
\end{bmatrix} = \begin{bmatrix}
-g \sin \theta \\
g \cos \theta \cos \phi \\
g \cos \theta \sin \phi
\end{bmatrix}
\]

where: $f_\alpha$ – acceleration measurement in axis $\alpha$.

Taking the ratios:
\[
\frac{f_y}{f_z} = \tan \phi, \quad \frac{f_x}{f_z} = \tan \theta \cos \phi, \quad \cos \phi \neq 0
\]

Implies that:
\[
\phi = \arctg \left(\frac{f_y}{f_z}\right)
\]
\[
\theta = \arctg \left(-\cos \phi \frac{f_x}{f_z}\right)
\]

5 Discrete-Time Systems
In order to perform calculation, the analog data has to be converted to digital form. It means, that discrete-time process arise from sampling a continuous process at discrete times.

Consider the following continuous process:
\[
\dot{x} = Fx + Gu
\]

For samples of this process at discrete times, state-space method may be used to obtain the difference equation relating the samples of $x$. Specifically, the solution may be written as [7]:
\[
x(t_{k+1}) = \Phi_k x(t_k) + \int_{t_k}^{t_{k+1}} \Phi(t, \tau) G(\tau) u(\tau) d\tau + w_k
\]

Or, in abbreviated notation,
\[
x_{k+1} = \Phi_k x_k + w_k
\]
where:
\[
x_k \quad \text{vector state of the process at time } t_k,
\Phi_k \quad \text{state transition matrix},
\quad w_k \quad \text{vector whose elements are white sequence with known covariance structure}.

The covariance matrix associated with $w_k$ can be determined by equation:
\[
Q_k = E [w_k w^*_k] = E \left[ \int_{t_k}^{t_{k+1}} \Phi(t, \xi) G(\xi) u(\xi) d\xi \right] =
\end{equation}
\[
= \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} \Phi(t, \xi) G(\xi) u(\xi) u^*(\eta) d\xi d\eta
\]

ISBN: 978-1-61804-260-6
and the transition matrix is easily determined as:

\[
\Phi_k = \left[ L^{-1} \left( (sI - F)^{-1} \right) \right]_{s=\Delta t}
\]

(8)

For the purpose of estimating underwater vehicle acceleration as well as position and velocity, the transition and covariance matrices for a step size of \(\Delta t\) and power spectral density of white noise \(W\) are given by [7]

\[
\Phi_k = \begin{bmatrix} 1 & \Delta t & \Delta t^2 / 2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}
\]

(9)

\[
Q_k = \begin{bmatrix} W\Delta t^5 & W\Delta t^4 & W\Delta t^3 \\ 8W & 6W & 2W \Delta t^2 \\ 2W & 6W & W\Delta t \end{bmatrix}
\]

(10)

6 Kalman filter

The Kalman filter is an efficient recursive filter enable to estimate the state of a linear or nonlinear systems from a series of noisy measurements. It is widely used in sensors since it can remove white and colored noise from the state estimates. In case of temporarily loss of measurements the filter behaves such as predictor. As soon as new measurements are available, the predictor is corrected and updated online to give the minimum variance estimate. This feature is particularly useful for inertial measurement units [6,8].

Assuming the random process to be estimated can be modeled in the form presented in Eq. 15, the observation (measurement) of the process is assumed to occur at discrete points in time in accordance with the linear relationship

\[
z_k = H_k x_k + w_k
\]

(11)

where:

- \(z_k\) – vector measurement at time \(t_k\),
- \(H_k\) – matrix giving the ideal connection between the measurement and the state vector at time \(t_k\),
- \(w_k\) – vector whose elements are white sequence

with known covariance structure.

The discrete-time Kalman filter algorithm is summarized in equations presented below.

Initialize

\[
\hat{x}_0 = x_0
\]

(12)

Gain

\[
K_k = P_k^{-1} H_k^T (H_k P_k^{-1} H_k^T + R_k)^{-1}
\]

(13)

Update

\[
\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-)
\]

(14)

Propagation

\[
P_{k+1} = \Phi_k P_k \Phi_k^T + Q_k
\]

(15)

where:

- \(K_k\) – Kalman gain at time \(t_k\),
- \(P_k\) – error covariance matrix at time \(t_k\),
- \(\hat{x}_k\) – state vector estimation at time \(t_k\),
- \("(\cdot)\)\) – indicates the a priori values of the variables (before the information in the measurement is used).

First, initial conditions for the state and error covariance are given. If a measurement is given at the initial time, the state and covariance are updated. Then, the state estimate and covariance are propagated to the next time step. If a measurement isn’t given at the initial time, then the estimate and covariance are propagated first to the next available measurement. The process is then repeated sequentially until all measurement times have been used in the filter [4].

6 Experiments

In order to test above approach, the following experiment was conducted. For this purpose the Video Ray Pro 4 micro ROV was used. It has built in the ADXL330 accelerometer and a 3-axes compass. The compass is used to determine the Micro ROV rotation about the \(z\) axis. The ADXL330 is a complete 3-axes acceleration measurement system. It contains a polysilicon surface micromachined sensor and signal conditioning circuitry to implement an open-loop acceleration measurement
architecture. The output signals are analog voltages that are proportional to acceleration.

The series of experiments were performed. The Video Ray Micro ROV was moving for a known distances while travel time was measured. The example of one acceleration measurement is shown in Fig. 1 and Fig. 2. In this case the vehicle was moving backward for distance 6 meters in 7.8 sec. The accelerometer measurement after gravity compensation and bias reduction is shown in Fig. 1. Fig. 2 presents acceleration measurement with additional Kalman filter estimation.

On Fig. 3 to Fig. 6 results of calculation velocity and displacement for above example were presented. It may be noticed that impact of dynamic bias of accelerometer is compensated with Kalman filter.
For every measurement the static bias was calculated as a mean of signal noise a short time before experiment. It was performed because the bias changes significantly in time. The example static bias drift in $x$-axis for the motionless vehicle is shown in Fig. 7.

Fig. 7. Static bias drift in $x$-axis

For every conducted experiment the error of distance measurement does not exceed 5% for time shorter than 10 sec.

There was also noticed strong impact of the gravity compensation for displacement calculation. Comparison of acceleration measurement with and without gravity compensation was shown in Fig. 8.

Fig. 8. Impact of gravity compensation on acceleration measurement

4 Conclusion

A procedure for short duration Micro ROV distance measurement using MEMS accelerometer was presented in this article. The experiments shown that the distance difference between hand-made and computed from acceleration measurement was less than 5% for time shorter than 10 sec. It appears promising usage of the presented approach in Micro ROV control applications for measure underwater vehicle covered distance when vision or other control system is unavailable for short time. Further work will concentrate on identifying and compensating accelerometer bias for real-time measurement.

References:


