Abstract: Game theory is a set of tools developed to model interactions between agents with conflicting interests [5]. It is a field of applied mathematics that defines and evaluates interactive decision situations. It provides analytical tools to predict the outcome of complicated interactions between rational entities, where rationality demands strict adherence to a strategy based on observed or measured results [13]. Originally developed to model problems in the field of economics, game theory has recently been applied to network problems, in most cases to solve the resource allocation problems in a competitive environment. The reason that game theory is an adapted choice for studying cooperative communications is various. Nodes in the network are independent agents, making decisions only for their own interests. Game theory provides us sufficient theoretical tools to analyze the network users’ behaviors and actions. Game theory, also primarily deals with distributed optimization, which often requires local information only. Thus it enables us to design distributed algorithms [14]. This article surveys the literature on game theory as they apply to wireless networks. First, a brief overview of classifications of games, important definitions used in games (Nash Equilibrium, Pareto efficiency, Pure, Mixed and Fully mixed strategies) and game models are presented. Then, we identified five areas of application of game theory in wireless networks; therefore, we discuss related work to game theory in communication networks, cognitive radio networks, wireless sensor networks, resource allocation and power control. Finally, we discuss the limitations of the application of game theory in wireless networks.

Keywords: Game theory; Nash equilibrium; Pareto efficiency; Wireless Networks; Power control; Cognitive radio network.

1. An overview of game theory

1.1 Introduction: brief game theory history

The ideas underlying game theory have emerged throughout history, apparent in the bible, the Talmud, the works of Descartes and Sun Tzu, and the writings of Charles Darwin. The basis of modern game theory, however, can be considered an outgrowth of a three seminal works [2].

Augustin Cournot's Researches into the Mathematical Principles of the Theory of Wealth in 1838, gives an intuitive explanation of what would eventually be formalized as the Nash equilibrium, as well as provides an evolutionary, or dynamic notion of best-responding to the actions of others. Francis Ysidro Edgeworth's Mathematical Psychics demonstrated the notion of competitive equilibria in a two-person (as well as two-type) economy.

Finally, Emile Borel, in “Algebre et calcul des probabilites, Comptes Rendus Academie des Sciences”, Vol. 184, 1927, provided the first insight into mixed strategies, or probability distributions over one's actions that may lead to stable play.

While many other contributors hold a place in the history of game theory, it is widely accepted that modern analysis began with John von Neumann and Oskar Morgenstern's book, Theory of Games and Economic Behavior and was given its modern methodological framework by John Nash building on von Neumann and Morgenstern's results.

1.2 Classification of game theory

There are diver’s game theory models which can be categorized on the basis of factors like the number of players involved, the sum of gains or losses, and the number of strategies employed in the game. The terminology used in game theory is inconsistent, thus different terms can be used for the same concept in different sources [3].

1.2.1 Non-cooperative and cooperative (coalition) games

In game theory, a cooperative game is a structure in which the players have the option of planning as a group in advance of choosing their actions. Unlike a cooperative game, a non-cooperative game is a game structure in which the players do not have the option of planning as a group in advance of choosing their actions. [1]

In non-cooperative game theory there are two alternative ways in which a game can be represented. The first type is called a normal form
game or strategic form game. The second type is called an extensive form game. A normal form game is any game where we can identify the following three things:

- The players: in a game are the individuals who make the relevant decisions.
- The strategies available to each player: is a complete description of how a player could play a game.
- The payoffs: is what a player will receive at the end of the game contingent upon the actions of all the players in the game.

In extensive form games greater attention is placed on the timing of the decisions to be made, as well as on the amount of information available to each player when a decision has to be made. This type of game is represented not with a matrix but with a decision, or game, tree.

1.2.2 Sequential / Simultaneous game

A sequential game is a game where one player adopts his action before the others adopt theirs. Importantly, the later players must have some information of the first's choice; otherwise the difference in time would have no strategic effect. Sequential games hence are governed by the time axis, and represented in the form of decision trees. An example is chess. When I am making my move I know what your last move was and can use that information to determine my own strategy.

Unlike sequential games, simultaneous games do not have a time axis as players adopt their actions without knowledge of the actions chosen by other players, and are usually represented in the form of payoff matrices.

An example is the Prisoner’s Dilemma, where the two players have to decide their strategy without knowing what the other player has chosen. Even though the police might not interview each of the prisoners at exactly the same time they are still making their decision without knowing what the other player has chosen.

The difference between a simultaneous and a sequential game is clear. As a sequential game the second player has the advantage, as a simultaneous game it is fair.

1.2.3 Games with perfect and imperfect information

Perfect information is a situation in which an agent has all the relevant information with which to make a decision. A game is defined as a game of perfect information if perfect information is available for all moves. Chess is an example of a game with perfect information as each player can see all of the pieces on the board at all times. If a player has no information about other players’ actions when it is her/his turn to decide, this game is called imperfect information game. Games with perfect information represent a small subset of games. Card games where each player's cards are hidden from other players are examples of games of imperfect information.

As it is hardly ever any user of a network knows the exact actions of the other users in the network, the imperfect information game is a very good framework in telecommunications systems. Nevertheless, assuming a perfect information game in such scenarios is more suitable to deal with [3].

1.2.4 Games with complete and incomplete information

In a game of complete information all players are perfectly informed of all other players’ payoffs for all possible action profiles. Examples will be the game of Prisoner's Dilemma, in this game, the players know about each others utility function/payoffs. If instead a player is uncertain of the payoffs to other players the game is one of incomplete information.

In an incomplete information setting players may not possess full information about their opponents. In particular players may possess private information that the others should take into account when forming expectations about how a player would behave. Examples would be situations such as buying auto insurance, playing blind poker etc.

In a game of imperfect information, players are simply unaware of the actions chosen by other players. However they know who the other players are, what their possible strategies/actions are, and the preferences/payoffs of these other players. Hence, information about the other players in imperfect information is complete. In incomplete information games, players may or may not know some information about the other players, e.g. their “type”, their strategies, payoffs or their preferences.

1.2.5 Zero-sum games

A zero–sum game is a mathematical representation of a situation in which a participant's gain (or loss) of utility is exactly balanced by the losses (or gains) of the utility of the other participant(s). If the total gains of the participants are added up, and the total losses are subtracted, they will sum to zero.

In contrast, non-zero–sum defines a situation in which the interacting parties' aggregate gains and losses are either less than or more than zero. A zero–sum game is also called a strictly competitive game while non-zero–sum games can be either competitive or non-competitive.

In telecommunications systems, it is quite hard to describe a scenario as a zero-sum game. However, in a bandwidth usage scenario of a single
link, the game may be defined as a zero-sum game [3].

1.2.6 Rationality in games

The most fundamental assumption in game theory is rationality [15]. It implies that every player is motivated by maximizing his own payoff. In a stricter sense, it implies that every player always maximizes his/her utility, thus being able to perfectly calculate the probabilistic result of every action. John V. Neumann and Morgenstern justified the idea of maximizing the expected payoff in their work in 1944 [4].

1.2.7 Evolutionary games

Evolutionary game theory started its development slightly after other games have been developed [18]. This type of game was originated by John Maynard Smith formalization of evolutionary stable strategies as an application of the mathematical theory of games in the context of biology in 1973 [19]. Evolutionary game theory defines game models in which players adopt their strategies through a trial-and-error process in which they learn over time that some strategies work better than others.

1.3 Nash Equilibrium

We suppose the players player 1, player 2, ..., player n.

For each i, let Si = {all possible strategies for player i}, si will always refer to a strategy in Si.

A strategy profile is an n-tuple S = (s1, ..., sn), one strategy for each player.

Utility Ui(S) = payoff for agent i if the strategy profile is S.

If S = (s1, ..., sn) is a strategy profile, then S-i = (s1, ...,si-1, si+1,..., sn), S-i is strategy profile S without agent i’s strategy.

If s’i is any strategy for agent i, then (s’i, S-i ) = (s1, ...,si-1, s’i, si+1,..., sn). Hence (si, S-i) = S.

If s’i is any strategy for agent i, then (s’i, S-i ) = (s1, ...,si-1, s’i, si+1,..., sn). Hence (si, S-i) = S.

A strategy profile s = (s1, ..., sn) is a Nash equilibrium if for every i, si is a best response to S-i if: Ui (si , S-i) ≥ Ui (s’i, S-i) for every strategy s’i available to agent i.

A strategy profile s = (s1, ..., sn) is a Nash equilibrium if for every i, si is a best response to S-i (no player can do better by unilaterally changing his/her strategy).

Nash [36] has expected that every game with a finite number of players and action profiles has at least one Nash equilibrium.

Finding the Nash equilibrium for any game requires two stages. First, we identify each player’s optimal strategy in response to what the other players might do. This involves working through each player in turn and determining their optimal strategies. This is done for every combination of strategies by the other players. Second, a Nash equilibrium is identified when all players are playing their optimal strategies simultaneously [35].

1.4 Pareto efficiency

Pareto efficiency is another important concept of game theory. This term is named after Vilfredo Pareto, an Italian economist, who used this concept in his studies and defined it as: “A situation is said to be Pareto efficient if there is no way to rearrange things to make at least one person better off without making anyone worse off” [37].

Strategy profile S Pareto dominates a strategy profile S’ if no player gets a worse payoff with S than with S’, i.e., Ui (S) ≥ Ui (S’) for all i and at least one player gets a better payoff with S than with S’, i.e., Ui (S) > Ui (S’) for at least one i.

Strategy profile S is Pareto efficient, if there’s no strategy S’ that Pareto dominates S.

1.5 Pure, Mixed and Fully mixed strategies

In any game someone will find pure and mixed strategies, a pure strategy has a probability of one, and will be always played. On the other hand, a mixed strategy has multiple pure strategies with probabilities connected to them [3].

Pure strategy selects a single action and plays it, each row or column of a payoff matrix represents both an action and a pure strategy.

Mixed strategies randomize over the set of available actions according to some probability distribution.

Let Ai = {all possible actions for player i}, and ai be any action in Ai.

Si (aj) = probability that action aj will be played under mixed strategy si.

The support of si is support (si) = {actions in Ai that have probability > 0 under si}.

A pure strategy is a special case of a mixed strategy; the support consists of a single action.

In fully mixed strategy, every action has probability > 0. i.e., support (si) = Ai.

In a game without a pure strategy Nash Equilibrium, a mixed strategy may result in a Nash Equilibrium.

1.6 Game Models

Game models are developed to be used for different types of applications. They also aim at finding the equilibrium states and deciding whether these states are acceptable for the application and the finding optimization parameters that force the system to reach the desired equilibrium states [34].

Every player (i) has its own action space (Ai) which is the set of actions which includes all possible actions that player can adopt. The total action space (A) is calculated by multiplying all action sets. A=A1×A2×A3×……×An.
U (Utility Set) is a set consists of utility or payoff functions for all players. 
U = {U1, U2, U3, … … , Un}.

In normal games, players adopt their actions in a way that will improve their personal benefit or payoff.

1.6.1. Repeated Game Model

Repeated game model is a multistage game where users move from stage to another in a repeated manner and each stage is the same as the normal game model. The order of players in the game is defined by a function called player function.

1.6.2. Potential Game Model

A game can be said to be potential when there is a function \( V: \{A\} \), and any independent variation in \( V (\Delta V) \) is seen by the corresponding independent player as \( (\Delta Ui) \). And if \( \Delta V = \Delta Ui \) the game is called exact potential game.

Mathematically a game has been proven as a potential game if it satisfies the following equation [28].

\[
\frac{\partial^2 u_i}{\partial a_i \partial a_j} = \frac{\partial^2 u_j}{\partial a_j \partial a_i}.
\]

It was also proved that any potential game has at least one steady state, and all states that maximize \( V \) are Nash equilibrium states [28].

1.6.3. Super Modular Game Model

When the action space of a game forms a lattice and the utility function is super-modular, the game is called super modular game.

A lattice \( (X) \) is defined as a partially order set for all \( a, b \in X \), and \( a \wedge b \in X \), and \( a \vee b \in X \) where \( a \wedge b = \inf \{a, b\} \) and \( a \vee b = \sup \{a, b\} \).

A super-modular function (f) is defined as \( f: X, \) where \( X \) is lattice, and for all \( a, b \in X \)

\[ f(a) + f(b) \leq f(a \vee b) + f(a \wedge b). \]

Mathematically, it was proven that a super-modular game should satisfy the following equation.

\[
\frac{\partial^2 u_i}{\partial a_i \partial a_j} \geq 0, \forall i \neq j \in N.
\]

It was also proven that all super-modular games have at least one Nash equilibrium state, and the Nash equilibrium states form a lattice.

2. Game theory in wireless network

A game is a set of there fundamental components: a set of players, a set of strategies, and a set of payoffs. Players or nodes are the decision takers in the game. The strategies are the different choices available to nodes. Finally, a utility function (payoffs) decides the all possible outcomes for each player. Table 1 shows typical components of a wireless networking game.

<table>
<thead>
<tr>
<th>Components of a game</th>
<th>Elements of a wireless network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Players</td>
<td>Nodes in the wireless network</td>
</tr>
<tr>
<td>A set of strategies</td>
<td>A modulation scheme, Coding rate, transmit power level, etc.</td>
</tr>
<tr>
<td>A set of payoffs</td>
<td>Performance metrics (e.g. Throughput, Delay, SNR, etc.)</td>
</tr>
</tbody>
</table>

Table 1: Components of a wireless networking game

Game theory has emerged in divers recent works related to communication networks, cognitive radio networks, wireless sensor networks, resource allocation and power control.

2.1. Game theory and power control

MacKenzie and al. [6] have presented applications of game theory to problems in random access and power control. In the case of random access, the authors examine the behavior of selfish users in a simplified Aloha system; surprisingly, rational selfish users do not implement the “always transmit” strategy that one might expect. In the case of power control, they show that game theoretic techniques can yield an optimal operating point without the intervention of an external controller.

Lin and al. [23] have addressed the crucial issue of how to design efficient MAC protocols in autonomous wireless networks with selfish users. They model the wireless medium access control problem as a non-cooperative two-person non-zero-sum game in which the MAC protocol can be regarded as distributed strategy update scheme approaching the equilibrium point.

Niyato and al. [17] have proposed an adaptive bandwidth allocation and admission control mechanism based on game theory for IEEE 802.16 broadband wireless networks. A non-cooperative two-person non-zero-sum game is formulated where the base station and a new connection are the players of this game. The solution of the game formulation provides not only the decision on accepting or rejecting a connection, but also the amount of bandwidth allocated to a new connection. A queueing model considering adaptive modulation and coding in the physical layer is used to analyze quality of service (QoS)
performances, namely, the delay performance for real-time and the throughput performance for non-real-time polling services and best effort service.

2.2. Game theory and resource allocation

In Bacci and al. [20], the authors focused their study on the particular issue of allocating power resources to optimize the receiver performance in terms of spreading code acquisition. The problem of initial signal acquisition is formulated as a non-cooperative game in which each transmitter-receiver pair in the network seeks to maximize a specifically chosen utility function.

Emmanouil and al. [21] have proposed a novel way of maximization of the network throughput and the provision of fairness which are key challenges in IEEE WLANs, using game theory. The authors examine two types of power control games, namely the non-cooperative and the cooperative power control game. In the case of non-cooperative power control game they find the Nash equilibrium in a distributed way. In the case of cooperative power control game they expect that there exists a central entity called coordinator which announces the calculated Nash bargaining solution to the access points.

Srivastava and al. [13] have described how various interactions in wireless ad hoc networks can be modeled as a game. This allows the analysis of existing protocols and resource management schemes, as well as the design of equilibrium-inducing mechanisms that provide incentives for individual users to behave in socially-constructive ways.

Zhou and al. [10] have developed a novel approach to encourage efficient behavior in solving the interaction between InPs (Infrastructure Providers) and SPs (Service Providers) by introducing economic incentives, in the form of game theory. Based on the non-cooperative game model, a bandwidth allocation scheme in the network virtualization environment is established, using the concept of the Nash Equilibrium. Then, the authors propose an iterative algorithm to find the Nash Equilibrium and solve the bandwidth allocation problem.

2.3. Game theory and wireless sensor networks

Shen and al. [12] have presented a survey of security approaches based on game theory in wireless sensor networks (WSNs). According to different applications, a taxonomy is proposed in the paper, which divides current existing typical game-theoretic approaches for WSNs security into four categories: preventing Denial of Services (DoS) attacks, intrusion detection, strengthening security, and coexistence with malicious sensor nodes. The main ideas of each approach are overviewed while advantages and disadvantages of various approaches are discussed. Then, the authors overview related work and highlights the difference from other surveys, and points out some future research areas for ensuring WSNs security based on game theory, including Base Station (BS) credibility, Intrusion Detection System (IDS) efficiency, WSNs mobility, WSNs Quality of Service (QoS), real-world applicability, energy consumption and sensor nodes learning.

Guan and al. [25] have introduced a novel routing algorithm to solve the obstacle problem in wireless sensor networks based on a game-theory model. Their algorithm forms a concave region that cannot forward packets to achieve the aim of improving the transmission success rate and decreasing packet transmission delays. Zheng [24] has also proposed a reliable routing model against selfish nodes in wireless sensor networks. Game theory is used in his model to find the balance between the reliability and resource limitation.

2.4. Game theory and cognitive radio networks

Almost all optimization problems in cognitive radio can be mapped into games. The following table shows the mapping cognitive applications into game models [34].

<table>
<thead>
<tr>
<th>Application</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Spectrum Allocation</td>
<td>Exact potential game</td>
</tr>
<tr>
<td>Distributed Power Control</td>
<td>Super-modular game</td>
</tr>
<tr>
<td>OFDM Channel Filling</td>
<td>Exact potential game</td>
</tr>
</tbody>
</table>

Table 2: Mapping cognitive applications into game models

Neel and al. [22] have defined how the components of the cognition cycle map into normal form game model and describe standard game theory techniques for investigating four important issues that game theory should address: steady state existence, steady state identification, convergence and steady state optimality. The authors defined also, three game models that can aid the analyst in addressing these issues and conclude with a discussion of additional ways in which the use of game models aids the analysis and development of cognitive and adaptive radios.

Nie and al. [27] have proposed a game theoretic framework to analyze the behavior of cognitive radios for distributed adaptive channel allocation. The authors have defined two different objective functions for the spectrum sharing games, which
capture the utility of selfish users and cooperative users, respectively. Based on the utility definition for cooperative users, the authors show that the channel allocation problem can be formulated as a potential game, and thus converges to a deterministic channel allocation Nash equilibrium point. Neel and al. [29] have addressed how the insertion of cognitive radio technology into a network will impact performance and demonstrates how techniques from game theory can be used to analyze the network as a first step of shaping the decisions of the radios to achieve optimal network performance. Scutari and al. [33] have proposed and analyze a totally decentralized approach, based on game theory, to design cognitive MIMO transceivers, which compete with each other to maximize their information rate. The formulation incorporates constraints on the transmit power as well as null and/or soft shaping constraints on the transmit covariance matrix, so that the interference generated by secondary users be confined within the temperature-interference limit required by the primary users. The authors provide a unified set of conditions that guarantee the uniqueness and global asymptotic stability of the Nash equilibrium of all the proposed games through totally distributed and asynchronous algorithms. Bloem and al. [30] have suggested a game theoretical approach that allows master-slave cognitive radio pairs to update their transmission powers and frequencies simultaneously. This is shown to lead to an exact potential game, for which it is known that a particular update scheme converges to a Nash Equilibrium (NE). A Stackelberg game model is also presented for frequency bands where a licensed user has priority over opportunistic cognitive radios. Xia and al. [31] have studied the power control of the transmitter in basic cognitive cycle. And game theory is applied for modeling. Non-cooperative power control game which is created by D. Goodman is used; however, they authors introduce a new sigmoid efficiency function only related to user's SIR. Ji and al. [32] have provided a game theoretical overview of dynamic spectrum sharing from diver’s aspects: analysis of network users' behaviors, efficient dynamic distributed design, and optimality analysis.

2.5. Game theory and communication networks

Saad and al. [7] have provided a comprehensive overview of coalitional game theory, and its usage in wireless and communication networks. For this purpose, they introduced a novel classification of coalitional games by grouping the sparse literature into three distinct classes of games: canonical coalitional games, coalition formation games, and coalitional graph games. For each class, they explained in details the fundamental properties, discussed the main solution concepts, and provided an in-depth analysis of the methodologies and approaches for using these games in both game theory and communication applications.

Xiao and al. [8] have proposed a game model to interpret the IEEE 802.11 distributed coordination function mechanism. In addition, by designing a simple Nash equilibrium backoff strategy, the authors have presented a fairness game model. Charilas and al. [9] have presented a collects applications of game theory in wireless networking and presents them in a layered perspective, emphasizing on which fields game theory could be effectively applied. Several games are modeled in this paper and their key features are exposed. Khan and al. [11] have presented the user-centric network selection decision mechanism, where negotiation between users and network operators is carried out using game-theoretic approach. They model the utility functions of users and network operators in terms of offered prices and service quality. The proposed approach builds on IEEE 802.21 standard. Session Initiation Protocol (SIP) and Mobile Internet Protocol (MIPv6).

Sundararaj and al. [26] have explored the theoretical approach to improve existing delay and disruption tolerant networking routing algorithms using game theory.

3. Conclusion

A wireless network is identified by a distributed, dynamic, self-organizing architecture. Each node in the network is capable of independently adapting its operation based on the current environment according to predetermined algorithms and protocols. Analytical models to estimate the performance of wireless networks have been scarce due to the distributed and dynamic nature of such networks [13].

This paper gives a detailed insight in the game theory definition, classifications, game models and applications of games in wireless networks. We have presented recent works related to game theory in communication networks, cognitive radio networks, wireless sensor networks, resource allocation and power control. Game theory offers a suite of tools that may be used effectively in modeling the interaction between independent nodes in wireless network. Because of these numerous benefits, adopting analytic approach that emphasizes the use of game theory over wireless networks is preferable for analyzing the users’ needs in such networks. However, game theory focuses on solving the Nash equilibrium and analyzing its properties and not to consider how players should interact to reach this equilibrium.
On contrary, multi agent systems seem to be a way to overcome this problem [16].

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