Design and Implementation of PI and PIFL Controllers for Continuous Stirred Tank Reactor System

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Abstract: Continuous stirred tank reactor system (CSTR) is a typical chemical reactor system with complex nonlinear characteristics where an efficient control of the product concentration in CSTR can be achieved only through accurate model. The mathematical model of the system was derived. Then, the linear model was derived from the nonlinear model. A conventional PI controller and PI fuzzy logic controller for continuous stirred tank reactor are proposed to control the concentration of the linear CSTR. The simulation study has been carried out in MATLAB SIMULINK workspace. The best controller has been chosen by comparing the criteria of the response such as settling time, rise time, percentage of overshoot and steady state error. From the simulation result the PIFL controller has a better performance than conventional PI controller.

Key-Words: Dynamic modeling, PI and PIFL controllers, Stirred tank system, Matlab and Simulink

1 Introduction

The best way to learn about control systems is to design a controller, apply it to the system and then observe the system in operation. One example of systems that use control theory is Continuous stirred tank reactor system (CSTR). It can usually be found in most university process control labs used to explain and teach control system engineering. It is generally linked to real control problems such as chemical factories, preparing of the antidotes in medicine and food processing too. It is widely used because it is very simple to understand; yet the control techniques that can be studied cover many important classical and modern design methods. Continuous stirred tank reactor system (CSTR) is a typical chemical reactor system with complex nonlinear characteristics. The system consists of two tanks as illustrated in Figure 1. The concentration of the outlet flow of two chemical reactors will be forced to have a specified response. It is assumed that the overflow tanks are well-mixed isothermal reactors, and the density is the same in both tanks. Due to the assumptions for the overflow tanks, the volumes in the two tanks can be taken to be constant, and all flows are constant and equal. It is assume that the inlet flow is constant. It is desired to control the second tank concentration based on the concentration in the first tank. One of the popular controllers both in the realm of the academic and industrial application is the PI and PIFL controllers. They have been applied in feedback loop mechanism and extensively used in industrial process control. Easy implementation of both controllers, made it in system control applications. It tries to correct the error between the measured outputs and desired outputs of the process in order to improve the transient and steady state responses as much as possible. In one hand, PI controller appear to have an acceptable performance in some systems, but sometimes there are functional changes in system parameters that need an adaptive based method to achieve more accurate response. Several researches are available that combined the adaptive approaches on PI controller to increase its performance with respect to the system variations. Limitations of traditional approaches in dealing with constraints are the main reasons for emerging the powerful and flexible methods. In this paper a PI and PIFL controllers for unstable continuous stirred tank reactor are proposed to control the concentration of the linear CSTR. The performance of the PIFL controller will be compared with a conventional PI controller using Matlab Simulink. Finally, the task of this paper is to design and select the best controller for the system that can control the concentration of the CSTR.
2 Modeling of the continuous stirred tank reactor system

The concentration of the outlet flow of two chemical reactors will be forced to have a specified response in this section. Figure 1 shows the simple concentration process control. It is assumed that the overflow tanks are well-mixed isothermal reactors, and the density is the same in both tanks. Due to the assumptions for the overflow tanks, the volumes in the two tanks can be taken to be constant, and all flows are constant and equal. It is assumed that the inlet flow is constant. Figure 2 shows the block diagram of two tanks of chemical reactor.

\[ V_1 \frac{dC_{A1}}{dt} = FC_{A0} - FC_{A1} - V_1 KC_{A1} \] (1)

Where \( V_1 \) is the volume of the first tank, \( F \) is the flow, \( C_{A0} \) is the inlet concentration of the first tank, \( C_{A1} \) is the outlet concentration of the first tank and \( C_{A2} \) is the inlet concentration of the second tank and \( K \) is the reaction rate. Equation 1 can be rearranged to be

\[ \frac{dC_{A1}}{dt} + \frac{1}{\tau_1} C_{A1} = \frac{F}{V_1} C_{A0} \] (2)

Where \( \tau_1 = \frac{V_1}{F + KV_1} \) is the time constant of the first tank.

By taking Laplace transform and rearranging equation 2, the transfer function of the first tank can be expressed as

\[ \frac{C_{A1}(s)}{C_{A0}(s)} = \frac{K_{p1}}{(\tau_1 s + 1)} \] (3)

Where \( K_{p1} = \frac{F}{F + KV_1} \) is the gain of the transfer function of the first tank. The transfer function of the second tank can be derived

\[ V_2 \frac{dC_{A2}}{dt} = FC_{A1} - FC_{A2} - V_2 KC_{A2} \] (4)

Where \( V_2 \) and \( C_{A2} \) are the volume and the inlet concentration of the second tank respectively. Equation 4 can be rearranged to be

\[ \frac{dC_{A2}}{dt} + \frac{1}{\tau_2} C_{A2} = \frac{F}{V_2} C_{A1} \] (5)

Where \( \tau_2 = \frac{V_2}{F + KV_2} \) is the time constant for the second tank. By taking Laplace transform and rearranging equation 5, the transfer function of the second tank can be obtained.

\[ \frac{C_{A2}(s)}{C_{A1}(s)} = \frac{K_{p2}}{(\tau_2 s + 1)} \] (6)

Where \( K_{p2} = \frac{F}{F + KV_2} \) is the gain of the transfer function of the second tank. The transfer function of the whole system can be obtained according to the following assumptions of parameters.

1- The flow rate is constant for the whole system \( F = 0.085 \text{m}^3/\text{min} \).
2- The volume of the two tanks is the same \( V_1 = V_2 = V = 1.05 \text{m}^3 \).
3- Reaction rate \( K = 0.04 \text{min}^{-1} \).

Since the time constants and the gains are equal for both tanks, they can be computed as follows:

\[ \tau = \frac{V}{F + KV} = 8.25 \text{min} \]

\[ K = \frac{F}{F + KV} = 0.669 \]

The transfer function of the combined two tanks with the assumed parameters can be obtained

\[ G(s) = \frac{C_{A2}(s)}{C_{A0}(s)} = \frac{K^2}{(\tau s + 1)^2} \] (7)

\[ G(s) = \frac{0.0066}{s^2 + 0.2424s + 0.0147} \] (8)

Figure 3 shows the block diagram of the open loop combined two tank system.
3 Control strategy for the two tank system

In the section a control strategy for the two-tank system will be discussed and presented. The control strategy will be based on the proportional plus integral (PI) and the proportional plus integral fuzzy logic controller (PIFLC) techniques. The proportional control mode produces a change in the controller output proportional to the error signal. Meanwhile, the integral control mode changes the output of the controller by an amount proportional to the integral of the error signal. Therefore, the integral mode is frequently combined with the proportional mode to provide an automatic reset action that eliminates the proportional offset. The combination is referred to as the proportional plus integral (PI) control mode. The proportional mode provides change in the controller output that is proportional to the error signal. The integral mode provides the reset action by constantly changing the controller output until the error is reduced to zero. The integral modes provide an additional change in the output that is proportional to the integral of the error signal. The reciprocal of integral action rate is the time required for the integral mode to match the change in output produced by the proportional mode. One problem with the integral mode is that it increases the tendency for oscillation of the controller variable. The gain of the proportional controller must be reduced when it is combined with the integral mode. This reduces the ability of the controller to respond to rapid load changes. The proportional plus integral control mode is used on processes with large load changes when the proportional mode alone is not capable of reducing the offset to an acceptable level. The integral mode provides a reset action that eliminates the proportional offset. The primary reason for the derivative action is to improve the closed loop stability. Controllers with proportional and derivative action can be interpreted in a way that the control is made proportional to the predicted output of the system. The time domain equation of the proportional plus integral mode is

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt$$  \hspace{1cm} (9)$$

The first step is to test the performance of the system for a step change in the output without a controller to examine the uncontrolled response. The step response of the closed loop of the linear CSTR system has been taken to see the behavior of this system in closed loop mode, where the response of this system without controller is carried out using SIMULINK. The block diagram in Figure 4 illustrates the construction of closed loop system for CSTR system in SIMULINK. Figure 5 shows the output response of the system for a step change without a controller.

Table 1 illustrates the Performance specification that defines the system response for the step response input for the system. The final value is 0.30943, but the desired value is 1 in order to make the error steady state zero. From this case, the response has high error compared with the desired value. So the controller is needed to eliminate this error.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>S-S Error</td>
<td>0.691</td>
</tr>
<tr>
<td>Over shoot</td>
<td>0%</td>
</tr>
<tr>
<td>Rise time</td>
<td>20 sec</td>
</tr>
<tr>
<td>Settling time</td>
<td>28</td>
</tr>
</tbody>
</table>

Fig. 4 Simulink block diagram of the system without controller

Fig. 5 Step response of system without controller

Table I Performance specification of step response for the system without controller
3.1 Design of the controller using Ziegler-Nichols step response method

The design of a controller for the system will be presented and investigated. There are many techniques to design and tune a PI controllers. Ziegler-Nichols [4] gave two methods to tune the controller parameters. These methods are based on experimental procedure on the system response to the step changes in the input. These methods are still widely used in many applications. This method is based on the step response of the open-loop of the dynamic system. In this method it has been noticed that many dynamic systems exhibit a process reaction curve from which the controller parameters can be estimated. This curve can be obtained from either experimental data or dynamic simulation of the model. This method is firstly used for continuous systems but it can also be used for discrete systems if the sampling rate is very fast. The output response of the open loop of the dynamic model of the system was obtained, as shown in figure 6, to determine the parameters.

The response will then reduced just to two main parameters, the time delay, \( L \), and the steepest slope for the response, \( R \), which defined in figure 5. The final values of the parameters for the PI controller can be calculated according to table II.

![Fig. 6 Process step response](image)

Table II Controller parameters tuning using transient response

<table>
<thead>
<tr>
<th>Type</th>
<th>( K_p )</th>
<th>( T_1 )</th>
<th>( T_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( 1/RL )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>( 0.9/RL )</td>
<td>( 3L )</td>
<td></td>
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</table>

3.2 Basic structure or FLC system

Fuzzy logic is the application of logic to imprecision and has found application in control system design in the form of Fuzzy Logic Controllers (FLCs). Fuzzy logic controllers facilitate the application of human expert knowledge, gained through experience, intuition or experimentation, to a control problem. Such expert knowledge of a system’s behaviour and the necessary intervention required to adequately control that behaviour which is described using imprecise terms known as “linguistic variables”. The vagueness of linguistic variables reflects the nature of human observation and judgment of objects and events within our environment, system-related information to actions observed to provide sufficient system control. In this way, FLCs obviate the need for complex mathematical descriptions of the system behaviour to the \( n \)th degree and thus offer an alternative method of system control. Figure 7 shows the components of a feedback control system that has an FLC in place of a classical controller. An FLC consists of three components; the fuzzification process, inference, and the defuzzification process. The fuzzification process interprets the inputs as linguistic values. Inference uses a knowledge base of rules to determine the output sets for the input linguistic values. Finally, the defuzzification process uses the output of the inference to derive a single “crisp” output value. In designing the controller, it is important to plan the characteristic of the controller. In designing FLC, the input and output of the system, number of Fuzzy partitions must be identified, type of membership functions should be chosen correctly, Fuzzy control rules-based must be derived. Moreover, the inference engine and the defuzzification method should be chosen correctly.

![Fig. 7 FLC in feedback control loop](image)

3.3 Design of the PIFLC system

In designing any fuzzy logic controller the some steps should be followed. Firstly, number of inputs and size of universes of discourses. Secondy number and shape of fuzzy sets. Finally, delimitate all accepted cases in the inlet and expect the outlet at all cases as shown in figure 3.19. According to the fuzzy structure, there are two inputs to fuzzy inference, which are error \( E(t) \) and changes rate of error \( CE(t) \) and two outputs for PI controller parameters respectively \( K_p \) and \( K_i \). Mamdani model
is applied as structure of fuzzy inference with some modification to obtain the best value for $K_p$ and $K_i$.

Fig. 8 Fuzzy inference block

The implementation of self-tuning PIFL controller for CSTR system in Simulink is shown in figure 9. It consists of fuzzy controller and PI block with some modification refers to the formula which is applied to calibrate the value of $K_p$ and $K_i$ from fuzzy block to obtain the value of $K_p$ and $K_i$. Each parameter has its own calibration.

Fig. 9 Block diagram of the system with PIFLC

3.2 Simulation and results

The PI controller parameters are first determined using the Ziegler-Nichols transient response method which produced coefficients ($K_p = 7$ and $K_i = 0.75$). These values were then fined tuned to produce a heuristic optimal response with coefficient values ($K_p = 6.5$ and $K_i = 0.65$). Figure 10 shows the output responses for the systems under PI controller. Meanwhile the output response for the PIFL controllers is shown in figure 11. Figure 12 shows the output comparison ion between the PI and PIFL controllers. It can be seen that the PIFL controller has improved the performance of the system over the PI controller. The percentage overshoot has been eliminated. The settling time is improved by approximately 7%. However the rise time was much less in the case of PI controller. The steady-state error is zero in both cares. Table III shows the comparison between the two controllers.

Fig. 10 The output response for PI controller

Fig. 11 The output response for PIFL controller

Fig. 12 The comparison between PI and PIFL controller
Table III Performance comparisons between PI and PI Fuzzy controller

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Type of controller</th>
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<tbody>
<tr>
<td></td>
<td>PI</td>
</tr>
<tr>
<td>Rise Time (T&lt;sub&gt;r&lt;/sub&gt;, sec)</td>
<td>8.59 sec</td>
</tr>
<tr>
<td>Overshoot (OS %)</td>
<td>19 %</td>
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<tr>
<td>Settling Time (T&lt;sub&gt;s&lt;/sub&gt;, sec)</td>
<td>45.35 sec</td>
</tr>
<tr>
<td>SS. error (e&lt;sub&gt;ss&lt;/sub&gt;)</td>
<td>0</td>
</tr>
</tbody>
</table>

4 Conclusion
In this paper, the dynamic modeling of the CSTR was successfully derived. Two control strategies have been designed to control the output response of the CSTR system. It is important to select the most significant criterions such as small settling and rise times, no steady-state error and no overshoot in order to choose the best control strategy. However, these criterions cannot be achieved at same time. Therefore, it is necessary to decide which criterion that is most important to be achieved. For the CSTR system, the most required criterion is that the system should have no overshoot and zero steady-state error. Simulation results show that the PIFL controller has the best performance compared to the conventional PI controller. It can be seen that the output response of the system with PIFL controller has no overshoot and no steady-state error at lowest time. It takes shorter time to reach the steady state. Hence, it can be concluded that the PIFL controller is the best controller for the continuous stirred tank reactor (CSTR) system.

References: