Chaos-Geometric Attractor and Quantum Neural Networks Approach to Simulation Chaotic Evolutionary Dynamics During Perception Process

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Abstract: - Nonlinear simulation and forecasting chaotic evolutionary dynamics during perception and tuition processes can be effectively performed using the concept of compact geometric attractors. We present a new approach to analyze and predict the nonlinear perception and tuition dynamics based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation. Using phase space information on the evolution of the perception and tuition processes in time and results of the of quantum neural network modelling techniques can be considered as one of the fundamentally new approaches in the construction of global nonlinear models of the most effective and accurate description of the structure of the corresponding attractor and in further optimal realizations of the perception and tuition processes.

Key-Words: - Perception and Tuition • Geometric Attractor Conception • Quantum neural Networks • Chaotic Dynamics • Optimal Realizations

1 Introduction

It is very well known now that multiple physical, chemical, biological, social systems could demonstrate the typical chaotic behaviour [1-2]. Here one could remind a great majority of different systems, which are formally very complex, and this feature is manifested at different spatial and temporal scale levels [1-15]. It is not difficult to understand that examples of such systems are the chemical systems, biological populations, cybernetical (neurocybernetical), communication, at last social, psychological and physiological etc systems and its subsystems.

Most important, the fundamental issue in the description of the dynamics of the system is its ability to forecast its future evolution, i.e. predictability of behavior. Recently, the theory of dynamical systems is intensively developed, and, in particular, speech is about application of methods of the theory to the analysis of complex systems that provide description of their evolutionary dynamics by means solving system of differential equations. If the studied system is more complicated then the greater the equations is necessary for its adequate description. Meanwhile, examples of the systems described by a small amount of equations, are known nevertheless, theses systems exhibit a complicated behavior.

Probably the best-known examples of such systems are the Lorenz system, the Sinai billiard, etc. They are described, for example, three equations (i.e., in consideration included three independent variables), but the dynamics of their behavior over time shows elements of chaos (so-called "deterministic chaos"). In particular, Lorentz was able to identify the cause of the chaotic behavior of the system associated with a difference in the initial conditions. Even microscopic deviation between the two systems at the beginning of the process of evolution leads to an exponential accumulation of errors and, accordingly, their stochastic divergence. During the analysis of the observed dynamics of some characteristic parameters of the systems over time it is difficult to say to what class belongs to the system and what will be its evolution in the future. Many interesting examples can be reminded in the modern statistical physics, physics of non-ordered semiconductors etc. In recent years for the analysis of time series of fundamental dynamic parameters there are with varying degrees of success developed and implemented a variety of methods, in particular, the nonlinear spectral and trend analysis , the study of Markov chains, wavelet and multifractal analysis, the formalism of the matrix memory and the method of evolution propagators etc. Most of the cited approaches are defined as the methods of a chaos theory. In the theory of dynamical systems methods have been developed that allow for the recording of time series of one of the parameters to recover some dynamic characteristics of the system. In recent years a considerable number of works, including an analysis from the perspective of the theory of dynamical systems and chaos, fractal sets, is devoted to time series analysis of dynamical characteristics of physics and other systems [1-11]. In a series of papers [10-16] the authors have attempted to apply some of these methods in a variety of the physical, geophysical, hydrodynamic problems. In connection with this, there is an extremely important task on development of new, more effective approaches to the nonlinear modelling and prediction of chaotic processes in different complex systems.

In this work we present a new, advanced approach to nonlinear simulation and forecasting chaotic evolutionary dynamics during perception and tuition processes. New approach to analyze and predict the nonlinear perception and tuition dynamics is based on the concept of geometric attractors, chaos theory methods and algorithms of the quantum neural network simulation. Using phase space information on the evolution of the perception and tuition processes in time and results of the of quantum neural network modelling techniques can be considered as one of the fundamentally new approaches in the construction of global nonlinear models of the most effective and accurate description of the structure of the corresponding attractor and in further optimal realizations of the perception and tuition processes.

2 New Conception to Analysis of Chaotic Processes in Complex Systems

The basic idea of the construction of our approach to prediction of chaotic processes in complex systems during perception and tuition processes is in the use of the traditional concept of a compact geometric attractor in which evolves the measurement data, plus the implementation of neural network algorithms. The existing so far in the theory of chaos prediction models are based on the concept of an attractor, and are described in a number of papers (e.g. [1-8]). The meaning of the concept is in fact a study of the evolution of the attractor in the phase space of the system and, in a sense, modelling ("guessing") time-variable evolution.

From a mathematical point of view, it is a fact that in the phase space of the system an orbit continuously rolled on itself due to the action of dissipative forces and the nonlinear part of the dynamics, so it is possible to stay in the neighborhood of any point of the orbit $y(n)$ other points of the orbit $y^r(n)$, $r = 1, 2, ..., N_B$, which come in the neighborhood $y(n)$ in a completely different times than n. Of course, then one could try to build different types of interpolation functions that take into account all the neighborhoods of the phase space and at the same time explain how the neighborhood evolve from $y(n)$ to a whole family of points about $y(n+1)$. Use of the information about the phase space in the simulation of the evolution of some physical (geophysical etc.) process in time can be regarded as a fundamental element in the simulation of random processes.

In terms of the modern theory of neural systems, and neuro-informatics (e.g. [11]), the process of modelling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations (miemo-dynamic equations). Imitating the further evolution of a complex system as the evolution of a neural network with the corresponding elements of the self-study, self-adaptation, etc., it becomes possible to significantly improve the prediction of evolutionary dynamics of a chaotic system. Considering the neural network (in this case, the appropriate term "geophysical" neural network) with a certain number of neurons, as usual, we can introduce the operators S_{ij} synaptic neuron to neuron u_i u_j, while the corresponding synaptic matrix is reduced to a numerical matrix strength of synaptic connections: $W = || w_{ij} ||$. The operator is described by the standard activation neuro-equation determining the evolution of a neural network in time:

$$s_i^{'} = sign(\sum_{j=1}^{N} w_{ij} s_j - \theta_i), \tag{1}$$

where $1<i<N$.

Of course, there can be more complicated versions of the equations of evolution of a neural network. Here it is important for us another proven fact related to information behavior of a neuro-dynamical system. From the point of view of the theory of chaotic dynamical systems, the state of the neuron (the chaos-geometric interpretation of the forces of synaptic interactions, etc.) can be represented by currents in the phase space of the system and its the topological structure is obviously determined by the number and position of attractors. To determine the asymptotic behavior of the system it becomes crucial a information aspect of the problem, namely, the fact of being the initial state to the basin of attraction of a particular attractor.

Modelling each physical attractor by a record in memory, the process of the evolution of neural network, transition from the initial state to the (following) the final state is a model for the reconstruction of the full record of distorted information, or an associative model of pattern recognition is implemented. The domain of attraction of attractors are separated by separatrices or certain surfaces in the phase space. Their structure, of course, is quite complex, but mimics the chaotic properties of the studied object. Then, as usual, the next step is a natural construction parameterized nonlinear function ***F** (x, **a**)*, which transforms:

$$\mathbf{y}(n) \to \quad \mathbf{y}(n+1) = \mathbf{F}(\mathbf{y}(n), \mathbf{a}),$$

and then to use the different (including neural network) criteria for determining the parameters a (see below). The easiest way to implement this program is in considering the original local neighborhood, enter the model(s) of the process occurring in the neighborhood, at the neighborhood and by combining together these local models, designing on a global nonlinear model. The latter describes most of the structure of the attractor.

Although, according to a classical theorem by Kolmogorov-Arnold -Moser, the dynamics evolves in a multidimensional space, the size and the structure of which is predetermined by the initial conditions, this, however, does not indicate a functional choice of model elements in full compliance with the source of random data. One of the most common forms of the local model is the model of the Schreiber type [3] (see also [10]).

3 Construction of the model prediction

Nonlinear modelling of chaotic processes during perception and tuition processes can be based on the concept of a compact geometric attractor, which evolve with measurements. Since the orbit is continually folded back on itself by the dissipative forces and the non-linear part of the dynamics, some orbit points $\mathbf{y}^r(k)$, $r = 1, 2, \ldots, N_B$ can be found in the neighbourhood of any orbit point $\mathbf{y}(k)$, at that the points $\mathbf{y}^r(k)$ arrive in the neighbourhood of $\mathbf{y}(k)$ at quite different times than k. Then one could build the different types of interpolation functions that take into account all the neighborhoods of the phase space, and explain how these neighborhoods evolve from y (n) to a whole family of points about y (n + 1). Use of the information about the phase space in modelling the evolution of the physical process in time can be regarded as a major innovation in the modelling of chaotic processes.

This concept can be achieved by constructing a parameterized nonlinear function F(x, a), which transform y(n) to y(n+1)=F(y(n), a), and then using different criteria for determining the parameters a. Further, since there is the notion of local neighborhoods, one could create a model of the process occurring in the neighborhood, at the neighborhood and by combining together these local models to construct a global nonlinear model that describes most of the structure of the attractor.

Indeed, in some ways the most important deviation from the linear model is to realize that the dynamics evolve in a multidimensional space, the size and the structure of which is dictated by the data. However, the data do not provide "hints" as to which model to select the source to match the random data. And the most simple polynomial models, and a very complex integrated models can lead to the asymptotic time orbits of strange attractors, so for part of the simulation is connected with physics. Therefore, physics is "reduced" to fit the algorithmic data without any interpretation of the data. There is an opinion that there is no algorithmic solutions on how to choose a model for a mere data.

As shown Schreiber [3], the most common form of the local model is very simple:

$$s(n+\Delta n) = a_0^{(n)} + \sum_{j=1}^{d_A} a_j^{(n)} s(n-(j-1)\tau) \qquad (2)$$

where Δ n - the time period for which a forecast .

The coefficients $a_j^{(k)}$, may be determined by a least-squares procedure, involving only points $s(k)$ within a small neighbourhood around the reference point. Thus, the coefficients will vary throughout phase space. The fit procedure amounts to solving $(d_A + 1)$ linear equations for the $(d_A + 1)$ unknowns. When fitting the parameters a, several problems are encountered that seem purely technical in the first place but are related to the nonlinear properties of the system. If the system is low-dimensional, the data that can be used for fitting will locally not span all the available dimensions but only a subspace, typically. Therefore, the linear system of equations to be solved for the fit will be ill conditioned. However, in the presence of noise the equations are not formally ill-conditioned but still the part of the solution that relates the noise directions to the future point is meaningless . Note that the method presented here is not only because, as noted above, the choice of fitting requires no knowledge of physics of the process itself. Other modelling techniques are described, for example, in [3,10].

Assume the functional form of the display is selected, wherein the polynomials used or other basic functions. Now, we define a characteristic which is a measure of the quality of the curve fit to the data and determines how accurately match y (k + 1) with F (y (k), a), calling it by a local deterministic error:

$$\varepsilon_D(k) = \mathbf{y}(k+1) - \mathbf{F}(\mathbf{y}(k), \mathbf{a}). \quad (3)$$

The cost function for this error is called W (ε). If the mapping F (y, a), constructed by us, is local, then one has for each adjacent to y (k) point, $y^{(r)}(k)$ ($r = 1, 2, ..., N_B$),

$$\varepsilon_D^{(r)}(k) = \mathbf{y}(r, k+1) - \mathbf{F}(\mathbf{y}^{(r)}(k), \mathbf{a}), \quad (4)$$

where y (r, k + 1) - a point in the phase space which evolves y (r, k). To measure the quality of the curve fit to the data, the local cost function is given by

$$W(\varepsilon, k) = \frac{\sum_{r=1}^{N_B} \left|\varepsilon_D^{(r)}(k)\right|^2}{\sum_{r=1}^{N_B} \left[\mathbf{y}(k) - \langle \mathbf{y}(r,k)\rangle\right]^2} \quad (5)$$

and the parameters identified by minimizing W (ε, k), will depend on a.

Furthermore, formally the neural network algorithm is launched, in particular, in order to make training the neural network system equivalent to the reconstruction and interim forecast the state of the neural network (respectively, adjusting the values of the coefficients).

The starting point is a formal knowledge of the time series of the main dynamic parameters of a chaotic system, and then to identify the state vector of the matrix of the synaptic interactions $||w_{ij}||$ etc. Of course, the main difficulty here lies in the implementation of the process of learning neural network to simulate the complete process of change in the topological structure of the phase space of the system and use the output results of the neural network to adjust the coefficients of the function display. The complexity of the local task, but obviously much less than the complexity of predicting the original chaotic processes in physical or other dynamic systems.

4 Dynamics of quantum neural networks on basis of photon echo: Some numerical realizations

This subchapter is devoted to description of our algorithm of the program realization of the photon echo based quantum neural networks and its using for simulation of the tuition process. Currently there is a considerable interest in the development of neuro-computers, i.e. physical realizations of neural network models [10,17]. Now the main features of the neural network are being actively developed . It is known that the optimal neural network must be multilayered one, with a possibility to implement learning, feedback and controlled noise. Key elements are as follows: matrix linkages, which should act as a one-dimensional or two-dimensional transducer image and model neuron, giving a binary or continuous sigmoid response to incoming stimulation. Although notable progress in the study of the features of quantum multilayer neural networks has been achieved, however, many important issues concerning their basic characteristics, operational models , information capacity, storage and recovery implementations chains induced sequentially in time , the possibility of learning, feedback , noise exposure etc. , until now are far from adequate resolution . This is especially true of neural networks based on photon echo [10]. Using the effect of photon echo (or multiphoton echo) is a new physical principle for implementation of a neural network to information processing. The basic aspects of theory of the photon echo based neural networks are stated previously (see, for example, [10,17-20]). So here we mention only the essential elements. Photon echo is a nonlinear optical effect, in fact this is the phenomenon of the four wave interaction in a nonlinear medium with a time delay between the laser pulses. Exciting sequence of optical pulses pass through the appropriate medium and call for the environment photon echo signal after a certain time interval. It is necessary that the medium was resonant , i.e. the carrier frequency of the optical pulses was close to the frequency of the excited transition. Interaction of light with a resonant medium should occur in a relatively short intervals , shorter than the "phase memory". Duration light pulses must be much less than the minimum relaxation time in the environment (condition of coherent interaction) . the effect of the three-pulse stimulated photon echo has the necessary properties for use as a photon echo new physical principle implementations of neural networks.

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One promising approach to the realization of an optical neural network is the inner product scheme [10,17]. Schematic diagram of the optical images for processing sequence is as follows: {↓Input→ Cumulative matrix F1→Correlation region→ Cumulative matrix F2→Output→ Threshold Device ↑}. The first pulse has an amplitude equal to unity over the entire plane of the medium, the second pulse defines the vectors of memory, incoming in the form of vertical columns and providing accumulation of the memory matrix $F_1 = F_2$ of the size $(N{\cdot}p)$ in the environment. The third pulse, whose amplitude is determined by a recognized one - dimensional image, comes to the input of the system and is uniformly distributed over the medium to the horizontal direction . As a result, there are arisen the stimulated echo-signals, which are collected in a horizontally disposed one-dimensional array in the correlation region . In the first phase there are calculated the inner product between the input vector and memory vectors. Expression for the stimulated photon echo signal is: $u(m) \sim \sum_j \xi_j^m \xi_j^{in}$. The amplitude of the first pulse is equal to unity and is omitted here. Further the inner product weigh the corresponding memory vectors $\xi^1,...,\xi^p$, accumulated in the matrix F2. This operation leads to a stimulated echo signals, which are then summed, resulting in a one-dimensional distribution of amplitude: $s_i \sim \sum_m u(m)\xi_{ii}^m = \sum_m (\xi_i^m) \sum_j \xi_j^m \xi_{ij}^{in}$.

This ratio, threshold conversion and feedback determine the dynamics of the Hopfield neural network with the Hebb coupling matrix. The photon echo based implementation allows to replace the resolution of images in the space by a time resolution. As result processing two-dimensional arrays is possible. Similarly, one has for the output signal amplitude as follows: $a_{kl}^{out} \sim \sum_m u(m) a_{kl}^m = \sum_m a_{kl}^m \sum_{j,i} a_{ij}^m a_{ij}^{in}$. To account for an effect of delay it is necessary to include the lag variables into network dynamics:

$$\xi_i(n+1) = f[\sum_{j=1}^{N} \sum_{l=0}^{Qk-1} J_{ij}^l \xi_j(n-l)\,], \quad (6)$$

where the onnection matrix (corresponding to the variables lag) have the form:

$$J_{ij}^l = \sum_{k=1}^{s} \sum_{m=1}^{Qk-l} \xi_\iota^{\kappa,\mu+\lambda-1} \xi_j^{k,m} ...and... \xi_j^{k,mk+1} = \xi_j^{k,l} \quad (7)$$

Here s is number of chains in the network , Q_k is the number of images in the k-th chain. If l=0, then we have a network with instant response. Note that the need for storing states in previous times in neural networks with l> 0 makes it difficult for their implementation by the known methods with the exception of the method, based on the effect of photon echo . In order to obtain an opportunity to make modeling invariant pattern recognition and get the most information capacity one should use the neural networks of higher orders. We have developed a software package for numerical modeling of the dynamics of the photon echo neural network. It has the following key features: multi-layering, possibility of introducing training, feedback and controlled noise. There are possible the different variants of the connections matrix determination and binary or continuous sigmoid response (and so on) of the model neurons. In order to imitate a tuition process we have carried out numerical simulation of the neural networks for recognizing a series of patterns (number of layers N=3-5, number of images p=320; the error function:

$$SSE = \sum_{p=1}^{p_{\max}} \{..\sum_{k=}^{k_{\max}} [t(p,k) - O(p,k)]^2\}, \quad (8)$$

where $O(p,k)$ – neural networks output k for image p and $t(p,k)$ is the trained image p for output κ; SSE is determined from a procedure of minimization; the output error is $RMS=sqrt(SSE/P_{max})$; As neuronal function there is used function of the form: $f(x) = 1/[1+\exp(-\delta x)]$. In our calculation there is tested the function $f(x,T)=\exp[(xT)^4]$ too. The results of the PC simulation (with using our neural networks package NNW-13-2003 [10]) of dynamics of the quantum multilayer neural networks with the input rectangular and sinusoidal pulses are listed in fig 1,2.

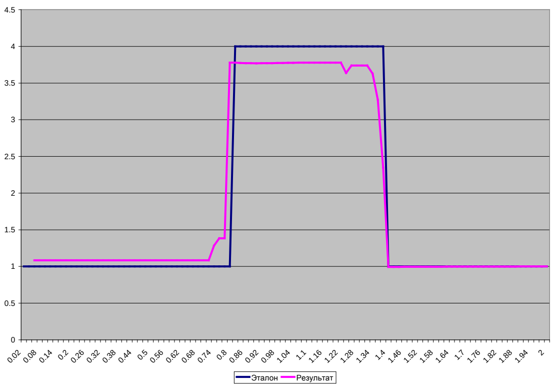

Fig. 1. The results of modeling the multi-layer neural networks with a rectangular pulse input

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Fig. 3 demonstrates the results of modeling the dynamics of multilayer neural network for the case of noisy input sequence. The input signal was the Gaussian-like pulse with adding a noise with intensity D. At a certain value of the parameter D (the variation interval .0001-0.0040) the network training process and signal playback is optimal. The optimal value of D is 0.0017 . A coherency of input and output is optimal for the indicated optimal noise level.

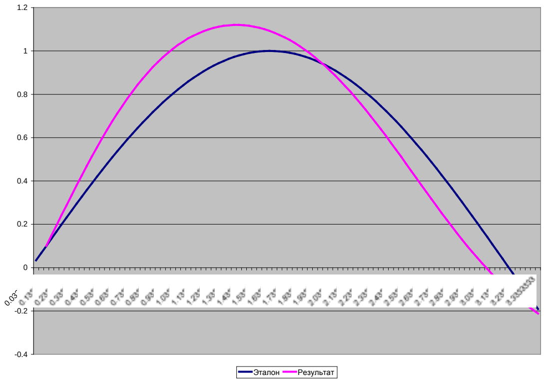

Fig. 2. The results of modeling the dynamics of multilayer neural networks with sinusoidal input pulse

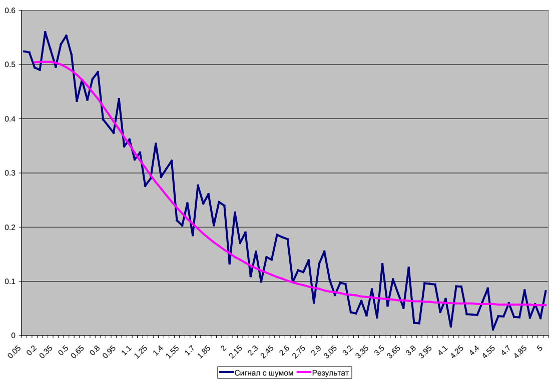

Fig. 3. The results of modeling the dynamics of multilayer neural for the case of noisy input sequence.

Thus, a stochastic resonance effect is in fact discovered in our PC experiment. In our view, this phenomenon is apparently typical for the neural network system. Obviously, one should search for the same effect for human tuition process. Analysis of the PC experiment results allows to make conclusion about sufficiently high-quality processing the input signals of very different shapes and complexity by a photon echo based neural network.

5 Conclusions

Here we have considered a new approach to nonlinear modelling and prediction of chaotic processes during perception and tuition processes, which is based on two key functional elements. Besides using other elements of starting chaos theory method, the proposed approach includes the application of the concept of a compact geometric attractor, and one of the neural network algorithms, or, in a more general definition of a model of artificial intelligence. The meaning of the latter is precisely the application of neural network to simulate the evolution of the attractor in phase space, and training most neural network to predict (or rather, correct) the necessary coefficients of the parametric form of functional display. In result one could get possibilities to analyze and predict the nonlinear (for example, perception and tuition) dynamics based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation [10,21]. Using phase space information on the evolution of the perception and tuition processes in time and results of the of quantum neural network modelling techniques can be considered as one of the fundamentally new approaches in the construction of global nonlinear models of the most effective and accurate description of the structure of the corresponding attractor and in further optimal realizations of the perception and tuition processes.

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