

Studying Interaction Dynamics of Chaotic Systems Within a Non-linear Prediction Method: Application to Neurophysiology

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Abstract: Paper is devoted to an employing a variety of techniques for characterizing dynamics of the nonlinear neuro-physiological systems identifying the presence of chaotic elements. To analyze measured time histories of the neurophysiological system responses the phase space of these systems was reconstructed by delay embedding. The mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's analysis, and surrogate data method are used for comprehensive characterization. The correlation dimension method provided a low fractal-dimensional attractor thus suggesting a possibility of the existence of chaotic behavior. Statistical significance of the results was confirmed by testing for a surrogate data. We present the concrete numerical results regarding the ensembles fluctuations of spontaneous Parkinsonian tremor of a few patients.

Key-Words: Chaotic Dynamics • Correlation integral analysis • Lyapunov exponent's analysis • Neuro-physiological systems • Parkinsonian tremor

1 Introduction

The task of studying the dynamics of chaotic dynamical systems arises in many areas of science and technology. We are talking about a class of problems of identifying and estimating the parameters of interaction between the sources of complex (chaotic) oscillations of the time series of experimentally observed values. Such problems arise in physics, biology, medicine, neuroscience, geophysics, engineering, etc.

Many studies in the cited and other fields of science and technique have appeared, where a chaos theory was applied to a great number of dynamical systems [1-16]. These studies show that chaos theory methodology can be applied and the short-range forecast by the non-linear prediction method can be satisfactory. Time series of the dynamical variables are however not always chaotic, and chaotic behaviour must be examined for each time series. In series of papers it has been developed an effective version of using a chaos theory method and non-linear prediction approach to studying chaotic behaviour of the different dynamical systems. In our opinion, using these methods has very attractive perspectives in medicine and physiology (neuro-physiology). As example, let us underline that an

ability to provide interaction between the different areas of the brain by using a multi-channel electroencephalograms helps determine the location of the foci of abnormal activity in brain of patients with epilepsy. Many diseases of the brain, including epilepsy, Parkinson's disease, are associated with abnormal synchronization large groups of neurons in the brain. Particular attention is paid to a non-linear signals as obvious is a typicality of a chaotic behavior of nonlinear systems.

This paper is devoted to an employing a variety of techniques [16-21] for characterizing dynamics of the nonlinear neuro-physiological systems identifying the presence of chaotic elements. To analyze measured time histories of the neurophysiological system responses the phase space of these systems was reconstructed by delay embedding. The mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's analysis, and surrogate data method are used for comprehensive characterization. The correlation dimension method provided a low fractal-dimensional attractor thus suggesting a possibility of the existence of chaotic behavior. Here we also present preliminary numerical results regarding the ensembles fluctuations of spontaneous Parkinsonian tremor of

a few patients. To implement this program, we follow the procedure set out in detail in [3].

2. New Method: Testing for chaos in time series

2.1 Data

Many diseases of the nervous system, including epilepsy and Parkinson's disease associated with abnormal synchronization large groups of neurons in the brain. A sign of Parkinson's disease is the synchronization of neurons in the ranks of the thalamus and basal ganglia. However, the functional role of synchronization in the generation of Parkinsonian tremor (involuntary limb regular oscillations with frequencies ranging from 3 to 6 Hz) remains a matter of debate (see [13]).

Standard therapy with no effect of medication - it's a deep electrical deep brain stimulation (DEBS) at high frequencies (above 100 Hz). Standard DEBS has been found empirically, the mechanism of its effect has not yet been elucidated, and it has restrictions, such as those associated with side effects. Confirmation that the tremor caused synchronous neuronal activity in nuclei of the thalamus and basal ganglia, would presumably result in a softer therapies with fewer side effects. In this connection of the relevance of the problem of determining the nature of the links between different areas of the brain and the muscles of patients.

The ensembles intervals of spontaneous Parkinsonian tremor three patients have been investigated in [13]. Fluctuations in the limbs were presented accelerometer signals recorded at the sampling rate of 200 Hz and 1 kHz. Information about the activity of the brain was presented recordings of local potentials (LP) of the four deep electrodes implanted in the thalamus and basal ganglia.

The data were obtained at the Department of Stereotactic and Functional Neurosurgery, University of Cologne and the Institute of Neurosciences and Biophysics, Research Center Juelich (Germany). Accelerometer signals and the LP with one of the electrodes during heavy Parkinsonian tremor are shown in Fig. 1.

The more detailed data can be found in [13] (and refs. therein). According to [13], the main conclusion is as the tests also showed that linear techniques do not reveal the activity of the thalamus and basal ganglia on the limb.

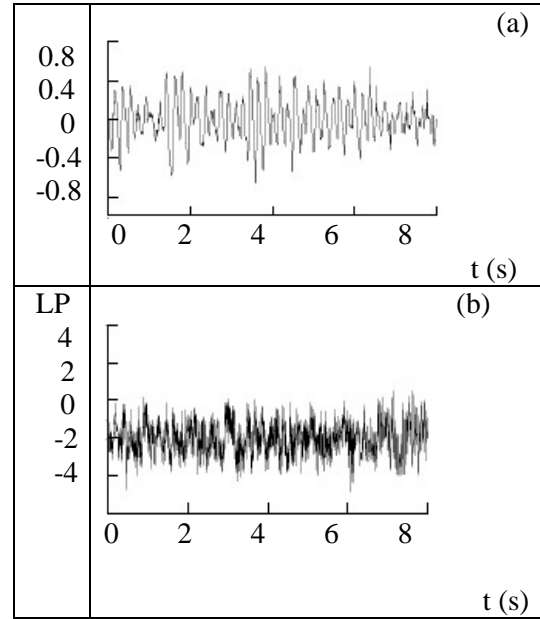


Fig. 1. Spontaneous interval Parkinsonian tremor (total duration 0.1x800) (a, b) and the accelerometer signal LP with one of the electrodes in arbitrary units (only the first 8 s shown);

Besides, it has been found that there are the fluctuations in the accelerometer signal, which correspond to a distinct peak in the power spectrum at a frequency of 5 Hz. The statistical significance of the findings has been confirmed by tests on surrogate data.

2.2 Testing for chaos in time series

On order to make testing for chaos in time series, we use the methodology [3,14-18]. As usually, let us consider scalar measurements $s(n)=s(t_0+n\Delta t)=s(n)$, where t_0 is a start time, Δt is time step, and n is number of the measurements. In a general case, $s(n)$ is any time series (f.e. atmospheric pollutants concentration). As processes resulting in a chaotic behaviour are fundamentally multivariate, one needs to reconstruct phase space using as well as possible information contained in $s(n)$. Such reconstruction results in set of d -dimensional vectors $\mathbf{y}(n)$ replacing scalar measurements. The main idea is that direct use of lagged variables $s(n+\tau)$, where τ is some integer to be defined, results in a coordinate system where a structure of orbits in phase space can be captured. Using a collection of time lags to create a vector in d dimensions,

$$\mathbf{y}(n)=[s(n), s(n+\tau), s(n+2\tau), \dots, s(n+(d-1)\tau)],$$

the required coordinates are provided. In a nonlinear system, $s(n+j\tau)$ are some unknown nonlinear combination of the actual physical variables. The dimension d is the embedding dimension, d_E .

2.3 Time lag

The choice of proper time lag is important for the subsequent reconstruction of phase space. If τ is chosen too small, then the coordinates $s(n+j\tau)$, $s(n+(j+1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. If τ is too large, then $s(n+j\tau)$, $s(n+(j+1)\tau)$ are completely independent of each other in a statistical sense. If τ is too small or too large, then the correlation dimension of attractor can be under- or overestimated. One needs to choose some intermediate position between above cases. First approach is to compute the linear autocorrelation function $C_L(\delta)$ and to look for that time lag where $C_L(\delta)$ first passes through 0. This gives a good hint of choice for τ at that $s(n+j\tau)$ and $s(n+(j+1)\tau)$ are linearly independent.

It's better to use approach with a nonlinear concept of independence, e.g. an average mutual information. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to 0 if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i, b_k)$. Usually it is necessary to choose that τ where the first minimum of $I(\tau)$ occurs.

2.4 Embedding dimension

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. The embedding dimension, d_E , must be greater, or at least equal, than a dimension of attractor, d_A , i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E , e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. According to [4], it is computed the correlation integral $C(r)$. If the time series is characterized by an attractor, then the correlation integral $C(r)$ is related to the radius r as

$$d = \lim_{\substack{r \rightarrow 0 \\ N \rightarrow \infty}} \frac{\log C(r)}{\log r},$$

where d is correlation exponent. If the correlation exponent attains saturation with an increase in the embedding dimension, then the system is generally considered to exhibit chaotic dynamics. The saturation value of correlation exponent is defined as the correlation dimension (d_2) of the attractor (see details in refs. [3,4]).

Another method for determining d_E comes from asking the basic question addressed in the embedding theorem: when has one eliminated false crossing of the orbit with itself which arose by virtue of having projected the attractor into a too low dimensional space? In other words, when points in dimension d are neighbours of one other? By examining this question in dimension one, then dimension two, etc. until there are no incorrect or false neighbours remaining, one should be able to establish, from geometrical consideration alone, a value for the necessary embedding dimension. Such an approach was described by Kennel et al. [7]. In dimension d each vector $\mathbf{y}(k)$ has a nearest neighbour $\mathbf{y}^{NN}(k)$ with nearness in the sense of some distance function. The Euclidean distance in dimension d between $\mathbf{y}(k)$ and $\mathbf{y}^{NN}(k)$ we call $R_d(k)$:

$$R_d^2(k) = [s(k) - s^{NN}(k)]^2 + [s(k+\tau) - s^{NN}(k+\tau)]^2 + \dots + [s(k+\tau(d-1)) - s^{NN}(k+\tau(d-1))]^2. \quad (1)$$

$R_d(k)$ is presumably small when one has a lot of data, and for a dataset with N measurements, this distance is of order $1/N^{1/d}$. In dimension $d+1$ this nearest-neighbour distance is changed due to the $(d+1)$ st coordinates $s(k+d\tau)$ and $s^{NN}(k+d\tau)$ so

$$R_{d+1}^2(k) = R_d^2(k) + [s(k+d\tau) - s^{NN}(k+d\tau)]^2. \quad (2)$$

We can define some threshold size R_T to decide when neighbours are false. Then if

$$\frac{|s(k+d\tau) - s^{NN}(k+d\tau)|}{R_d(k)} > R_T, \quad (3)$$

(the nearest neighbours at time point k are declared false).

Kennel et al. [7] showed that for values in the range $10 \leq R_T \leq 50$ the number of false neighbours identified by this criterion is constant. In practice,

the percentage of false nearest neighbours is determined for each dimension d . A value at which the percentage is almost equal to zero can be considered as the embedding dimension.

2.5 Nonlinear prediction model

As usually, the predictability can be estimated by the Kolmogorov entropy, which is proportional to a sum of positive Lyapunov exponents. The spectrum of the Lyapunov exponents is one of dynamical invariants for non-linear system with chaotic behaviour. The limited predictability of the chaos is quantified by the local and global Lyapunov exponents, which can be determined from measurements.

The Lyapunov exponents are related to the eigenvalues of the linearized dynamics across the attractor. Negative values show stable behaviour while positive values show local unstable behaviour. For chaotic systems, being both stable and unstable, Lyapunov exponents indicate the complexity of the dynamics. The largest positive value determines some average prediction limit. Since the Lyapunov exponents are defined as asymptotic average rates, they are independent of the initial conditions, and hence the choice of trajectory, and they do comprise an invariant measure of the attractor. An estimate of this measure is a sum of the positive Lyapunov exponents. The estimate of the attractor dimension is provided by the conjecture d_L and the LE are taken in descending order. The dimension d_L gives values close to the dimension estimates discussed earlier and is preferable when estimating high dimensions. To compute the Lyapunov exponents, we use a method with linear fitted map, although the maps with higher order polynomials can be used too.

2.6 Final remarks

Summing up the review, it is useful to summarize the key points of the investigating system for a chaos availability and wording the forecast model (evolution) of the system. The above methods are just part of a large set of approaches (see our versions in [16-24]), which is used in the identification and analysis of chaotic regimes in the time series. Generally speaking, the short technique of processing any time series can be formulated as follows:

- a) check for the presence of a chaotic regime (the Gottwald-Melbourne's test; the method of correlation dimension);
- b) reducing the phase space (choice of the time

delay, the definition of the embedding space by methods of correlation dimension algorithm and false nearest neighbor points);

c) determination of the dynamic invariants of a chaotic system (global Lyapunov exponents);

d) forecasting evolution of the dynamical system.

Algorithm for calculating the characteristics of the chaotic time series and use it to forecast the non-linear method is presented in Fig.2

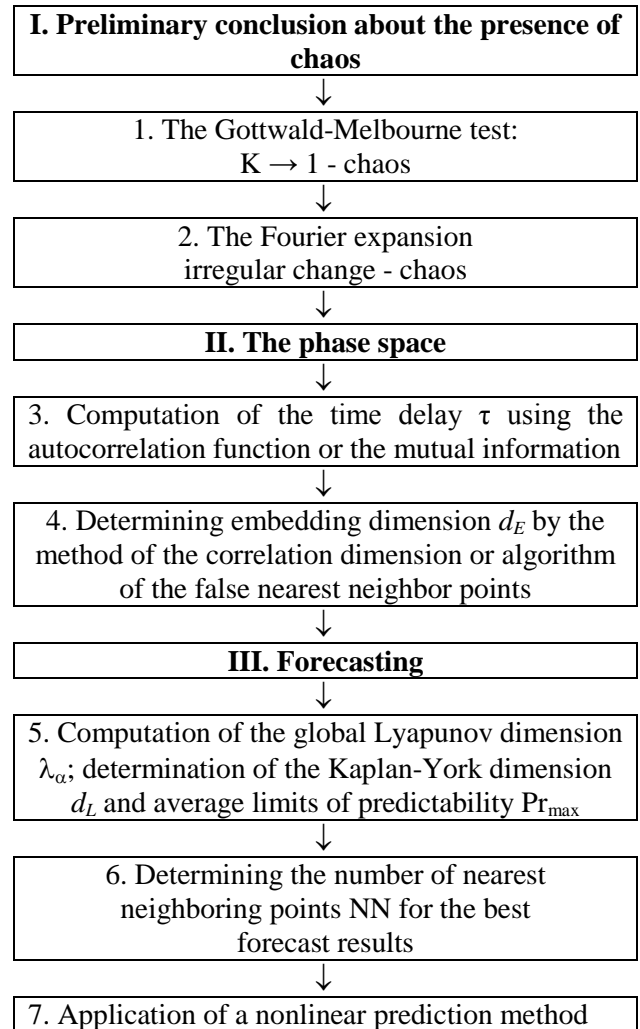


Fig. 2. Algorithm for computation of the characteristics of the chaotic time series and application of the non-linear prediction method to it [24].

The most important stage of this technique are the first two points, as the accuracy of the recovery will depend on the dimension of the attractor chaotic classification system and forecast its evolution. Therefore it is preferable not to use any one method, and several compare results. There is another very

important aspect related to the invariants of the system. The fact is that if the aggregate and dynamic topological invariants (see details in [1-3]), the two systems are identical, then we can say that the evolution of these systems are also subject to the same laws. Further, if one of these systems is known differential equation (or system of equations) describing its dynamics, it can be assumed that an analogous equation (or system) and the other describes the evolution of the system.

3. Results and conclusions

In our studying we have analyzed the time series of of the LP signal using methodology from chaos theory. Table 1 summarizes our preliminary results for the time lag calculated for first 800 values of time series of the LP signal. The values, where the autocorrelation function first crosses 0.1, can be chosen as τ , as an attractor cannot be adequately reconstructed for very large values of τ .

Table 1. Time lag (τ), correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L), average limit of predictability (Pr_{max}) and the Gottwald-Melbourne chaos availability parameter

τ	d_2	d_E	λ_1
9	5.61	6	0.0143
λ_2	d_L	Pr_{max}	K
0.0039	4.07	8	0.63

Let us note that the Kaplan-Yorke dimensions, which are also the attractor dimensions, are smaller than the dimensions obtained by the algorithm of false nearest neighbours.

Our results show that the time series is resulted from the low-dimensional chaos. The embedding dimension for the time series is $d_N = 6$. Also, the correlation dimensions were calculated using the algorithm of Grassberger and Procaccia. It is noteworthy that the nearest integer above the saturation value provides the minimum or optimum embedding dimension for reconstructing the phase-space or the number of variables necessary to model the dynamics of the system. This concept can be applied, since the embedding dimension determined by both the correlation dimension method and the algorithm of false nearest neighbours are identical. In this case, the number of variables necessary to model the dynamics of the system equals six

(preliminary estimate).

From the other hand, the analysis of correlation dimension provides only the number of variables, but not their physical meaning. At last, let us comment regarding the Lyapunov exponents.

Fistly, our data show that the Kaplan-Yorke dimensions, which are also the attractor dimensions, are smaller than the dimensions obtained by the algorithm of false nearest neighbours. There are the two positive λ_i for the time series under consideration.

Since the Lyapunov exponents determine conversion rate from a sphere into an ellipsoid, then the smaller sum of positive exponents results in the more stable dynamical system and, correspondingly, the higher predictability. The further work in application of the chaos theory methods to neuro-physiological problems requires the availability of reliable empirical data and the corresponding time series of measured values.

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