Abstract: - The study focuses on real estate expertise in Italy considering: the market surface ratios, proposing a series of tests designed to predict the influence of these surfaces on the result of the valuation; and the coefficients of non-surface characteristics providing a measure of the estimation error induced by their use. Overall, the study aims to bring out on a rational level the deficiencies, inaccuracies and errors of the valuations based on the expertise when it is applied to the real estate market. In Italy the application of valuation standard is in the early stages of launching.

Key-Words: -Surface ratio, diagnostic test, expertise, real estate, appraisal, relation economic

1 Introduction

The real estate appraisal is developed in relation to the economic, legal and social system in each country. In Italy, the real estate market has poor transparency and a deficient level of information, consequently in the appraisal profession it is common to make use of empirical valuation based on subjective judgments formulated by experts in the form of a real estate expertise. In this way, the lack of market information is compensated with experience and ability (animusa estimandi). The valuation of the expert cannot be proved or ascertained, nor is it repeatable, but can only be confirmed by the valuations of other experts.

Different are the international valuation standards that are based on the collection of data for real estate valuations (IVSC, 2011) (RICS, 2014) (TEGoVA, 2012) and the standards of measurement (International Property Measurement Standards Coalition, 2014) (RICS, 2007). For residential real estate (apartments, houses, etc.) (single-family houses, condos and town houses), the appraisal of market value is based on the surface of the property and on a set of coefficients reflecting the effect of other characteristics different from the surface (state maintenance, floor level, exposure, etc.). The surface of the property is composed of the main surface and the secondary areas (balconies, terraces, etc.), considered as...
fractions of the main surface. This surface is called commercial area. The coefficients of non-surface characteristics are expressed with pure numbers, higher or lower depending on whether the variation of the amount of the characteristic increase or decrease the market value of the property being appraised. In practice, a value (€/sqm), assigned by expertise is multiplied by the commercial area (sqm) and a certain number of coefficients. In general, the market value of residential property is the following:

\[ \text{Market value} = \text{Unit value} \cdot \text{Commercial surface} \cdot \text{Coefficients}. \] [1]

The unit value is estimated by the appraiser or detected by databases of quotations prepared by the public administration, magazines of real estate sector and other generic sources (list of interval values, list of price, etc.). The calculation of the commercial surface considers the ratios of the secondary surfaces (surface ratios) with the main surface. The ratio of the price of a secondary surface over the price of the main one can assume values less than, greater than or equal to the unit at the discretion of the market.

The surface ratios are detected directly from the housing market.

The criterion of secondary surface is applied to particular purposes relevant to: the determination of the cadastral area of the property assessed in ordinary use (DPR 23 March 1998, n. 138), the allocation of costs among the members of the buildings of co-operative housing (Circular 26 March 1966 n. 12480 of the Ministry of Public Works); the UNI 10750 norm that defines the homogenization ratios for the services of estate agents. These are usually fixed ratios uniquely determined. At the local level there may be agreements between operators and agencies and professional organizations to fix ratios as constant and uniform practice.

The coefficients of the non-surface characteristics are subjectively established by the evaluator and often with the help of commercial manuals, where their values are reported for each characteristic in the form of an interval between a minimum and a maximum.

2. Surface ratio

In the ratios of the secondary surfaces it is generally supposed that these are worth less than the main surface (surface ratio less than one), such as an open surface compared to a covered one by reason of a lower construction cost or a minor use. However, it is not considered that secondary surfaces can sometimes require work with structural complications or be more useful than the main surface. In addition, these secondary areas may characterize architecturally and typologically the property and be subject to significant estate transformations. Consequently, the distinction between the main surface and secondary surfaces is weak and sometimes contradictory, namely when a surface is considered secondary instead of a main surface for a particular or special type of property (surface ratio greater than one) whereas it represents the spaces of greater importance for the property. The market ratio \( \pi_i \) of the generic secondary surface \( i = 2, 3, ..., n \) is equal to the ratio between the average price \( p_i \) of the secondary surface and the average price \( p_1 \) of the main surface as follows:

\[ \pi_i = \frac{p_i}{p_1} \] (2)

In the expertise the market value \( V \) of the property being appraised is calculated by multiplying the unit value \( v \) by the commercial area and the coefficients, according to[1], as follows:

\[ V = v \cdot \left( x_{01} + \sum_{i=2}^{n} x_{0i} \cdot \pi_i \right) \cdot \alpha_1 \cdot \alpha_2 \cdot \ldots \cdot \alpha_h \cdot \ldots \cdot \alpha_l \] (3)

where \( x_{0i} \) is the main surface of the property being appraised; \( x_{01} \) is the generic secondary surface of the property being appraised; \( \pi_i \) is the ratio of the generic secondary surface; \( \alpha_h \) (with \( h=1, 2, ..., l \)) is the coefficient of a generic
property’s characteristic different from the surface. In the real estate expertise the surface ratios are indispensable because their omission leads to an error of overestimation or underestimation of the market value of the property.

In the approaches based on the data collection and on their comparison, such as the market comparison approach, the adjustments lead to correct prices for each property of comparison (IVSC, 2011) (RICS, 2014) (TEGoVA, 2012). The calculation of the correct price $V_j$ of the generic comparable property $j$ (with $j = 1, 2, ..., m$) can be referred to the comparison with the property to be estimated(0), highlighting the adjustments to surfaces as follows (Simonotti, 1985):

$$V_j = P_j + p_1 \cdot (x_{o1} + x_{j1}) + p_2 \cdot \sum_{j=2}^{n} (x_{o1} + x_{ji}) \cdot \pi_i + A_j, (4)$$

Where $P_j$ is the known market price of the comparable property, $x_{j1}$ is the main surface and $x_{ji}$ is the generic secondary surface of the generic comparable and $A_j$ the set of adjustments for non-surface characteristics (with $h = 1, 2, ..., l$). These adjustments are equal to:

$$A_j = \sum_{h=1}^{l} (x_{0h} + x_{jh}) \cdot p_h, (5)$$

Denoting by $X_0$ the commercial area of the property being appraised and by $X_j$ the commercial area of the property, in general:

$$X_0 = x_{o1} + \sum_{i=2}^{n} x_{0i} \cdot \pi_i, (6)$$

$$X_j = x_{j1} + \sum_{i=2}^{n} x_{ji} \cdot \pi_i, (7)$$

the price $p_1$ of the main surface can be calculated as the ratio between the price partially corrected $P_j^*$ for the adjustments of the non-surface characteristics ($P_j^* = P_j + A_j$) and the relative commercial surface, as follows:

$$p_1 = \frac{P_j + A_j}{X_j} = \frac{P_j^*}{X_j}. (8)$$

The correct price of the generic property is calculated by replacing the[8] in the [4] as follows:

$$V_j = P_j^* \cdot \frac{X_0}{X_j}. (9)$$

The correct price is equal to the price corrected for the non-surface characteristics of the comparable property multiplied by the ratio between the commercial surfaces of the building being appraised and that of the comparable property. In order to isolate, in terms of surface ratios, the effect of the secondary areas, it is indicated with $k_0$ the ratio between the sum of the secondary surfaces and the main surface of the building, i.e. the typological ratio of the property being appraised:

$$\frac{\sum_{i=2}^{n} x_{0i}}{x_{o1}} = k_0, (10)$$

And with $k_j$ the ratio between the sum of the secondary surfaces and the main surface of the comparable property:

$$\frac{\sum_{i=2}^{n} x_{ji}}{x_{j1}} = k_j; (11)$$

$\Pi_0$ denotes the surface ratio weighted by the surfaces of the building:

$$\Pi_0 = \frac{\sum_{i=2}^{n} x_{0i} \cdot \pi_i}{\sum_{i=2}^{n} x_{0i}}, (12)$$

And $\Pi_j$ indicated the surface ratio weighted by the areas of the comparable property:

$$\Pi_j = \frac{\sum_{i=2}^{n} x_{ji} \cdot \pi_i}{\sum_{i=2}^{n} x_{ji}}, (13)$$
Substituting in (9) the (12) and (13), the market value for the set of secondary area is equal to:

\[ V_j = P_j^* \cdot \frac{x_{01} + \Pi_0 \cdot \sum_{i=2}^{n} x_{0i}}{x_{j1} + \Pi_j \cdot \sum_{i=2}^{n} x_{ji}}. \quad (14) \]

When the ratio \( \pi_j \) is equal for all the secondary surfaces, then \( \Pi_0 \) and \( \Pi_j \) are equal to \( \pi_i \) though the secondary areas may differ in the property being appraised and in the comparable building. This means that the respective ratios between the sum of the secondary areas and the main surface are in the same ratio despite having different sizes. In practice, this circumstance occurs for serial or modular properties, such as apartments in multistory buildings, and more generally for similar properties belonging to the same market segment where the typological surface ratios are almost equal. The higher the difference between the surface ratios of two or more secondary surfaces (e.g., \( \pi_i \) and \( \pi_{i+1} \)), the smaller that between the weighted ratios (\( \Pi_0 \) and \( \Pi_j \)). In principle, if the number of the secondary surfaces increases, this relationship is more valid. This means that with good approximation can be considered a unique surface ratio for all secondary surfaces.

Ultimately, in the approaches based on the data collection, the surface ratios have minor importance compared to procedures based on the expertise.

3. Diagnostic indexes

In order to measure the effect of induced changes in the market value from the surface ratios, it is possible to use some diagnostic indexes for the comparable property and the property being appraised. A diagnostic index for surface ratios is the double typological ratio \( K_j \) between the typological ratios (10) and (11):

\[ K_j = \frac{k_0}{k_j} = \frac{P_j^* \cdot \frac{x_{j1}}{x_{01}} \cdot \sum_{i=2}^{n} x_{0i}}{\sum_{i=2}^{n} x_{ji}}. \quad (15) \]

For \( K_j < 1 \) when the value of the surface ratio increases the estimated value decreases, while for \( K_j > 1 \) when the value of the surface ratio increases the value increases (for the same price adjusted for non-surface characteristics). If the amounts of the surfaces of the building being appraised are equal to the corresponding amounts of the surfaces of the comparable building, then the market value is equal to the price adjusted for the objective condition of equality. When the comparison is on more than one comparable property, the diagnostic index must be presented in general terms. The double ratio \( K \) referred to the sample of comparable properties may be set equal to the weighted average of the simple indices for market prices collected (or adjusted prices) as follows:

\[ K_j = \frac{\sum_{j=1}^{m} K_j \cdot P_j}{\sum_{j=1}^{m} P_j}. \quad (16) \]

The overall index is diagnostically interpreted in the same terms as the indices of each comparable property in the presence of secondary surfaces. This index can be approximated by the simple average of the individual indices.

The series of diagnostic indexes can be applied as a first step to the sample of comparable properties (Table 1). The diagnostic indices of each comparable property are based exclusively on the surfaces of property; the indices referred to the sample of the properties can be related to the simple average of the indices or to the weighted average of the prices collected (or adjusted prices) (Table 2).

4. Diagnostic test

The surface ratios vary from market segment to market segment and over time. Sometimes in the same market segment information may indicate more than one surface ratio related to the same situation, usually because of a lack of the same
information and circulation of ratios taken from non-commercial sources.
In these circumstances it may be useful to compare the surface ratio from different sources to check the consequences of the use of fixed coefficients compared to those of the market, and vice versa, and of different fixed coefficients. This verification may also refer to ambiguous situations in which two or more amounts of the market surface ratio are simultaneously found. In general, in the situation in which the secondary surfaces are considered equal to the main one, it is no longer necessary to distinguish between them($\pi_i=1$). The difference between the estimated value in this situation and the corresponding estimated value without taking into account the secondary surfaces($\pi_i=0$) identifies the interval of variation calculated a priori in these two extreme hypotheses. The range of percentage variation $d_{j(0.1)}$ of the generic property, calculated by applying the[9] in both situations, is equal to:

$$d_{j(0.1)} = \frac{x_{01} + \sum_{i=2}^{n} x_{0i}}{x_{j1} + \sum_{i=2}^{n} x_{ji}} \cdot \frac{x_{j1}}{x_{01}} - 1. (17)$$

It is easily pointed out that the interval of variation is not dependent on the price of the comparable but it is linked to the physical surfaces of the properties. The index can assume positive and negative values depending on whether the estimated value, with the secondary surfaces equivalent to the main one, is greater or less than the value calculated without secondary surfaces for null surface ratios. A surface ratio verification index is the interval of percentage variation $D_{(\pi', \pi'')}$ of the sample of comparable properties detected that is proposed according to[19] and referred to the collected prices in the following way:

$$D_{(\pi', \pi'')} = \frac{X''_0 \cdot \sum_{j=1}^{m} \frac{P_j}{X''_j} - X'_0 \cdot \sum_{j=1}^{m} \frac{P_j}{X'_j}}{\sum_{j=1}^{m} P_j \cdot \frac{X''_j}{X'_j}}. (20)$$

This index can also be approximated by the simple average of the individual indices.

When the collection of the surface ratios is uncertain between two or more values, an index of practical interest is represented by the difference between the appraised values with different ratios. Indicated the sets of surface ratios $\pi'$ and $\pi''$, the percentage range of relative variation $d_{j(\pi', \pi'')}$ of the generic property can be then calculated by applying the[9] in the two hypotheses as follows:

$$d_{j(\pi', \pi'')} = \frac{X''_0}{X''_j} \cdot \frac{X'_j}{X'_0} - 1. (19)$$

it should easily be noted that the interval of variation is not dependent on the price of the comparable but is tied to the surface ratios. The index can take positive and negative values. The percentage range expresses the relative variability of the surface ratios tested. An index of verification of surface ratios is the relative percentage variation range $D_{(\pi', \pi'')}$ of the sample of comparable properties detected that is proposed according to[19] and reported to the collected prices in the following way:

$$D_{(\pi', \pi'')} = \frac{X''_0 \cdot \sum_{j=1}^{m} \frac{P_j}{X''_j} - X'_0 \cdot \sum_{j=1}^{m} \frac{P_j}{X'_j}}{\sum_{j=1}^{m} P_j \cdot \frac{X''_j}{X'_j}}. (20)$$

This index can also be approximated by the simple average of the individual indices.

It is possible to calculate two partial indices of practical interest. The first index refers to the range of variation between the appraised value with $\pi_i$ and the appraised value with $\pi_i=1$; the second index refers to the range of variation between the estimated value with $\pi_i$ and the value with $\pi_i=0$.

The first range of percentage variation $d_{j(\pi, 1)}$ of the generic property can then be calculated by applying the [9] in the two corresponding hypotheses as follows:

$$d_{j(\pi, 1)} = \frac{x_{01} + \sum_{i=2}^{n} x_{0i}}{x_{j1} + \sum_{i=2}^{n} x_{ji}} \cdot \frac{x_{j1}}{x_{01}} - 1. (21)$$

The second range of percentage variation $d_{j(\pi, 0)}$ of the generic property can be calculated by applying the [9] in the two corresponding hypotheses as follows:
The percentage range of variation $D_{(\pi,1)}$ of the collected sample of comparable properties can then be calculated according to [21] and referred to the collected prices in the following way:

$$D_{(\pi,1)} = \frac{(x_{01} + \sum_{i=2}^{n} x_{0i}) \cdot \sum_{j=1}^{m} \frac{P_j}{X_j} + \sum_{i=2}^{n} x_{0i} \cdot \frac{P_j}{X_j}}{\sum_{j=1}^{m} \frac{P_j}{X_j} \cdot X_j}.$$ \hspace{1cm} (22)

This index can also be approximated by the simple average of the individual indices.

The second percentage range of variation $d_{j(0,\pi)}$ of the generic property can then be calculated by applying the [9] in the two corresponding hypotheses as follows:

$$d_{j(0,\pi)} = \frac{X_j}{X_0} \cdot x_{j1} - 1.$$ \hspace{1cm} (23)

The percentage range of variation $D_{(0,\pi)}$ of the collected sample of comparable properties can then be calculated according to [23] and referred to the collected prices in the following way:

$$D_{(0,\pi)} = \frac{x_{01} \cdot \sum_{j=1}^{m} \frac{P_j}{X_j} - x_{02} \cdot \sum_{j=1}^{m} \frac{P_j}{X_j}}{\sum_{j=1}^{m} \frac{P_j}{X_j} \cdot X_j}.$$ \hspace{1cm} (24)

This index can also be approximated by the simple average of the individual indices.

The set of tests can be applied as a first step to the sample of comparable properties (Table 1). The tests for each comparable property are based exclusively on the surfaces of property; the tests referred to the sampling of properties can be referred to the weighted average of the collected prices (or adjusted prices) and to the simple average of the indices (Table 3).

The ranges of variation can be further proposed for surface ratios higher than one. The ranges of variation are presented as diagnostic indices that regard the collected data and surface ratios considered in the preliminary step of appraisal.

5. Coefficients

In the expertise the coefficient of a real estate characteristic is the ratio between the value that it is believed the property being appraised should have and the value as signed until that moment without taking into account other characteristics that are uncorrelated, or taking it into account in some way (in the first instance, they are considered subjective).

In the market survey, the coefficient of a property characteristic is the ratio between the price of a property $P_h$ that has this characteristic and the price of a property $P$ that does not have this characteristic (or does not possess the same level) other things being equal. The coefficient of the generic characteristic $\alpha_h$ (with $h=1, 2, ..., l$) is equal to:

$$\alpha_h = \frac{P_h}{P}.$$ \hspace{1cm} (25)

In the traditional form, the market value of the property being appraised is equal to the product of the unit value, the commercial area and the coefficients of the property being appraised according to [3]:

$$V = \nu \cdot \left( x_{01} + \sum_{i=2}^{n} x_{0i} \cdot \pi_i \right) \cdot \prod_{h=1}^{l} \alpha_h.$$ \hspace{1cm} (26)

For the real estate characteristics measured on nominal scale (absence, presence) there is only one coefficient for the characteristic calculated with the [25].

For real estate characteristics measured on ordinal scale, there are many coefficients as the number of levels of the characteristic minus one. The coefficient $\alpha_{h(f)}$ of the generic characteristic is given by the ratio between the price of a property $P_{(f)}$ with $f=1, 2, ..., g$ that possesses the characteristic at the $f$ level and the price of a property $P_{(1)}$ which has the characteristic on the first level:

$$\alpha_{h(f)} = \frac{P_{(f)}}{P_{(1)}}, \quad \alpha_{h(1)} = \frac{P_{(1)}}{P_{(1)}}, \quad \alpha_{h(2)} = \frac{P_{(2)}}{P_{(1)}}, \ldots, \alpha_{h(1)} = \frac{P_{(1)}}{P_{(1)}}.$$ \hspace{1cm} (27)
For the real estate characteristics measured on cardinal scale, the $\alpha_h$ coefficient of the generic characteristic considers the mode of $x_h$ of the property with the price $P_h$, the mode $x$ of the property with the price $P$ and the mode $x_0$ for the property being appraised, in the following way:

$$\alpha_h = 1 + \frac{P_h - P}{P} \cdot \frac{x_0 - x}{x_h - x}. \quad (28)$$

In the approaches based on the comparison of the data, the adjustment is set equal to the difference between the prices of properties with and without the characteristic (or with different level of the characteristic) assuming that all the other characteristics are identical. The market value $V$ of the property to be estimated according to the comparison function[4] is equal to the adjusted price calculated in the following way:

$$V = P + \sum_{h=1}^l (P_h - P) = P \cdot \left(1 + \sum_{h=1}^l \alpha_h + 1\right). \quad (29)$$

For the property of commercial surface $X_0$ being appraised, the price $P$ is equal to:

$$P = \nu \cdot X_0. \quad (30)$$

And the absolute error $E$ can be measured by the difference between the market value according to the expertise and the market value according to the market approach and, respectively, with[26] and[29], yielding:

$$V = \nu \cdot X_0 \cdot \prod_{h=1}^l \alpha_h - \nu \cdot X_0 \cdot \left(1 + \sum_{h=1}^l \alpha_h + 1\right). \quad (31)$$

The percentage error $e$ can be referred to the property being appraised equal to:

$$e = \frac{E}{P} = \prod_{h=1}^l \alpha_h - \sum_{h=1}^l \alpha_h + l - 1. \quad (32)$$

The percentage error can be studied by placing $\alpha_1 = \alpha_2 = \ldots = \alpha_h = \alpha$ thereby obtaining by[32]:

$$e = \alpha^l - l \cdot \alpha + l - 1, \quad (33)$$

that is a polynomial of degree $l$ in $\alpha$. In the observable field usually between 0.8 and 1.2, the error percentage increases more than proportionally and asymmetrically respect to the unit coefficient and is greater the more numerous the coefficients considered are (Exhibit 1).

In general for $\alpha>1$ and for $\alpha<1$ the error is positive ($E>0$ and $e>0$). In practice, any alternation of coefficients major and minor than 1 can determine compensation.

6. Conclusions

The real estate expertise does not obey to definitions, limits and rules approved, checked and verified. For the expertise is not possible to define a best practice as a minimum acceptable benchmark.

In Italy the real estate expertise is based on market quotation of the surface ratios and unit coefficients. Quotes are related to wide areas of the market with approximate boundaries, identified by zone (central, semi-central, peripheral, etc.) destination (residential, commercial, etc.) and real estate typology (new, used, refurbished, etc.). The sources of real estate quotes are numerous and heterogeneous: the interview of technical intermediaries, announcements in economic newspapers and magazines, lists of judicial auctions, brochures. The collection in the market area of secondary surface ratios in respect to the main one can be difficult, because many contracts are made lump sum and direct interviews of operators frequently slow to different outcomes (Salvo, 2001). In practice, often folded improperly on the fixed coefficients from manuals and circulars of public administration. The unit coefficients of non-surface characteristics are different from those typically detected in commercial handbooks, where are
reported for a single feature in the form of an interval between a minimum and a maximum.
In the expertise it is not possible to introduce any correction to counter the effects of the use of the surface ratios and of the coefficients and much less the use of quotes instead of market prices.
While in the expertise the omission of surface ratios would lead to an unacceptable error of overestimation or underestimation, since the market value of the property increases if the ratios of the secondary surfaces increase, in the valuation procedures based on the market comparison, the surface ratios operate on the differences between the secondary surfaces, rather than on the entire surfaces. Their impact on the outcome of the valuation is thereby minor, and sometimes negligible in particular when the secondary surfaces of the property being appraised are practically equal to those of the comparable property.
The study proposes a series of diagnostic indices that provide a priori information about changes in the market value due to increases or decreases of the surface ratios without to know their amount.
The surface ratios, recognized in the market, vary from segment to segment of the market and over time, compared with fixed coefficients whose sources are usually static. Under these conditions it may be useful to compare the surface ratios with fixed coefficients of the various sources and their effect on the result of the valuation, or between fixed coefficients from different sources.
For the purposes of comparison it has been proposed a series of test that compares the variations in the value induced by changes in the surface ratios, examining the extreme and intermediate terms. The interval expresses the variability induced by the tested surface ratios. The elementary index does not depend on prices of comparable properties. For the coefficients of the real estate characteristic different from surface a measure of the error related to their use in the expertise is proposed, as in the valuations carried out according to the valuation standard the coefficients do not find any application.

References:

### Table 1 - Data of the property units

<table>
<thead>
<tr>
<th>Price and characteristic</th>
<th>Unit 1</th>
<th>Unit 2</th>
<th>Unit 3</th>
<th>Subject0</th>
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</thead>
<tbody>
<tr>
<td>Total Price (euro)</td>
<td>355,000.00</td>
<td>366,000.00</td>
<td>318,000.00</td>
<td>-</td>
</tr>
<tr>
<td>Date</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>Main surface (mq) (i=1)</td>
<td>121.3</td>
<td>128.6</td>
<td>113.0</td>
<td>115.0</td>
</tr>
<tr>
<td>Balconies (mq) (i=2)</td>
<td>12.0</td>
<td>12.3</td>
<td>9.9</td>
<td>10.3</td>
</tr>
<tr>
<td>Terraces (mq) (i=3)</td>
<td>28.0</td>
<td>33.5</td>
<td>31.9</td>
<td>29.1</td>
</tr>
<tr>
<td>Not surface characteristics</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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</tbody>
</table>

### Table 2 - A priori diagnostic indices

<table>
<thead>
<tr>
<th>Unit</th>
<th>(K_j) Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>(\frac{121.3}{115.0}, \frac{10.3+29.1}{12.0+28.0}) 1,0390</td>
</tr>
<tr>
<td>Unit 2</td>
<td>(\frac{128.6}{115.0}, \frac{10.3+29.1}{12.3+33.5}) 0,9620</td>
</tr>
<tr>
<td>Unit 3</td>
<td>(\frac{113.0}{115.0}, \frac{10.3+29.1}{9.9+31.9}) 0,9262</td>
</tr>
<tr>
<td>Simple index</td>
<td>(\frac{1,0390+0,9620+0,9262}{3}) 0,9757</td>
</tr>
<tr>
<td>Weighted index</td>
<td>(\frac{1,0390 \cdot 355.000 + 0,9620 \cdot 366.000 + 0,9262 \cdot 318.000}{355.000 + 366.000 + 318.000}) 0,9773</td>
</tr>
</tbody>
</table>

### Table 3 - Diagnostic indexes and a priori test

<table>
<thead>
<tr>
<th>Unit</th>
<th>(d_{j(0.1)})%</th>
<th>(d_{j(x1)})%</th>
<th>(d_{j(0.5)})%</th>
<th>(d_{j(\pi=0.3;0.5,\pi'=0.4;0.8)})%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>0,966</td>
<td>-0,093</td>
<td>1,060</td>
<td>0,159</td>
</tr>
<tr>
<td>Unit 2</td>
<td>-0,998</td>
<td>-0,283</td>
<td>-0,717</td>
<td>-0,186</td>
</tr>
<tr>
<td>Unit 3</td>
<td>-1,993</td>
<td>-0,231</td>
<td>-1,767</td>
<td>-0,352</td>
</tr>
<tr>
<td>Simple index</td>
<td>-0,675</td>
<td>-0,202</td>
<td>-0,474</td>
<td>0,975</td>
</tr>
<tr>
<td>Weighted index</td>
<td>-0,631</td>
<td>-0,202</td>
<td>-0,431</td>
<td>-0,119</td>
</tr>
</tbody>
</table>
Exhibit 1 – Percentage error of coefficients