INVESTING IN PHOTOVOLTAICS IN A SMART GRID CONTEXT: THE PROSUMER'S PERSPECTIVE

CHIARA D'ALPAOS
DICEA (Department of Civil, Architectural and Environmental Engineering)
University of Padova
Via Venezia 1, 35131, Padova
Italy
chiara.dalpaos@unipd.it

MARINA BERTOLINI
DICEA (Department of Civil, Architectural and Environmental Engineering)
University of Padova
Via Venezia 1, 35131, Padova
Italy
marina.bertolini.ita@gmail.com

MICHELE MORETTO
DSEA (Department of Economics and Management)
University of Padova
Via del Santo 22, 35123, Padova
Italy
michele.moretto@unipd.it

Abstract: - In Italy - and in the countries working for GHG emissions reduction - the last decade was characterized by a large development of distributed generation power plants. Private investments have been heavily boosted by monetary incentives, such as guaranteed feed-in tariffs, especially in the photovoltaic sector. These incentives, on the one hand, allowed for developing photovoltaic technology faster and guaranteed payoffs for huge initial investments, but on the other hand they determined new critical issues for the design and management of the overall energy system and the electric grid especially in the presence of discontinuous sources. Contingent problems that affect local grids (e.g. inefficiency, congestion rents, power outages, etc.) may be solved by the implementation of a "smarter" electric grid. Smart grids represent de facto the evolution of electrical grids and their implementation is challenging the electric market organization and management.
The main feature of smarts grid is the great increase in production and consumption flexibility. Smart grids give de facto producers and consumers, the opportunity to be active in the market and strategically decide their optimal production/consumption scheme. The paper provides a theoretical framework to model the prosumer's decision to invest in a photovoltaic power plant, assuming it is integrated in a smart grid. To capture the value of managerial flexibility, a real option approach is implemented.

Key-Words: - Photovoltaic, smart grid, investment under uncertainty, investment timing, flexibility, optimal sizing, prosumer

1 Introduction

Growing concern about GHG emissions and future availability of traditional energy sources motivated national governments to promote renewable energy distributed generation.

In Italy, the last decade was characterized by a large development of distributed generation power plants, and particularly biomass and photovoltaic power plants: private investments in these sectors have been boosted through incentives, that made them particularly attractive for both institutional investors and (small) private investors.

In particular, private participation was favored in the photovoltaic sector, by the implementation of high feed in tariff remuneration schemes [1] . These incentives, on the one hand, allowed for a faster development photovoltaic technology faster, by guaranteeing free-risk payoffs for large initial investments, but on the other hand, they caused an increase in public expenditure, due to both monetary disbursement to pay incentives and to additional system costs born to manage a significant number of energy production sources not efficiently integrated. Photovoltaic plants, actually, have a relevant responsibility for grid costs increase: in 2012, the installed photovoltaic capacity reached a power amount of more than 16,4 GW, through 478.331 plants, that generated an increase in power of 28,5% with respect to 2011 [1][2]. Total power amount and fragmented subdivision of the plants have a considerable impact on the electric system, provided that the grid is not design to support peripheral inflows, and especially those instable coming from unpredictable production.

It is undeniable that photovoltaics shall have a considerable role in future energy supply in Italy, given the particularly favorable geographical conditions. The increasing number of investments in photovoltaic power plants, as other discontinuous and distributed energy production sources, generated problems that affected local grids (e.g. inefficiency, congestion rents, power outages, etc.), part of which can be solved by the implementation of a “smarter” electric grid. Smart grids represent de facto the evolution of electrical grids and their implementation is challenging the electric market organization and management. To favor the development of photovoltaic energy production in a sustainable way, the electric system need to be balanced and efficiently managed.

This objective could be reached by implementing the so called Smart Grid. The Smart Grid environment allows for an instantaneous interaction between the agent and the grid: depending on its needs, the grid can send signals (through prices) to the agents, and the agents have the possibility to respond to the signals and obtain a monetary gain. In this way, the system can allow for better integration of the renewables - that contribute to keep the grid stable - and for a photovoltaic development in the absence of costly monetary incentives. In addition, the possibility to gain revenues by direct grid management the investor has more positive flows to compute in investment evaluation and this accelerates the process towards private investments sustainability.

The main feature of the smart grid is the great increase in production and consumption flexibility. Smart grids give de facto producers and consumers, the opportunity to be active in the market and, eventually, to match their needs with the neighbors’ ones in a complementary way, through the implementation of a micro smart grid. smart grids generate managerial flexibilities that prosumers (i.e. subjects that both produces and consumes electric energy) can exploit when deciding to invest in photovoltaics.

This flexibility gives prosumers the option to strategically decide the optimal production/consumption scheme and can significantly contribute to energy saving and hedging of investment risk. In other words, if optimally exercised, operational flexibility can be economically relevant and its value is strongly related to the prosumers’s ability to decide their investment strategy and planned course of action in the future, given then-available information.

Traditional capital budgeting techniques fail to capture the value of this managerial flexibility.

It is widely recognized that the Net Present Value rule (or Discounted Cash Flow Analysis) fails because it cannot properly capture managerial flexibility to adapt and revise later decisions in response to unexpected market events.

As new information arrives and uncertainty about future cash flows is gradually resolved, management may have valuable flexibility to alter its initial operating strategy in order to capitalize on favorable future opportunities. The real option approach, by

[1] The "Conto Energia" programme, started in Italy in 2005.

[2] Other non dispatchable energy sources - wind – provide for less than half of photovoltaic power capacity (8,1 GW) and this power is concentrated in 807 plants.

endogenizing the optimal operating rules and explicitly capturing the value of flexibility, provides contextually for a consistent treatment of investment risk. The paper provides a theoretical framework to model the prosumer's decision to invest in a photovoltaic power plant, assuming it is integrated in a smart grid, The paper is organized as follows. Section 2 briefly defines the model setting by defining potential prosumers behavior and investment strategy in a smart grid context. Section 3 and 4 provide the model on the optimal investment size timing respectively. Section 5 provides numerical simulations to illustrate theoretical results. Section 6 concludes.

2 The smart grid and the prosumer

A critical aspects deriving from the pervasive presence of non dispatchable sources is that energy flows on local grid segments become highly unpredictable. Photovoltaic production varies within the day, suffers from seasonality, and can be reduced or interrupted depending on weather conditions. As a consequence, it is not possible to rely exclusively on Photovoltaics to supply energy demand. As long as there is a lack in photovoltaic energy inflows, it is required power capacity of stable power plants, that are always available, in order to balance the grid. Capacity service is therefore a considerable cost in the actual management of the grid.

Small or medium producers, domestic and industrial consumers and a growing number of prosumers interact with respect to the local grid segment and generate energy from photovolatics.

Different agents and loads should be organized and coordinated in order to locally balance the grid, promoting energy consumption in short distance (in order to limit losses due to Joule effect) and thus reducing negative effects on the entire system.

In our setting, we consider a photovoltaic power plant which can react to smart grid signals.

the fundament assumption is that the grid able to deliver information to the grid manager, to consumers, producers and prosumers that react to information acquired. Information from the grid to prosumers are delivered through prices, determined according to instantaneous technical needs registered by the grid, (e.g. local balancing needs, congestions, provision of ancillary services). The different agents connected to the grid could contribute to shape loads patterns depending on grid necessities, provide ancillary services and react instantaneously to grid requests.

In this paper, we consider an investment decision in a photovoltaic power plant from the prosumer's perspective.

The prosumer whenever connected to the smart grid can *de facto* decide its own consumption and production schemes. The prosumer can decide to change its energy consumption path, but also the source of consumed energy: the prosumer can switch from photovoltaic production to main grid provision constrained to a fix-price contract. The prosumer can decide whether to directly use energy for self-consumption to satisfy its individual demand, or to sell it to the grid when receiving a proper signal from the grid.

3 The Model

We consider a consumer that must decide whether to invest in a photovoltaic (PV) plant. The decision is based on the possibility to take advantage of consuming self-produced energy, selling energy to the grid when it is convenient, holding the possibility to call for energy from the main grid at a contractually fixed price.

In this respect, the consumer's objective is cost minimization. Being interested to reduce energy costs, the agent wants to evaluate the opportunity of investing in a private energy source to be able to use self-production if buying energy from the main market becomes too expensive, and given this assumption we expect that agent's energy consumption needs will guide investment decisions.

Let introduce some simplifying assumptions:

1) The agent's demand of energy per unit of time (day, week, month, year, etc.) is normalized to one (MWh); the agent can shift the consume within the time period, but not between different time periods;
2) We indicate with α_1 the expected production (expressed in MWh), of the power plant per unit of time, α_2 is the energy bought from the main grid and $\xi \in [0,1]$ is the expected production quota used for self-consumption[3];
3) For the sake of simplicity, we assume that the prosumer makes the decision to consume the self-produced energy or to sell it in the market at the beginning of each period. That means that the prosumer receives price information at the beginning of the time interval t, and he takes the

[3]This implies that $(1-\xi)$ is the quota the agent can sell on the energy market.

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decision between consuming his own energy or buying it from the market;

4) No batteries are included in the model: this simplifying assumption reduces managerial flexibilities, since energy must be used as produced. Further studies on this kind of investment shall be performed including batteries, and evaluating their value considering new opportunities for the producer;
5) the grid allows for exchange of energy flows and information on instantaneous energy prices.

Then, by Assumptions 1-3, if we indicate by *c* the per-unit market price of energy, *a* the per-unit cost paid to generate energy from the PV plant where v the per unit selling price, we can define the agent's net cost of energy per unit of time as:

$$\begin{aligned} C = \min\{c - \alpha_1(v-a),\ \xi\alpha_1 a + (1-\xi\alpha_1)c - (1-\xi)\alpha_1(v-a)\} \\ \min\{c - \alpha_1(v-a),\ c - \alpha_1(v-a) + \xi\alpha_1(v-c)\} \end{aligned} \quad (1)$$

where $\xi\alpha_1 + \alpha_2 = 1.$

The first term is the net cost without self-consumption, the second term indicated the net cost in presence of self-consumption. Notice that the energy costs sustained by the agent is modified by the possibility of choosing between selling energy on the market or consuming self-produced energy: in the first case, the agent pays *c* and earns α_1, minus the cost of production of α_1, by selling the entire photovoltaic energy production on the market; in the second case, part of the energy produced from the photovoltaic plant is used for self-consumption ($\xi\alpha_1$, at the production cost a), while part of the energy α_2 is bought in the market at the price *c* and the energy produced but not self-consumed is sold on the market at price v. It is always possible for the agent to switch from one mix to the other, depending on energy prices.

Whenever $v > c$ the agent minimizes energy costs selling outside all the production, i.e. $\xi = 0.$ Otherwise, when $v < c$ the agent minimizes energy cost applying for a positive self-consumption quota. Further, since the photovoltaic production is technically limited and it can't fully satisfied the daily energy demand of the agent, we must impose: $\xi\alpha_1 \le \overline{\alpha} < 1$ where $\overline{\alpha}$ is the maximum energy demand's quota that can be satisfied by the photovoltaic production[4]. Since $\overline{\alpha}$ is given by environmental conditions we get $\xi = \frac{\overline{\alpha}}{\alpha_1} < 1.$ Dimensions higher than α_1 allow for an increase in the photovoltaic production that can be sold in the market.

We assume that the market price of energy $v(t)$, is uncertain, since it reflects instantaneous grid needs at local level, while we consider as constant both the marginal cost of internal production of energy *a* and the buying price *c*. Then, we can re-write equation (1) as follows:

$$C(t) = c - \alpha_1(v(t) - a) + \min[\xi\alpha_1(v(t) - c),\ 0] \quad (2)$$

Finally, we introduce the following assumption that says that the photovoltaic plant cannot be used to make profits.

$c - \alpha_1(v(t) - a) > 0$ for all $t > 0,$

This condition is related to our setting: in the prosumer's perspective the investment is made to reduce energy cost[5].

Dynamic uncertainty in the market price of energy, $v(t),$ can be described by a geometric Brownian motion[6]:

$$dv(t) = \gamma v(t)dt + \sigma v(t)dz(t) \quad (3)$$

where $dz(t)$ is the increment of a Wiener process (or Brownian motion) uncorrelated over time, σ is the volatility of the market price and γ is the drift, lower than the risk-free discount rate *r*, i.e., $\gamma \le r$.[7] The process $dz(t)$ satisfies the conditions that $E(dz) = 0$ and $E(dz^2) = dt.$ Therefore,

[4] This is also connected to the impossibility to storage energy we formulated as the premises: if we had batteries or other storing possibilities, the process should have been developed differently.

[5] It is worth note that $v(t)$ can assume any value including that of prices that makes $\alpha_1(v-a)$ higher than c. In this case installing the plant for self-consumption will be not profitable. We assume that the probability for this to happen is very low and negligible in our analysis.

[6] Alternative dynamic frameworks could be used, such as mean reverting process. Conclusions would not change, but a closed solution would not be possible.

[7] Alternatively, we could use an interest rate that includes an appropriate adjustment for risk and the expectation operator with respect to a distribution of c adjusted for risk neutrality (see [2]; [3]; [4]).

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$E(dv(t))/v(t) = \gamma dt$ and $E(dv(t)/v(t))^2 = \sigma^2 dt,$ i.e., starting from the initial value $v_0,$ the random position of the price $v(t)$ at time $t > 0$ has a normal distribution with mean $v_0 e^{\gamma t}$ and variance $v_0^2(e^{\sigma^2 t} - 1),$ which increases as we look further and further into the future. Notice that the process has no memory (i.e., it is Markovian), and hence *i*) at any point in time $t,$ the observed $v(t)$ is the best predictor of future energy price, *ii*) $v(t)$ may next move upwards or downwards with equal probability, *iii*) the random position of $v(t)$ has a lognormal distribution.

3.1 The value of flexibility

Note that from (2), once installed, the photovoltaic plant allows for a flexible choice between two opposite scenarios:

- If $v(t) > c$, the prosumer decides to satisfy the entire energy demand by the national grid provision, and to sell the energy produced by the PV plant in the market;
- If $v(t) < c$, the prosumer decides to self-consume the energy produced by the PV plant.

Therefore, for any given $\xi > 0$, the value of flexibility to the prosumer is given by the solution of the following free boundary dynamic programming problems ([5]; [6]; [7]):

$$\Gamma C^0(v(t);\xi,\alpha_1) = -[c - \alpha_1(v(t) - a) + \xi\alpha_1(v(t) - c)], \textit{for } v(t) < c \quad (4.1)$$

and

$$\Gamma C^1(v(t);\xi,\alpha_1) = -[c - \alpha_1(v(t) - a)], \textit{for} \quad v(t) > c, \qquad (4.2)$$

where Γ indicates the differential operator: $\Gamma = -r + \gamma v \frac{\partial}{\partial v} + \frac{1}{2}\sigma^2 v^2 \frac{\partial^2}{\partial v^2}$. The solution of the differential equations (4.1) and (4.2) requires to impose the following boundary conditions:

$$\lim_{v\to 0}\left\{C^0(v(t);\xi,\alpha_1) - \frac{(1-\xi\alpha_1)c + \alpha_1 a}{r} + \frac{(1-\xi)\alpha_1 v(t)}{r-\gamma}\right\} = 0 \quad (5.1)$$

and

$$\lim_{v\to\infty}\left\{C^1(v(t);\xi,\alpha_1) - \frac{c + \alpha_1 a}{r} + \frac{\alpha_1 v(t)}{r-\gamma}\right\} = 0, \qquad (5.2)$$

where $\frac{(1-\xi\alpha_1)c+\alpha_1 a}{r} - \frac{(1-\xi)\alpha_1 v(t)}{r-\gamma}$ indicates the present value of operating costs while the prosumer use the PV for self-consumption and $\frac{c+\alpha_1 a}{r} - \frac{\alpha_1 v(t)}{r-\gamma}$ is the present value of operating costs while selling energy in the market. Then, from the assumptions and the linearity of the differential equations (4.1) and (4.2), using (5.1) and (5.2), we get:

$$C^0(v(t);\xi,\alpha_1) = \frac{(1-\xi\alpha_1)c + \alpha_1 a}{r} - \frac{(1-\xi)\alpha_1 v(t)}{r-\gamma} + \hat{A}v(t)^{\beta_1}$$

and:

$$C^1(v(t);\xi,\alpha_1) = \frac{c + \alpha_1 a}{r} - \frac{\alpha_1 v(t)}{r-\gamma} + \hat{B}v(t)^{\beta_2},$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive roots of the characteristic quadratic equation [8]: $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) + \gamma\beta - r$.

By combining C^0 and $C^1,$ we get the net discounted flows of costs that take into account the value of reversing the vertical setting, i.e. the value of flexibility:

$$C(v(t);\xi,\alpha_1) = \begin{cases} \frac{(1-\xi\alpha_1)c+\alpha_1 a}{r} - \frac{(1-\xi)\alpha_1 v(t)}{r-\gamma} + \hat{A}v(t)^{\beta_1} & \text{if } v(t) < c \\ \frac{c+\alpha_1 a}{r} - \frac{\alpha_1 v(t)}{r-\gamma} + \hat{B}v(t)^{\beta_2} & \text{if } v(t) > c. \end{cases} \qquad (6)$$

The additional terms $\hat{A}v(t)^{\beta_1}$ and $\hat{B}v(t)^{\beta_2}$ indicate respectively the value of the option to switch from self-consumption to pure energy selling plus the value of the option to switch the other way round. Therefore, the constants $\hat{A}$ and $\hat{B}$ must be negative. Finally, under the exclusively selling option, the impossibility of making profits rules out any closure option.

To evaluate the constants the value matching and the smooth pasting conditions must be satisfied at $v(t) = c$. That is:

$$C^0(c;\xi,\alpha_1) = C^1(c;\xi,\alpha_1),$$

and

$$C_v^0(c;\xi,\alpha_1) = C_v^1(c;\xi,\alpha_1),$$

whose solutions give:

$$\begin{cases} \hat{B} = \xi\alpha_1 cB \equiv \xi\alpha_1 c\frac{1}{(r-\gamma)}\frac{r-\gamma\beta_2}{r(\beta_2-\beta_1)} \\ \hat{A} = \xi\alpha_1 cA \equiv \xi\alpha_1 c\frac{1}{(r-\gamma)}\frac{r-\gamma\beta_1}{r(\beta_2-\beta_1)}. \end{cases} \tag{7}$$

Note that the constants $\hat{A}$ and $\hat{B}$ are always non-positive and both are linear in $\xi\alpha_1 c$.

3.2 The Optimal PV size

The prosumer can always satisfy its energy demand, by buying it in the market at a price c. That is, the agent will take the decision to invest only if the value of the opportunity to higher generates a payoff greater than the status quo condition, i.e. buying the energy from the grid.

The value of the investment is terms of energy cost reduction is given by:

$$\Delta C(v(t);\xi,\alpha_1) \equiv \frac{c}{r} - C(v(t);\xi,\alpha_1) =$$

$$= \begin{cases} \frac{\xi\alpha_1 c-\alpha_1 a}{r} + \frac{(1-\xi)\alpha_1 v(t)}{r-\gamma} - \hat{A}v(t)^{\beta_1} & \text{if } v(t) < c \\ -\frac{\alpha_1 a}{r} + \frac{\alpha_1 v(t)}{r-\gamma} - \hat{B}v(t)^{\beta_2} & \text{if } v(t) > c. \end{cases} \tag{8}$$

Where:

1) whenever $v(t) < c$, the prosumer still buys part of the energy from the market at price c, but can self-consume part of the energy obtaining a decrease in energy costs; the prosumer can also decide to sell energy on the market, and flexibility opportunity is still present;

2) whenever $v(t) > c$, the prosumer decides to buy the energy needed from the market and to sell the entire quota of energy self-produced at a price v, still mantaining the possibility to switch to self-consumption.

The investor's problem is to choose the optimal PV size (in terms of production per unit of time) by maximizing (8) with respect to α_1, net of the investment cost. As we focus on an agent that mainly invests for self-consumption, the optimal size is given by:

$$\alpha_1^* = \arg\max\left[NPV^0(v(t);\alpha_1)\right] \tag{9}$$

where $NPV^0(v(t);\alpha_1) \equiv \Delta C^0(v(t);\xi,\alpha_1) - I(\alpha_1)$, and $I(\alpha_1)$ represents the direct sunk costs of photovoltaic plant installation.

$I(\alpha_1)$ deserves some comments. The cost of a PV plant is generally related to the maximum power at standard conditions (power at peak, that can be measured in kWp [8]). This cost includes the costs of the panels, the components such as inverters and cables, the need for surface etc. However, referring to the path of the agent's instantaneous energy demand and the main characteristics of the production curve, it is possible to switch from kWp to the kWh per time interval t.[9]

We model $I(\alpha_1)$ as a Cobb-Douglas with increasing cost-to-scale and we translate it into a total cost function, which is quadratic in α_1, multiplied by a constant K which represents all the costs, including inefficiencies and transformations from the measure of power to the corresponding forecasted/expected production, i.e.:[10]

$$I(\alpha_1) = \frac{K}{2}\alpha_1^2. \tag{10}$$

Substituting (11) into (10) and given the constraint $\xi = \frac{\overline{\alpha}}{\alpha_1}$, we get:

$$\begin{aligned} NPV^0(v(t);\alpha_1) &\equiv \Delta C^0(c(t);\xi,\alpha_1) - I(\alpha_1) \\ &= \frac{\xi\alpha_1 c - \alpha_1 a}{r} + \frac{(1-\xi)\alpha_1 v(t)}{r-\gamma} - \hat{A}v(t)^{\beta_1} - \frac{K}{2}\alpha_1^2 \\ &= \frac{\overline{\alpha}c - \alpha_1 a}{r} + \frac{(\alpha_1 - \overline{\alpha})v(t)}{r-\gamma} - \overline{\alpha}cAv(t)^{\beta_1} - \frac{K}{2}\alpha_1^2 \end{aligned}$$

The plant's optimal size is given by:

$$\alpha_1 \equiv \alpha_1(v(t)) = \max\left[\frac{\frac{v(t)}{(r-\gamma)} - \frac{a}{r}}{K},\ 0\right] \tag{11}$$

[8] kWp stands for "kilowatt peak", and indicates the nominal power of the plant (or of the panel). It is calculated with respect to specific standard environmental conditions: 1000 W/m2 light intensity, cell positioned at latitude 35°N, reaching a temperature of 25°C.

[9] Notice that $\alpha_1 = \int_{t_0}^{t_1}[l(t)+s(t)]dt$, where $s(t)$ is the instantaneous energy sold and (t_1,t_0) is the real production period of the plant. If we set $p^* = \arg\max[l(t)+s(t)]$ and the output load of a panel is sufficiently stable in (t_1,t_0) we can approximate α_1 by $p^*(t_1-t_0)$.

[10] The sunk costs to build up the PV plant is assumed quadratic only for the sake of simplicity. None of the results will be altered if the investment cost is of a more general type $I(\alpha) = K\alpha^{\delta}$ with $\delta > 1$.

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i.e. the optimal dimension is given by the ratio between the expected discounted flow of revenues by an additional unit of capacity (i.e. kWh), divided by the marginal cost of an additional kWh.

Note also that α_1 is a function of the current value of the selling price $v(t)$, that should be sufficiently high to make worthy investing in the plant. In particular if $v(t)$ is lower than $\hat{v} = \frac{r-\gamma}{r} a$ it is better not to invest and buy energy form the main grid, while, as $v(t)$ increases, $\alpha_1 (>0)$ increases and the prosumer decides to install a plant of capacity that exceeds its needs.

By substituting (11) into $NPV^0(v(t);\alpha_1)$ and re-arranging the state-contingent net present value of a plant that contemplates self-consumption with the option to sell all the production to the grid, we get**:**

$$NPV^0(v(t);\alpha_1) = \\ = \frac{1}{K}\left(\frac{v(t)}{r-\gamma} - \frac{a}{r}\right)^2 + \frac{\bar{\alpha}c}{r} - \frac{\bar{\alpha}v(t)}{r-\gamma} - \bar{\alpha}cAv(t)^{\beta_1} \text{ for } v(t) > \hat{v} \tag{12}$$

4 The optimal investment timing

In this section we determine the value of the option to invest in timing. the PV plant as well as its optimal investment.

When $v(t) < c$, the option of waiting to invest is positive, then the value, $F(v(t))$, is given by the solution of the following free boundary dynamic programming problem:

$$\Gamma F(v(t)) = 0, \qquad \text{for } \hat{v} < v(t) < v^* \tag{13}$$

where v^* is the threshold that triggers the investment in the PV plant.[11]. It is straightforward that the option value to acquire the PV system is increasing in energy market price $v(t)$: when $v(t)$ tends to zero, the option value tends to zero either. Then, the solution of the differential equation (13) requires the following boundary condition: $\lim_{v \to 0} F(v(t)) = 0$.

By the linearity of the differential equation (13) and using the boundary condition, the value of the option to invest becomes:

$$F(v(t)) = Mv(t)^{\beta_1}. \tag{14}$$

where $\beta_1 > 1$ is the positive root of $\Phi(\beta)$ and M is a constant to be determined. In order to determine the constant M and the optimal trigger v^*, according to the standard literature on optimal investment timing, we have to impose to boundary conditions (matching value and smooth pasting conditions), i.e.:

$$F(v^*) = NPV^0(v^*, \alpha_1^*(v^*)), \tag{15.1}$$

$$F'(v^*) = NPV_v^0(v^*, \alpha_1^*(v^*)), \tag{15.2}$$

where (15.2) follows from $NPV_\alpha^0(c^*, \alpha_1^*(v^*)) = 0.$[12]

Furthermore, a necessary condition to induce the agent to invest as a prosumer is that the optimal entry trigger v^* is lower than c. In other words, at the investment time the price of energy must be higher than the selling price. Otherwise it would be optimal to switch immediately to selling entirely the energy produced.

Substituting (12) and (14) in (15.1-2) we obtain:

$$\frac{\beta_1 - 2}{\beta_1} y^2 - \frac{\beta_1 - 1}{\beta_1}\left(2\frac{a}{r} + \bar{\alpha}K\right) y + \left[\left(\frac{a}{r}\right)^2 + \frac{\bar{\alpha}c}{r}K\right] = 0 \tag{16}$$

where $y = \frac{v^*}{r-\gamma}$. The optimal trigger is given by the solution of (16):

$$Mv^{*\beta_1} = \frac{1}{K}\left(\frac{v^*}{r-\gamma} - \frac{a}{r}\right)^2 + \frac{\bar{\alpha}c}{r} - \frac{\bar{\alpha}v^*}{r-\gamma} - \bar{\alpha}cAv^{*\beta_1}$$

$$M\beta_1 v^{*\beta_1 - 1} = \frac{2}{K}\left(\frac{v^*}{r-\gamma} - \frac{a}{r}\right)\frac{1}{r-\gamma} - \frac{\bar{\alpha}}{r-\gamma} - \bar{\alpha}c\beta_1 Av^{*\beta_1 - 1}$$

imposing $y = \frac{v^*}{r-\gamma}$

$$\frac{1}{K}\left(y - \frac{a}{r}\right)\left[\frac{a}{r} - \frac{\beta_1 - 2}{\beta_1} y\right] = \frac{\bar{\alpha}c}{r} - \frac{\beta_1 - 1}{\beta_1}\bar{\alpha}y$$

$$y\frac{a}{r} - \frac{\beta_1 - 2}{\beta_1} y^2 - \left(\frac{a}{r}\right)^2 + \frac{\beta_1 - 2}{\beta_1}\frac{a}{r} y = \frac{\bar{\alpha}c}{r}K - \frac{\beta_1 - 1}{\beta_1}\bar{\alpha}Ky$$

$$f(y) = \frac{\beta_1 - 2}{\beta_1} y^2 - \frac{\beta_1 - 1}{\beta_1}\left(2\frac{a}{r} + \bar{\alpha}K\right) y + \left[\left(\frac{a}{r}\right)^2 + \frac{\bar{\alpha}c}{r}K\right] = 0$$

[11] If $v^* < v(t)$ it is optimal for the agent to invest immediately (i.e., the agent follows the *NPV* rule and the value of the option to wait is null).

[12] $F'(c^*) = NPV_c^{VI}(c^*, \alpha^*(c^*)) + NPV_\alpha^{VI}(c^*, \alpha^*(c^*))\frac{d\alpha^*}{dc}$.

We have two alternatives.

1) If $\beta_1 - 2 > 0$ then the parabola has an upside concavity, and if calculated in $y = \frac{a}{r}$ we obtain:

$$f(\frac{a}{r}) = -\frac{\beta_1 - 1}{\beta_1}(\overline{\alpha}K)(\frac{a}{r}) + \frac{\overline{\alpha}c}{r}K$$
$$= \frac{\overline{\alpha}}{r}K\left[c - \frac{\beta_1 - 1}{\beta_1}a\right] > 0$$

moreover

$$f'(y) = 0 \rightarrow \frac{\beta_1 - 2}{\beta_1}y - \frac{\beta_1 - 1}{\beta_1}\left(2\frac{a}{r} + \overline{\alpha}K\right) = 0$$
$$\tilde{y} = \frac{\frac{\beta_1 - 1}{\beta_1}\left(2\frac{a}{r} + \overline{\alpha}K\right)}{\frac{\beta_1 - 2}{\beta_1}} = \frac{\beta_1 - 1}{\beta_1 - 2}\left(2\frac{a}{r} + \overline{\alpha}K\right) > \frac{a}{r} > 0$$
$$f(\tilde{y}) = \left[(\frac{a}{r})^2 + \frac{\overline{\alpha}c}{r}K\right] > 0$$

No solutions are found in this case.
We can interpret this result by saying that if the option to sell energy and participate to the market is too high, then the prosumer becomes a PV producer and keeps buying energy from the grid, and maintaining as well the possibility to switch to self consumption if selling prices decreases.

2) If $\beta_1 - 2 < 0,$ the parabola has a downside concavity. Then:

$$f(\frac{a}{r}) > 0$$

$$\tilde{y} = \frac{\beta_1 - 1}{\beta_1 - 2}\left(2\frac{a}{r} + \overline{\alpha}K\right) < 0$$

$$f(\tilde{y}) = \left[(\frac{a}{r})^2 + \frac{\overline{\alpha}c}{r}K\right] > 0$$

and the positive solution, $y^* > \frac{a}{r}$, must be higher than c, otherwise we must use the other NPV formula to find out α_1.

$$y^* = \frac{\frac{\beta_1 - 1}{\beta_1}\left(2\frac{a}{r} + \overline{\alpha}K\right) + \sqrt{(\frac{\beta_1 - 1}{\beta_1})^2\left(2\frac{a}{r} + \overline{\alpha}K\right)^2 - 4\frac{\beta_1 - 2}{\beta_1}\left[(\frac{a}{r})^2 + \frac{\overline{\alpha}c}{r}K\right]}}{2\frac{\beta_1 - 2}{\beta_1}}$$
$$= \frac{\beta_1 - 1}{\beta_1 - 2}\left(\frac{a}{r} + \frac{1}{2}\overline{\alpha}K\right) + \sqrt{(\frac{\beta_1 - 1}{\beta_1 - 2})^2\left(\frac{a}{r} + \frac{1}{2}\overline{\alpha}K\right)^2 - \frac{\beta_1}{\beta_1 - 2}\left[(\frac{a}{r})^2 + \frac{\overline{\alpha}c}{r}K\right]}$$

5 Model calibration

In order to test the model, we performed some numerical simulations based on the following assumptions:

- c, fixed buying price, is set at 100 (€/MWh);
- K costs equation, provided a levelised cost of energy produced by the PV plant equal to 180 €/MWh;
- γ and v are equal to 0.

We analyze different scenarios:

- $\overline{\alpha}$, i.e. the percentage of the total load that the prosumer can satisfy through the photovoltaic production, is equal to 50% or 75%;
- T, i.e the investment life, is equal to 20 or 25 years respectively;
- r, i.e. the investment discount rate, is equal to 4% in the first scenario and to 6% in the second scenario;
- σ, i.e. volatility of v, assumes the following values: 20%, 30%, 40% and 50%.

The following tables (Table 1 and Table 2) summarize the results we obtained for v^* and α_1^* in different scenarios.

r=4%		σ=20%	σ=30%	σ=40%	σ=50%
T=20	α^{bar} =0,5	nd	α_1* =0,68; v* =135,33	α_1 *=0,61; v*=120,30	α_1* =0,57; v*=113,36
	α^{bar} =0,75	nd	α_1* =0,78; v* =154,22	α_1 *=0,75; v*=141,14	α_1* =0,75; v*=134,85
T=25	α^{bar} =0,5	nd	α_1* =0,62; v* =141,68	α_1 *=0,56; v*=127,14	α_1* =0,53; v*=120,33
	α^{bar} =0,75	nd	α_1* =0,75; v* =160,76	α_1 *=0,75; v*=148,73	α_1* =0,75; v*=142,85

Table 1: v^* and α_1^* for different values of σ, α*, r=4% and T=20 years and T=25 years respectively

r=6%		σ=20%	σ=30%	σ=40%	σ=50%
T=20	α^{bar} =0,5	nd	α_1* =1,34; v* =337,32	α_1 *=0,75; v*=188,46	α_1* =0,52; v*=131,55
	α^{bar} =0,75	nd	α_1* =1,74; v* =438,43	α_1 *=0,92; v*=232,35	α_1* =0,75; v*=153,80
T=25	α^{bar} =0,5	nd	α_1* =0,60; v* =167,74	α_1 *=0,52; v*=146,51	α_1* =0,5; v*=137,14
	α^{bar} =0,75	nd	α_1* =0,75; v* =181,70	α_1 *=0,75; v*=167,01	α_1* =0,75; v*=159,96

Table 2: v^* and α_1^* for different values of σ, α*, r=6% and T=20 years and T=25 years respectively

For increasing σ, the entry price v^* decreases: when prices are highly volatile, it increases the possibility to have a gain in investing and selling energy in the market (i.e. the prosumer turns to be a producer).
Regarding the investment size, what emerges is that, according to investment costs and buying prices, we often arrived at a corner solution when the percentage of load satisfied by the photovoltaic power plant is higher: when $\overline{\alpha} = 0,75$, we obtain

$\alpha_1^* = \overline{\alpha}$ in most of the cases, this occur in particular when T=25 years. In this the prosumer decides the plant's size according to its consumption needs. When σ increases, the optimal size of the plant rapidly decreases: when r=4% and σ =20%÷30% we observe a reduction in size of -14%; when r=4% and σ=30%÷40% we observe a reduction in size of -8%. When the discount rate is higher, the effect on over-dimensioning the plant is much higher as well (-118% and -46%). That means that, changing parameters, the prosumer decides to invest, but it is more oriented to self-conumption instead of being more active in the market.

6 Conclusions

The development of distributed power plants shall be managed through a system that allow for a better integration of renewable energy plants, calling for private actions helping in grid management. The smart grid environment allows for an instantaneous interaction between the agent and the grid: depending on its needs, the grid can send signals (through prices) to the agents, and the agents have the possibility to respond to the signals and get a monetary gain. In this event, the system can better integrate renewables – able to keep the grid stable -- and PV production in the absence of costly monetary incentives.

In some specific cases, and in particular in those area where the grid suffers from congestions or high degrees of production unpredictability, the prosumers involvement in grid management might boost investments, and induce agents to extra effort to provide the grid with private services in response to price signals. On the other hand, areas where grid calls for energy inflows are not frequent or poorly remunerated will face lack of private investments in distributed energy generation.

According to the model's calibration results, the threshold selling price v^* and the investment size α_1^* move in opposite directions for increasing values of the expected variance of the selling price. This result is of major importance when considering how the market generated by the implementation of smart grids. agents might enter the market relatively early, but with small size investments, and this in turn will result in a minor participation of the agents to the market.

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