Toward Damage Tolerance Design of Nonlinear Cracked Laminated Composite Shell Structures under Internal Pressure

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Abstract: - Currently, the commonly adopted approach of damage tolerance design for shell structures under static or steady state internal pressure is the application of bulging factors. As pointed out in a recent paper by the authors, bulging factors available in the literature have several important shortcomings for damage tolerance design and characterization of cracked cylindrical shell structures under internal pressure. The authors have recently proposed the equivalent bulging factors for cracked laminated composite shell structures under internal dynamic pressure. While the equivalent bulging factors have potential to be applied to the damage tolerance design of nonlinear cracked laminated composite shell structures under internal dynamic pressure they still suffer the important limitation that they are only applicable to specific ranges of ratios of crack length to diameter of the shell structures. In order to circumvent this important limitation the maximum nonlinear transversal or out-of-plane response at the center of the flange or edge of the crack is proposed as a viable alternative to damage tolerance design of laminated composite shell structures which are commonly employed in many aircrafts, and aerospace and automotive systems. Computed results for various laminated composite cylindrical shell structures under internal dynamic pressure, and their implications are included in this presentation.

Key-Words: - Nonlinear, cracked laminated composite shells, internal pressure, finite element method

1 Introduction
The occurrence of cracks in existing and aging aircraft fuselages is a major problem in the aerospace industry. Those fatigue cracks can be caused by corrosion, incorrect maintenance or impact damages from accidents, which may lead to unexpected disasters. To avoid any catastrophic failure due to these causes, the aircraft fuselage must be designed with some damage tolerance capabilities. The philosophy of damage tolerance design requires determination of realistic stress state in the neighborhood of cracks in airframe fuselages.

Currently, the commonly adopted approach of damage tolerance design for isotropic shell structures under static or steady state internal pressure is the application of bulging factor [1-15]. As pointed out in a recent paper by the authors [16], the bulging factors have several important shortcomings for damage tolerance design and characterization of cracked cylindrical shell structures under internal pressure. Firstly, the bulging factor is hinged on stress intensity factor at the crack tip and therefore numerical results obtained by techniques such as the finite element method (FEM) are very sensitive to the mesh around the crack tip. Secondly, it does not include explicitly the influence of crack width. Thirdly, individual bulging factor only applies to a range of ratios of crack length to radius of shell. Consequently, an approach, based on the application of nonlinear responses at the centers of edges of cracks or the so-called crack flange deflection [17], that has no such limitations for the damage tolerance design and characterization of cracked isotropic cylindrical shells under internal pressure was proposed in [16]. As many parts of modern aircraft bodies are made of laminated composite shell structures it is logical and important to apply the approach presented in Ref. [16] to laminated composite shell structures.
with crack and under internal step pressure. For completeness, before the application of the approach in Ref. [16] to cracked laminated composite cylindrical shell structures under internal step pressure the finite element (FE) models is briefly introduced in Section 2. In Section 3 responses of two-layer and eight-layer laminated composite cylindrical panel under internal step pressure will be examined so as to validate the FE representations and their accuracy since results for these two-layer and eight-layer laminated composite panels under applied step internal pressure are available in the literature for comparison. To further verify the FE models adopted and explore their usefulness, and again since results are available for comparison, responses of two-layer cylindrical shells under step internal pressure are studied in Section 3. The two-layer cracked cylindrical shell structures under similar internal pressure are investigated in Section 4. Different crack lengths and widths are considered. The difference between constraining and releasing the drilling degrees-of-freedom (ddof) of the two-layer shell structures is examined in this section. It may be appropriate to mention that four-layer laminated composite shells have also been investigated but not included in the present report for brevity. Section 5 is concerned with the studies of the relationship between nonlinear responses of cracked cylindrical shells and internal step pressure. This relationship is proposed as a viable approach to the damage tolerance design and characterization of cracked laminated composite shell structures under internal step pressure. Summary and concluding remarks are presented in Section 6.

2 Nonlinear System Equations
In the present investigation, every laminated composite shell structure with and without crack is discretized by the laminated composite triangular shell finite element developed by To and Wang [18]. This shell finite element is identified as HLCT\textsuperscript{3d}. It has three nodes and every node has six degrees-of-freedom (dof). The latter include three translational and three rotational dof. The element is sketched in Fig.1. The global coordinate system is denoted by \((X,Y,Z)\) while the local coordinate system consists of \((r,s,t)\). The plane containing the local axes, \(r\) and \(s\) is located at the mid-surface of the laminated composite shell element. For identification purposes, the nodal dof at Node 2 are \(u_2, v_2, w_2, \theta_{2r}, \theta_{2s}, \text{ and } \theta_{2t}\) in which \(u_2, v_2, \text{ and } w_2\) are the displacements at Node 2 along the \(r\), \(s\), and \(t\) axes, respectively, while \(\theta_{2r}, \theta_{2s}, \text{ and } \theta_{2t}\) are the angular displacements or rotations about the \(r\), \(s\), and \(t\) axes, respectively. This laminated composite triangular shell element is based on the first order lamination theory and mixed formulation. Thus, transverse shear is included in the formulation. The main features of HLCT\textsuperscript{3d} include simplicity, drilling dof (ddof), ability to deal with large deformation of finite strain and finite rotation, elimination of shear locking, and ability to represent correctly the six rigid-body modes implying, mathematically, the element is rank sufficient. In addition, explicit expressions of the element consistent mass and stiffness matrices have been derived and therefore no numerical integration and numerical matrix inversion are required. This, in turn, improves drastically the computational time over those elements employing numerical integration and numerical matrix inversion or transformation [19]. This element has been applied in a relatively detailed investigation of highly nonlinear static and dynamic problems of laminated composite plate and shell structures [18]. Vibration analysis of plate and shell structures has also been reported in Ref. [20].

\[ M\ddot{x} + C\dot{x} + Kx = F \]  

(1)

where \(M, C, \text{ and } K\) are, respectively the assembled mass, damping, and stiffness matrices while \(x\) and \(F\) are respectively, the generalized displacement and force vectors. The over-dot and double over-dot are respectively, denote the first and second time derivatives.
In the present investigation, the displacement responses in Eq. (1) are numerically evaluated by the trapezoidal rule. A digital computer program written in Fortran language has been developed for all the computations reported in this paper. For simplicity, in the present investigation damping in the system has been disregarded.

3 Shell Structures without Crack
In this section a segment of a finite cylindrical shell and a cylindrical shell with two different ply-orientations are presented. The geometry of the segment of a finite cylindrical shell analyzed in the present investigation is shown in Fig. 2. The shell shown is a segment of a finite cylindrical shell. In this section the shell panels and whole cylindrical shell structures without crack are studied. The computed results in this section serve two main purposes. First, there are data in the literature for comparison and therefore these computed results are applied to validate the correctness of the input and output data. Second and for more important purpose, they are used to show that even for the coarse meshes adopted in the present investigation the computed results are very accurate compared with those published in the literature.

The shell structure shown in Fig. 2 has a radius, \( R \), an axial length, \( L_a \), a circumferential length, \( L_c \), a wall thickness, \( h \), and a crack length, \( 2a \). The crack is centrally located and is oriented longitudinally (parallel to the \( y \)-axis) or circumferentially (whose projection on the \( xy \)-plane is parallel to the \( x \)-axis). All thickness, radius, axial, circumferential length, crack length can be varied. The internal dynamic pressure, \( p \), which generates a circumferential or hoop stress, \( \sigma_h \), and an axial stress, \( \sigma_a \), is a step load.

3.1 Cylindrical shell structures
To and Wang [16] have analyzed several laminated composite shell structures by using the hybrid or mixed formulation-based laminated composite triangular shell elements HLCT\(^{nd}\) and demonstrated the attractive features of this element. Typical finite element model of the laminated composite cylindrical shell structure is shown in Fig. 3. In order to better represent the responses along the crack edge, the mesh scheme, \( 2 \times 2 \) D, shown in Fig. 3 needs to be appropriately modified and a finer mesh in the crack region is applied. The new mesh for computing the responses of cracked and intact or non-cracked shells is shown in Fig. 4(a). There is a one to one correspondence between nodes of the upper layer and nodes of the lower layer. This means that a similar mesh refinement pattern is used for both layers if the shell structure is made of two-layer composite materials. Figs. 4(b) and 4(c) show the FE mesh for one-eighth of the cylindrical shell with diamond and ellipse crack, respectively.
3.2 Two-layer cylindrical panel

In this sub-section, the two-layer cross-ply [0/90] panel shown in Fig.4 is studied. This panel is selected because their results are available in the literature for comparison. The two-layer cross-ply laminated composite cylindrical panel considered here has the following pertinent geometrical data: cylindrical radius $R = 2.54$ m (100 in), arc length $L_c = 0.508$ m (20 in), axial length $L_a = 0.508$ m (20 in) while $\varphi = 0.10$ rad. The total thickness $h = 0.00127$ m (0.05 in) and equal layer thickness is assumed. The material selected is graphite-epoxy. Its properties are: $E_1 = 137.90$ GPa ($2.0 \times 10^7$ psi), $E_2 = 9.8599$ GPa ($1.43 \times 10^6$ psi), $G_{12} = G_{13} = G_{23} = 5.2402$ GPa ($0.76 \times 10^6$ psi), Poisson’s ratio $= 0.3$, and density $\rho = 1562.2$ kg/m$^3$ ($5.644 \times 10^{-2}$ lb/in$^3$). The shell panel is clamped at all edges. By taking advantage of symmetry, only one-quarter panel under suddenly applied step internal pressure is modeled. The boundary conditions imposed on the FE model are: $V = \Theta_x = \Theta_y = 0.0$ at CD, $U = \Theta_y = \Theta_z = 0.0$ at AD due to the symmetry and all of dof are constrained at AB and BC since the panel is clamped at these two edges. The shear correction factors for this case are $k_4 = k_5 = (5/6)^{1/2}$. The internal step pressure applied is $p = 6.895$ kPa (1.0 psi). The results shown in Fig.5 are dynamic responses along the $Z$-axis at the center of the two-layers cross-ply [0/90] panel. That is, point D in Figure 4(a). In this figure, the results from To and Wang [18] obtained by employing the HLCTS$^d$ element with a $4 \times 4D$ mesh are included for comparison. The present results are obtained by using the mesh shown in Fig.4. Both set of results in the figure are based on the trapezoidal integration scheme for the numerical integration of the governing matrix equation of motion, Eq. (1), with a time step size, $\Delta t = 0.05$ ms. Since there is no crack in the cylindrical panel for this case, the directors version [18] of the element was employed. Also, large strain deformation has been taken into account. Clearly, as shown in the figure, the present results agree very well with those from To and Wang [18]. It may be appropriate to note that the results of To and Wang have already been compared very well with those of Wu and Yang [22]. Therefore, it is concluded that the relatively coarse mesh used in this section is capable of providing accurate nonlinear responses.

3.3 Eight-layer cylindrical panel

In order to further substantiate the correctness of input data and the accuracy of computed results, an eight-layer cylindrical panel is considered. The eight-layer symmetrically laminated shell panel has the following fiber arrangement [0/-45/90/45]$_{sym}$. This shell panel is clamped at all edges. Thus, the boundary conditions applied to the quarter shell are the same as those imposed on the two-layer panel described in Sub-section 3.2 above. The dynamic responses at the center of the panel and along the $Z$-axis which is perpendicular to the surface of the panel, henceforth, referred to as the central deflections are evaluated by using the HLCTS$^d$ element [18] with the options of director formulation and large strain. The trapezoidal integration scheme with a time step size, $\Delta t = 0.05$ ms is employed. The obtained results are compared with those from To and Wang [18] in Fig.6. In this
case, the agreement between the present results and those of To and Wang is even better than that of the two-layer cross-ply cylindrical panel examined in Sub-section 3.2. Thus, it is believed that the current FE mesh can be confidently employed to study the nonlinear dynamic responses of laminated composite shell structures.

3.4 Two-layer cross-ply cylindrical shell

In reality, an aircraft fuselage is a complete cylindrical shell rather than a cylindrical panel. Thus, the whole cylinder is required to be studied to better understand the behavior of laminated composite cylindrical shell structure under internal pressure. The computed results can also be used to further validate the input data and the correctness of the output responses. Only one-eighth of the cylindrical shell needs to be modeled because of geometrical symmetry. The two-layer cross-ply, denoted by [0°/90°] or simply as [0/90], laminated composite cylindrical shell structure under internal pressure is studied and included in this sub-section. Similar to those presented in the foregoing, the mesh employed is that shown in Fig.4(a). The following geometrical properties are used: cylindrical radius \( R = 0.508 \) m (20 in), length \( L_a = 0.508 \) m (20 in). The total thickness \( h = 0.0254 \) m (1.0 in) and equal layer thickness was assumed. The material parameters are: \( E_1 = 51.711 \) GPa (7.5×10⁶ psi), \( E_2 = 13.790 \) GPa (2.0×10⁶ psi), \( G_{12} = G_{13} = G_{23} = 8.6185 \) GPa (1.25×10⁶ psi), Poisson’s ratio = 0.25, and density \( \rho = 27680 \) kg/m³ (1.0 lb/in³).

With reference to Fig.4, the boundary conditions imposed on the FE model are: \( V = \Theta_x = \Theta_y = 0.0 \) at CD, \( U = \Theta_y = \Theta_z = 0.0 \) at AD, \( W = \Theta_x = \Theta_z = 0.0 \) at BC due to the symmetry and all of dof are constrained at AB since the shell structure is clamped at both ends. An internal step pressure with intensity \( p = 34.474 \) MPa (5000 psi) is applied to the laminated composite cylindrical shell. The time step size used is 5.0 ms. Fig.7 contains the plots of central deflections in the Z-direction. The results from Reddy and Chandrashekhara [23], which were obtained by using 2 × 2 mesh of nine-node shell elements and the trapezoidal rule for numerical integration, are included for comparison. As shown in Fig.7, both sets of results agree with each other very well. It is noted that the largest response of the first peak is 24.51 mm (0.965 in) which is almost as large as the thickness of the shell, meaning the response is relatively nonlinear.

3.5 Two-layer angle-ply cylindrical shell

A two-layer angle-ply \([-45°/45°]\) laminated composite cylindrical shell is studied in this sub-section. The geometrical and material parameters, and internal pressure are the same as those used in the last sub-section. The same trapezoidal rule for the numerical integration of the matrix equation of motion is also applied to obtain the dynamic responses at the center of the cylindrical shell structure. The computed results and those from Reddy and Chandrashekhara [23] are presented in Fig.8. It is observed from the latter figure that the responses are in very good agreement. However, it is interesting to note that although the deflection of cross-ply shell structure is smaller than that of angle-ply shell structure, the first natural frequency of angle-ply shell structure, 59.74 rad/s, is smaller than that of its cross-ply counterpart, 65.34 rad/s. This observation is consistent with the fact that in the cross-ply case with a higher first natural frequency implying that the structure is stiffer and therefore the deflection is smaller than the angle-ply example. Of course, one can also estimate the natural frequencies from the periods of oscillation in Figs. 7 and 8. In this respect, one can find that the period of oscillation for the cross-ply shell structure is slightly smaller than that of the angle-ply shell structure.
4 Two-layer Shells with Cracks

The relatively coarse FE mesh without crack shown in Fig.4(a) that can be applied to provide accurate computed results has been verified in the last section. In the present section the same FE mesh is employed in the computation of geometrically nonlinear dynamic responses of laminated composite shell structures with cracks and under internal step pressure. Of course, the meshes with crack, such as those shown in Figs.4(b) and 4(c), are slightly modified from that in Fig.4(a). The influences of various layer configurations and crack types as well as crack sizes on the nonlinear dynamic responses are studied.

Although the internal step pressure range in a transport aircraft seldom exceeds 68.950 kPa (10 psi) [7], analysis in this section has included pressure up to the atmosphere pressure of 101.325 kPa (14.7 psi). Two-layer laminated composite cracked shell structures under internal step pressure are studied. The boundary and symmetry conditions as well as the FE mesh are identical to those in Sub-section 4.1, except that now the symmetry condition does not hold for the nodes along the crack edge any more. However, the node at D is still symmetric about the line CD. The shell is of Carbon/Epoxy (AS4/3501-6). The material properties are: \(E_1 = 147\) GPa (2.13 × 10^7 psi), \(E_2 = 10.3\) GPa (1.50 × 10^6 psi), \(G_{12} = G_{13} = G_{23} = 7.0\) GPa (1.0 × 10^6 psi), Poisson’s ratio \(\nu_{12} = 0.27\), and density \(\rho = 1600.0\) kg/m^3 (5.8 × 10^3 lb/in^3). The shell has a radius of 1.98 m, which is representative of a narrow-body transport aircraft fuselage geometry, length of 1.98 m, and skin thickness of 1.0 mm. Equal layer thickness is assumed in the present studies. The applied internal step pressure is 101.325 kPa (14.7 psi).

4.1 Cross-ply laminated composite shells

In this sub-section two-layer cross-ply laminated composite shell structures are studied. The shell structure has a diamond crack with \(2a/L_w = 0.375\) and \(b/a = 0.10\). The time step size is examined first in order to obtain accurate responses. The time step sizes considered are: \(\Delta t = 0.05\) ms, 0.01 ms, and 0.005 ms. The results presented in Fig.9 show an excellent agreement between the data for \(\Delta t = 0.01\) ms and \(\Delta t = 0.005\) ms, suggesting the time step size, \(\Delta t = 0.01\) ms can be applied to obtain accurate results. The corresponding first natural frequency is 974.8 rad/s, which is much lower than that of the single layer cylindrical shell, 2227 rad/s [16]. The time step size \(\Delta t = 0.01\) ms has also been proved excellent for single layer shell whose natural frequency is higher. Thus, \(\Delta t = 0.01\) ms is chosen for the computation of dynamic responses. The dynamic responses of cross-ply laminated cylindrical shells without crack and with different crack sizes are compared in Fig.10 through 12. Fig.10 is concerned with computed responses of the cross-ply two-layer shell with crack configuration of \(2a/L_w = 0.375\) and \(b/a = 0.10\). Apparently, the deflection at the center of the crack edge is dramatically higher than that without crack. This is reasonable in that the cracked shell is softer than the one without crack. The results shown in Fig.11 confirm that the responses become larger as the crack length is increased, while the influence of crack width is not as dramatic as that of crack length.

![Fig.8 Responses of two-layer angle-ply shell: —, Ref. [23]; - - -; present model.](image)

![Fig.9 Responses of cross-ply shell: —, \(\Delta t = 0.05\) ms; - - -, \(\Delta t = 0.01\) ms; xxx, \(\Delta t = 0.005\) ms.](image)

4.2 Angle-ply [−45/45] shells

Two-layer angle-ply laminated composite shells with crack are now studied in this sub-section. The geometrical and material properties are similar to those investigated in Sub-section 4.1. Results for of various crack lengths and widths are provided in Figs.13 and 14. As can be observed in the latter figures, the behavior trends of the two-layer angle-ply laminated composite shell structure are similar to their cross-ply counterparts in the sense that the influence of crack lengths has a more pronounced effect on the central deflection of the crack than that of the crack widths.
structures with and without ddof included in the computation. To limit the scope of the present studies, the two-layer cross-ply laminated composite cylindrical shell, which has a diamond crack of $2a/L_a = 0.375$ and $b/a = 0.10$, is considered. The time step size, $\Delta t = 0.01$ ms is applied in the dynamic response computation. The geometrical and material properties are the same as those presented above. That is, the material for this two-layer shell structure is Carbon/Epoxy (AS4/3501-6). The computed results are included in Fig.15. Clearly, setting $\Theta_z$ free results in larger amplitude of the central deflection of the shell structure since by constraining the ddof the strain energy of the structure is increased and therefore the shell structure is stiffer. The difference between the amplitudes of the responses is very significant. Therefore, in order to obtain more accurate responses the ddof of the aircraft fuselage model that contains crack(s) cannot be disregarded and should not be constrained.

4.3 Two-layer cross-ply shells with ddof
In the parametric studies performed and discussed thus far the ddof of every finite element were constrained. That is, in the foregoing every rotation about $Z$-axis, $\Theta_z = 0$. However, in reality, the aircraft fuselage under uniform internal pressure and contains crack(s) cannot disregard the ddof in the FE model if accurate responses and correct representation of the actual shell structure are desired. Besides, it is important, from the point of understanding the behavior of the shell structure, to study the difference between results of shell structures with and without ddof included in the computation. To limit the scope of the present studies, the two-layer cross-ply laminated composite cylindrical shell, which has a diamond crack of $2a/L_a = 0.375$ and $b/a = 0.10$, is considered. The time step size, $\Delta t = 0.01$ ms is applied in the dynamic response computation. The geometrical and material properties are the same as those presented above. That is, the material for this two-layer shell structure is Carbon/Epoxy (AS4/3501-6). The computed results are included in Fig.15. Clearly, setting $\Theta_z$ free results in larger amplitude of the central deflection of the shell structure since by constraining the ddof the strain energy of the structure is increased and therefore the shell structure is stiffer. The difference between the amplitudes of the responses is very significant. Therefore, in order to obtain more accurate responses the ddof of the aircraft fuselage model that contains crack(s) cannot be disregarded and should not be constrained.
4.4 Laminated composite aero-plane

In order to provide dynamic response studies of more ‘realistic’ cylindrical shells, an Airbus A320 fuselage section having length of 8.0 m is considered here. The remaining properties and loading condition are similar to those used for the shorter shell structures examined and discussed above. The number of shell elements representing the model is similar to that shown in Fig.4. The element size near the crack region (labeled as ‘m’ in Fig.16) is not changed, while larger elements are adopted for the remaining part of the cylindrical shell (labeled as ‘n’ in Fig.16). As observed in the foregoing sections, this coarse FE model can provided very accurate responses and it is very efficient, computationally. This also reflects the superior features of the shell finite element employed. Both the cylindrical shells with $\Theta_z$ released and constrained are considered in this sub-section. The shell consists of two layer cross-ply laminated composite material. The clamped-clamped (CC) boundary conditions are applied. The computed responses are presented in Fig.17. Inspection of the latter figure reveals that the peak values of the transverse deflection at the center of the edge of crack or central deflection, $W$ for the shell with $\text{dof}$ constrained have reached about 90.0 mm, which is 90 times of the thickness of the cracked shell, whereas responses of the shell with $\Theta_z$ released have higher peaks than that of the shell with $\Theta_z$ constrained. The amplitudes of the central deflections for both cases are about 1/8 of the crack length. It may be observed that if longer time history is plotted, the responses of cylindrical shell with $\Theta_z \neq 0$ will reach another peak as can be seen in Figure 20 later in this sub-section. Two other boundary conditions at the ends of the laminated composite cylindrical shell in addition to the one considered above are applied. These are the simply-supported (SS) at both ends and the free-free (FF) cases.

$m$

$\Theta_z = 0$ and $\Theta_z \neq 0$ are included in the studies. Computed results are presented in Figs. 18 and 19.

Fig.16 FE model of a cracked fuselage section.

Fig.17 Responses of cracked fuselage section: —, $\Theta_z = 0$; - - -, $\Theta_z \neq 0$.

Fig.18. Responses of long cylindrical shell with $\Theta_z = 0$: —, CC; - - -, SS; - - - - , FF.

Fig.19 Responses of long cylindrical shell with $\Theta_z \neq 0$: —, CC; - - -, SS; - - - - , FF.
5 Relationship between Responses and Internal Pressures

As stated in Section 1 and discussed in Ref. [16], the bulging factor $\beta$ is based on the stress intensity factor at the crack tip and therefore the numerical results are generally very sensitive to the mesh around the crack tip. The dynamic responses from the center of the edge of crack are, however, obtained relatively far away from the crack tip and therefore they are essentially not affected by the accuracy of the stress intensity factor at the crack tip. Furthermore, the definitions of bulging factor for isotropic cylindrical shells [16] cannot be applied to laminated composite shell structures with cracks. Consequently, the concept of equivalent bulging factors [24] has recently been proposed by the authors as a means for the damage tolerance of cracked laminated composite shell structures under internal pressure. Owing to the fact that the equivalent bulging factors are based on the bulging factors $\beta$ for isotropic materials, and naturally it still has an important limitation in that every bulging factor equation is applicable within a certain range of $a/R$. For example, the equivalent bulging factor defined by Eq. (7) of [16] is applicable between $a/R = 0.017$ and $0.18$ since the later equation was based on the formula developed in [11] for isotropic cracked shells. On the other hand, dynamic responses at the centers of the crack edges are not confined to a particular range of ratios of crack length to radius of shell, $a/R$. Thus, it is believed that the nonlinear dynamic responses at the centers of the cracked edges can be applied as a potentially better alternative to the equivalent bulging factors for the damage tolerance design of cracked laminated composite shell structures under internal pressure. It is interesting to note that a similar concept was introduced in [17]. “Similar” in the sense that crack flange deflection is applied. More specifically, a factor, $\beta^*$ defined as the ratio of the nonlinear crack flange deflection of the cylindrical shell structures to the linear crack flange deflection of the same shell structures was employed in [17]. However, $\beta^*$ is a ratio simply expressing the difference between the nonlinear and linear crack flange deflections of an isotropic shell structure under internal pressure. It does not give the actual nonlinear responses and therefore does not reflect directly the related stress state at the crack.

Accordingly, the relationship of nonlinear dynamic responses of cracked laminated composite cylindrical shells and applied step internal pressure is studied in this section. The nonlinear dynamic crack flange deflection is believed to be a more appropriate quantity in that it is the nonlinear deflections rather than the difference between the nonlinear and linear crack flange deflections. Further, it does reflect directly the related stress state at the crack.

To limit the scope of the presentation, the two-layer shell structures with a diamond crack modeled in this section have a length of 8.0 m, a radius of 1.98 m, thickness of 1.0 mm, the ratio of the half crack length to shell radius, $a/R = 0.1875$ and the ratio of the half crack width to half crack length $b/a = 0.10$. These properties correspond to those of the Airbus A320 fuselage section investigated in the foregoing. The range of applied internal step pressure is selected from 6.895 kPa (1 psi) to 101.325 kPa (14.7 psi). The cylindrical laminated composite shell structures are CC at both ends. Similar to that in Section 3, only one-eighth of each shell is modeled due to geometrical symmetry. More specifically: $V = \Theta = \Theta = 0.0$ at CD, $U = \Theta = \Theta = 0.0$ at AD, $W = \Theta = \Theta = 0.0$ at BC and all degrees of freedom are constrained at AB. In other words, the boundary and symmetry conditions as well as the FE mesh are identical to those in Sub-sections 3.1 and 3.2, except that symmetry condition does not hold for the nodes along the crack edge. The nodes at D in Fig.4, however, is still symmetric about the line CD. In addition, all $\Theta_x$ not on the boundaries and lines of symmetry are free or not constrained in this section.

The material is Carbon/Epoxy (AS4/3501-6) and therefore, its properties are identical to those included in Section 4. The time step size selected for the response computation is $\Delta t = 0.01$ ms.

5.1 Cross-ply [0/90] shell structures

The relation between the central deflections and applied step internal pressures of two-layer cross-ply laminated composite shells with crack is considered. Computed results are presented in Figs.20 and 21. In these figures the dynamic responses are the largest peak values to their corresponding applied step pressures. That is, for...
every internal step pressure applied to the shell structure the largest peak of the displacement response is selected from the corresponding time history of central deflection or dynamic central deflection similar to that in Fig.10. It should be emphasized that this only provides one point in Fig.20, for example. In other words, every plot in the latter figure consists of many runs of the developed finite element computer program and the results presented are not trivial. The results in Fig.20 are concerned with shells having crack parameters, $b/a = 0.10$ and $aR = 0.0625$, 0.125 and 0.1875. They show that the central deflections increase with the internal step pressure. The shell structure with shorter crack length has smaller central deflection. The central deflection of shell with value of $aR = 0.0625$ levels off as pressure increases while the central deflections of shells with longer cracks keep increasing. The results in Fig.21 are concerned with $aR = 0.1875$ and various crack widths. The applied high internal step pressure induces large deformation. It is obvious that the shell with a smaller crack width has larger response comparing with those having larger crack widths. This seems to be contradictory to intuition in that as the crack width is increased the shell structure should be less stiffness and therefore the responses should be increased. The explanation for the observation is that when the crack width is increased the mass of the shell structure is significantly reduced to the extent that the ratio of stiffness to mass of the shell structure is increased instead of decreased. Thus, as the crack width is increased the first natural frequency of the shell structure is higher. In other words, the shell structure is stiffer. This finding agrees with that for cracked isotropic shell structures [16].

5.2 Angle-ply [-45/45] shell structures
The relationship between central deflection and applied internal step pressure for angle-ply laminated composite shell structures are provided in Fig.22 for the cases of cracked shells with $b/a = 0.10$. The construction of Fig.22 is similar to those in Figs.20 and 21 in that the largest peak values of nonlinear responses to their corresponding applied pressures are used in these plots. As can be observed in the figures the central deflections initially increase rapidly with increasing load, and as the load is increased, the rate of change of increase in deflection reduces. Fig.22 reveals that the shells with shorter cracks have smaller response for a given internal pressure. Central deflection results for shells with different crack widths and $aR = 0.1875$ are presented in Fig.23. Similar trends to that observed in the last sub-section are also found in Fig.23 in the sense that the central deflection increases as the internal pressure is increased, and shells with smaller crack width has larger response.
5.3 Shells with different boundary conditions
The properties as well as applied internal pressures are the same as those included above. The cross-ply cylindrical shells have crack parameters, $a/R = 0.1875$ and $b/a = 0.10$. The computed results for the cracked two-layer cross-ply laminated composite pressurized shell structures with different boundary conditions, such as, CC, SS, and FF at both ends of the cylindrical shell, are presented in Fig.25. As shown in the latter figure, the central deflection for cracked shell with FF boundary conditions is much larger than those with CC and SS boundary conditions. For the range of internal pressure studied the central deflection of the shell with FF boundary conditions increases as the applied internal step pressure is raised. This trend reverses at the internal pressure slightly passes 85 kPa.

6 Summary and Concluding Remarks
Nonlinear responses of two-layer and eight-layer laminated composite cylindrical panels without crack and under internal step pressure have been studied in Section 3. They have been applied to validate the FE representations and their accuracy since results for these two-layer and eight-layer laminated composite panels under internal step pressure are available in the literature. To further verify the FE models adopted and explore their usefulness, and since results are available for comparison, responses of two-layer cylindrical shells under internal step pressures have been included in Section 4. Different crack lengths and widths have been considered. The difference between constraining and releasing the drilling degrees-of-freedom (ddof) of the two-layer shell structures is also studied in this section. Section 5 is concerned with the studies of the relationship between nonlinear central deflections of cracked cylindrical shells under internal step pressures. This relationship is proposed as a viable approach to the damage tolerance design and characterization of cracked pressurized laminated composite shell structures. It is believed that the proposed approach is: (1) capable of providing information about the influence of crack lengths as well as crack widths on responses of laminated composite shell structures under internal step pressures, (2) not limited to a given range of ratios of crack length to radius of shell, (3) conceptually simple, and (4) the nonlinear dynamic responses at the centers of edges of cracks of laminated composite shell structures are relatively easy to obtain accurately and efficiently even by using very coarse meshes of the mixed formulation based triangular shell FE [18].

References: