Stabilization of a Modified Slotine-Li Adaptive Robot Controller by Robust Fixed Point Transformations

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Abstract: The "*Adaptive Slotine-Li Robot Controller (ASLRC)*" of the nineties of the past century was designed by a sophisticated process based on the use of Lyapunov's 2^{nd} method. In the possession of the *exact analytical form of the system model* it generally can achieve *global asymptotic stability by learning the system's exact dynamic parameters*. However, it is not robust to friction effects and unknown external disturbances. In contrast to that the adaptive controllers designed by the use of "*Robust Fixed Point Transformations (RFPT)*" are only locally stable, work on the mathematical basis of Banach's Fixed Point Theorem, cannot learn the system's analytical model parameters but they are very robust to modeling deficiencies (e.g. abandoned friction effects) and unknown external forces. In this paper it is shown that by evading the use of Lyapunov function in the adaptive control design an appropriate modification of the ASLRC can be elaborated that is able to properly learn the exact model parameters if external disturbances are missing. It can be combined with the RFPT-based controller that makes it robust to formal modeling inconsistencies and external forces, though in this case it cannot learn the appropriate system parameters. It is also shown that the symbiosis with the RFPT-based method does not disturb the parameter identification process if modeling inconsistencies and disturbances are absent.

Key–Words: Adaptive control; Lyapunov function, Lyapunov's direct method, Slotine-Li Adaptive Robot Controller; Robust Fixed Point Transformation; Global stability; Asymptotic Stability; Robustness.

1 Introduction

On the basis of the translations of A.M. Lyapunov's PhD Thesis of 1892 [1], from the sixties of the past century [2] Lyapunov's "direct" method became the prevailing mathematical design tool for constructing stable controllers for strongly nonlinear systems in which the motion is not limited to the close vicinity of some "working point". Industrial robots of open kinematic chains are excellent paradigms to represent such systems: as the various links of e.g. a PUMA robot are rotated by considerable angles the dynamics of the whole system suffers very drastic variation so such systems cannot be "linearized" for the aims of controller design. As early examples from the nineties of the past century the "*Adaptive Inverse Dynamics Controller (AIDSC)*" and the ASLRC controllers can be mentioned [3]. Both were designed by the use of some Lyapunov functions. The latter one utilized more subtle analytical details therefore it allowed faster parameter tuning by avoiding the use of the inverse of a tuned inertia matrix that generally may become singular or ill-conditioned. It also contained less number of arbitrary adaptive parameters as the AIDSC. Both methods were critically analyzed in details in [4]. In [5] two modifications were introduced to improve these classical methods: a) in the 1st step the feedback term was modified by the inclusion of integrated tracking error terms but essentially the same Lyapunov function and parameter tuning techniques was used for proving global asymptotic stability of the controlled system; b) following that the parameter tuning processes were modified on the basis of simple geometric interpretation, in this manner the use of the Lyapunov function in the design was evaded. In connection with this latter step it was recognized that *formally insisting on the use of a Lyapunov function means very significant handicap* as far as the possible parameter tuning process is concerned, and it was shown that reasonable tuning rules can be deduced without the use of any Lyapunov function.

The difficulties related to the Lyapunov function based techniques inspired the research for alternative adaptive control solutions in which instead of the use of any analytical model and related parameter tuning a fixed approximate system model was used in the calculation of the force to be exerted, and the result of this calculation was *adaptively deformed* by the RFPT to achieve precise trajectory tracking. The essence of the idea was outlined in [6] and [5]. In [11] it became clear that this simple method containing only altogether 3 adaptive parameters can be developed competitive with the Lyapunov function based technique because an appropriate, model-independent observer was developed for tuning its one parameter to maintain global convergence. In [7] it was shown that the RFPT-based method can cooperate with a modification of the AIDSC that was designed with an evasion of the Lyapunov function. For this purpose a 1 Degree of Freedom (DoF) modified version of the van der Pol Oscillator [10] was utilized. In the present paper our aim is to show a similar possibility for the ASLRC controller. For starting point we go back to its modification introduced in [5]. For simulation purposes and illustrations the same paradigm (a cart+beam+hamper system) will be used here.

2 Cooperation of the Modified Adaptive Slotine-Li Controller and the RFPT-based Design

Let the starting point be the modification of the ASLRC containing an integrated feedback as used in [5].

2.1 The Lyapunov Function Based Tuning

The *integrated tracking error* can be introduced as $\xi(t) \overset{def}{=} \int_{t_0}^{t} \left[q^N(\zeta) - q(\zeta)\right] d\zeta$. If $\Lambda > 0$ (constant symmetric positive definite matrix) an "*error metrics*" can be introduced as $S(t) \overset{def}{=} \left(\frac{d}{dt} + \Lambda\right)^2 \xi(t)$. Furthermore, for the feedback the quantity $v \overset{def}{=} \dot{q}^N + 2\Lambda\dot{\xi} + \Lambda^2\xi$ also is practically defined. Evidently $v - \dot{q} = S$.

As it was shown by Slotine and Li, the *approximate model* of the robot can be described by the *positive definite symmetric inertia matrix* $\hat{H}(q)$, the *special matrix* $\hat{C}(q,\dot{q})$, the approximation of the gravitational term $\hat{g}(q)$, and a positive symmetric matrix K_D, in which variable q denotes the "*Generalized coordinates*" of the robot. Regarding the definition of matrix C this method takes into account the fact that in the Euler-Lagrange equations of motion this matrix is composed from the inertia matrix:

$$
\begin{array}{c}
L \overset{def}{=} \frac{1}{2}\sum_{ij} H_{ij}\dot{q}_i\dot{q}_j - U(q), \\
Q_k = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k}, \\
Q_k = \sum_j H_{kj}\ddot{q}_j + \sum_{ji}\frac{\partial H_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j \\
-\frac{1}{2}\sum_{ij}\frac{\partial H_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j + \frac{\partial U}{\partial q_k},
\end{array}
\tag{1}
$$

in which the product $\dot{q}_i\dot{q}_j$ is *symmetric* in the indices i, j, therefore only the symmetric part of its coefficient yields contribution as

$$
\begin{array}{c}
Q_k = \sum_j H_{kj}\ddot{q}_j + \frac{\partial U}{\partial q_k} + \\
\sum_{ji}\left(\frac{1}{2}\frac{\partial H_{kj}}{\partial q_i} + \frac{1}{2}\frac{\partial H_{ki}}{\partial q_j} - \frac{1}{2}\frac{\partial H_{ij}}{\partial q_k}\right)\dot{q}_i\dot{q}_j \\
C_{kj} \overset{def}{=} \frac{1}{2}\sum_i\left(\frac{\partial H_{kj}}{\partial q_i} + \frac{\partial H_{ki}}{\partial q_j} - \frac{\partial H_{ij}}{\partial q_k}\right)\dot{q}_i
\end{array}
\tag{2}
$$

Let the controller exert the generalized force Q according to (3). An important assumption of the method is that neither unknown external disturbances nor other modeling inaccuracies may exist, therefore the generalized force Q as calculated in the first line of (3) is related to the motion of the system as given by its 2nd line:

$$
\begin{array}{c}
Q = \hat{H}(q)\dot{v} + \hat{C}(q,\dot{q})v + \hat{g}(q) + K_D S \\
Q = H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \\
= Y(q,\dot{q},v,\dot{v})\Theta,
\end{array}
\tag{3}
$$

in which the "exact model values" are denoted by $H(q)$, $C(q,\dot{q})$, and $g(q)$, and it is also utilized that the array of the dynamic model parameters Θ can be written in a separated form in which Y is *exactly known*.

The equality of the left hand sides of the equations in (3) traditionally is utilized as follows. Following the elimination of Q from both sides the *unknown quantities* (the exact matrices are not known) $H\dot{v}$, Cv, g, and $K_D S$ can be subtracted. Since $-H\dot{v} + \ddot{q} = -H\dot{S}$, and $C(-v + \dot{q}) = -CS$, it is obtained that

$$
\begin{array}{c}
(\hat{H} - H)\dot{v} + (\hat{C} - C)v + (\hat{g} - g) = \\
-H\dot{S} - CS - K_D S = Y(\hat{\Theta} - \Theta).
\end{array}
\tag{4}
$$

The Lyapunov function is $V = \frac{1}{2}S^T H(q) S + \frac{1}{2}(\Theta - \hat{\Theta})^T \Gamma (\Theta - \hat{\Theta})$. For guaranteeing negative time-derivative for the Lyapunov function

$$
\begin{array}{c}
\dot{V} = S^T H\dot{S} + \frac{1}{2}S^T\dot{H}S + \\
(\dot{\Theta} - \dot{\hat{\Theta}})^T\Gamma(\Theta - \hat{\Theta})
\end{array}
\tag{5}
$$

must be made negative. From (4) $H\dot{S}$ can be expressed and substituted into (5):

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$$\dot{V} = S^T\left(-Y(\hat{\Theta}-\Theta) - CS - K_D S\right) + S^T\tfrac{1}{2}\dot{H}S + (\dot{\Theta}-\dot{\hat{\Theta}})^T\Gamma(\Theta-\hat{\Theta}). \tag{6}$$

Taking into account that according to (2) $\frac{1}{2}\dot{H}_{kj} - C_{kj} = \frac{1}{2}\sum_i\left(-\frac{\partial H_{ki}}{\partial q_j} + \frac{\partial H_{ij}}{\partial q_k}\right)\dot{q}_i$ is *skew symmetric* in the indices (k,j), $S^T\left(\frac{1}{2}\dot{H} - C\right)S = 0$, and the the condition of the stability is

$$0 > \dot{V} = -S^T K_D S + \left[S^T Y + (\dot{\Theta}-\dot{\hat{\Theta}})^T\Gamma\right](\Theta-\hat{\Theta}). \tag{7}$$

Since normally $\dot{\Theta} \equiv 0$ and K_D is positive definite the appropriate parameter tuning rule can be: $\dot{\hat{\Theta}}^T = S^T Y \Gamma^{-1}$. It worths noting that:

- since in this approach no matrix inversion happens, the speed of parameter tuning can be quite high;
- the actual value of $\dot{V}$ is *independent of* $(\Theta-\hat{\Theta})$ and $\frac{d}{dt}(\Theta-\hat{\Theta})$, therefore if the $S=0$ state is achieved *the parameter tuning process is stopped even if the estimation error is not zero*, and the consequence of any instant disturbance that kicks out S from zero is an immediate decrease in $\|S\|$;
- this method cannot properly compensate the effects of unknown external disturbances and friction forces since in the 1st two lines of (3) the same Q generalized force must occur;
- further problems arise with the systems for which the model cannot be separated as a multiplication of the array of the dynamical parameters and known functions.

The above statements are trivial and do not require illustration via simulation. In the next subsection it will be shown that consistent parameter tuning can be invented without the use of any Lyapunov function.

2.2 Modified Parameter Tuning

Let us return to (3) and observe that if the aim is not the construction of any Lyapunov function the *known terms* as $\hat{H}\ddot{q}$, $\hat{C}\dot{q}$ and $\hat{g}$ can be subtracted from both sides of the equation that was obtained after the elimination of Q. In the result we again obtain the *modeling error* multiplied by *known quantities* at one side, and known quantities will appear at the other side:

$$\begin{aligned}\hat{H}(q)(\dot{v}-\ddot{q}) + \hat{C}(q,\dot{q})(v-\dot{q}) + K_D S = \\ \left[H-\hat{H}\right]\ddot{q} + \left[C-\hat{C}\right]\dot{q} + [g-\hat{g}] = \\ = Z(q,\dot{q},\ddot{q})\left(\Theta-\hat{\Theta}\right)\end{aligned} \tag{8}$$

in which $Z(q,\dot{q},\ddot{q})$ is a *known quantity*. This is a great advantage with respect to (4) in which the left hand side of the 2nd equation is not known since H and C are unknown. Equation (8) has *simple geometric interpretation that directly can be used for parameter tuning* as follows: if *exponential decay rate* could be realized for the parameter estimation error the *array equation* $\frac{d}{dt}\left(\Theta-\hat{\Theta}\right) = -\alpha\left(\Theta-\hat{\Theta}\right)$ $(\alpha > 0)$ should be valid. If we multiply both sides of this equation with a *projector* determined by *a few pairwisely orthogonal unit vectors* as $\sum_i e^{(i)}e^{(i)T}$ the equation $\sum_i\left(\dot{\Theta}_i - \dot{\hat{\Theta}}_i\right) = -\alpha\sum_i e^{(i)}\left(\Theta_i-\hat{\Theta}_i\right)$ is obtained. This situation can well be approximated if we use the Gram-Schmidt algorithm (e.g. [8], [9]) for finding the *orthogonal components* of the rows of matrix Z in (8). Assuming that the speed of variation of Z is not too significant, we can apply the tuning rule *only for the known components* in the form: $\frac{d}{dt}(\Theta-\hat{\Theta}) = -\alpha\sum_i\frac{\tilde{z}^{(i)}\tilde{z}^{(i)T}}{\|\tilde{z}^{(i)}\|^2+\varepsilon}(\Theta-\hat{\Theta})$ in which $\tilde{z}^{(i)}$ denotes the transpose of the orthogonalized rows of matrix Z, and a small $\varepsilon > 0$ evades division by zero whenever the norm of the appropriate row is too small. Since the scalar product is a *linear operation* during the orthogonalization process the appropriate linear combinations of the scalar products in the 3rd row of (8) can be computed.

2.3 Further Modification in the Exerted Force/Torque Components

It is evident that all the above considerations remain valid if in the place of $\hat{H}\dot{v}$ some different term is written in (8). (Obtaining exactly $\dot{S}$ was important only for the construction of a Lyapunov function.) So useful information can be obtained for model parameter tuning if in the exerted forces this term is replaced by its iterative variant obtained from the RFPT-base design as follows:

$$\begin{aligned} h := f(r_n) - r^d_{n+1}, \quad e := h/\|h\|, \\ \tilde{B} = B_c\sigma(A_c\|h\|) \\ r_{n+1} = (1+\tilde{B})r_n + \tilde{B}K_c e \end{aligned} \tag{9}$$

in which $\sigma(x) \stackrel{def}{=} \frac{x}{1+|x|}$, $r^d_{n+1} \stackrel{def}{=} v_{n+1}$, r_n denotes the adaptively deformed control signal used instead of v_n control in control cycle n, and $f(r_n) \equiv \ddot{q}_n$, i.e. the *observed system response* in cycle n. It is evident that if $f(r_n) = r^d_{n+1}$ then $r_{n+1} = r_n$, that is the solution of the control task (i.e. the appropriate adaptive deformation) is the fixed point of the mapping defined in (9). Since the details of the convergence were discussed in ample literature references in the sequel only simulation results will be presented to reveal the cooperation of the FRPT-based adaptivity and model parameter tuning.

3 Comparative Simulation Results

For the simulations the same cart+beam+hamper system was used as in [4] with the Euler–Lagrange equations of motion

$$\begin{bmatrix} (ML^2+\theta) & \theta & mLcosq_1 \\ \theta & \theta & 0 \\ mLcosq_1 & 0 & (m+M) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \\ + \begin{bmatrix} -mgLsinq_1 \\ 0 \\ -mLsinq_1\dot{q}_1^2 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}. \tag{10}$$

in which $M = 30\,kg$ and $m = 10\,kg$ denote the masses of the cart and the hamper respectively (the mass of the beam connecting the hamper to the cart is neglected), $\theta = 20\,kg\cdot m^2$ describes the momentum of the hamper referenced to its rotary axle on which its mass center point is located, $L = 2\,m$ denotes the length of the beam, and $g = 10\,m/s^2$ in this case denotes the gravitational acceleration. With the definition $\Theta \stackrel{def}{=} [mL, mL^2+\theta, \theta, M+m, mgL]^T$ matrix Z easily can be constructed. The *approximate model parameters* were $\hat{M} = 60\,kg$ and $\hat{m} = 20\,kg$, $\hat{\theta} = 50\,kg\cdot m^2$, $\hat{L} = 2.5\,m$ (in the dynamical calculations), and $\hat{g} = 8\,m/s^2$. These settings correspond to $\hat{\Theta}_{ini} = [50, 175, 50, 80, 400]^T$, and $\Theta = [20, 60, 20, 40, 200]^T$.

3.1 Cooperation in the Lack of External Disturbances

In the 1st step it will be illustrated that the RFPT-based design can well coexist with the dynamical parameter tuning in the absence of disturbances. The control parameters were as follows: $\Lambda = 10/s$, $\alpha = 1/s$, $K_D = 100/s$, $K_c = -10^7$, $B_c = 1$, and $A_c = 10^{-8}$, the cycle time and the time-resolution of the numerical (Euler-type) integration was $\delta t = 10^{-4}\,s$. According to Fig. 1 the application of the RFPT considerably improved the tracking precision. As it is displayed by Fig. 2 the initially strongly over-estimated parameters are tuned in similar manner.

3.2 Cooperation under the Effect of a LuGre Friction at Axle 3

For disturbances a LuGre-type friction was introduced at axle 3 as it was done in [4]. This model cannot be

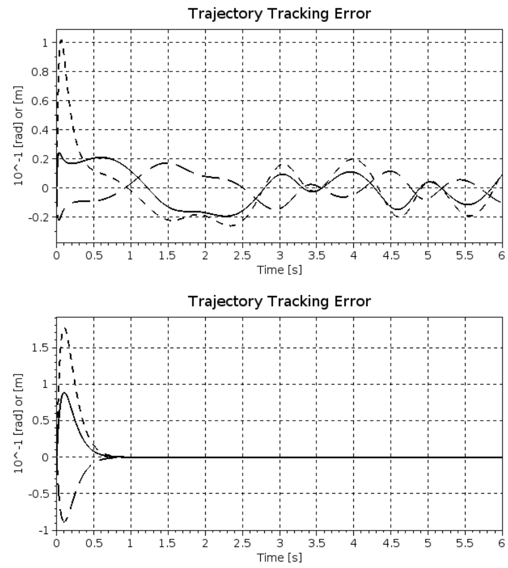

Figure 1: The tracking error in the lack of unknown disturbances: with modified tuning *without RFPT* (upper chart), and modified tuning *with RFPT* (lower chart)[q_1: solid, q_2: dashed, q_3: dense dash lines]

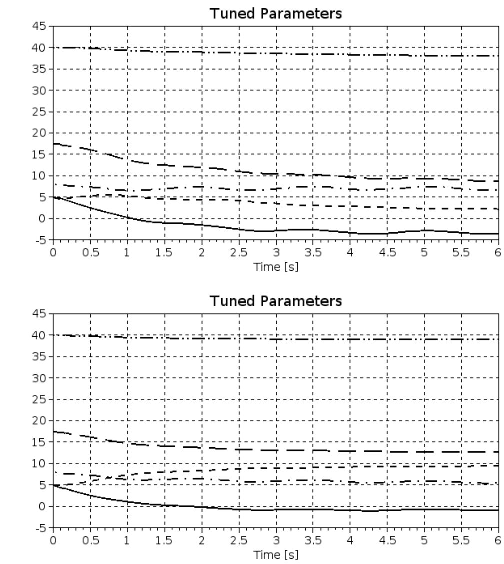

Figure 2: Tuning of the adaptive parameters in the lack of unknown disturbances: with modified tuning *without RFPT* (upper chart), and modified tuning *with RFPT* (lower chart)[Θ_1: solid, Θ_2: dashed, Θ_3: dense dash, Θ_4: dash-dot, and Θ_5: dash-dot-dot lines]

taken into account in a "separated form" and also contains an internal dynamic variable that is not modeled by our controller (it is used only in the simulations. Figure 3 reveals that the application of the RFPT again considerably improved the tracking error, with the exception of the initial "transient" section.

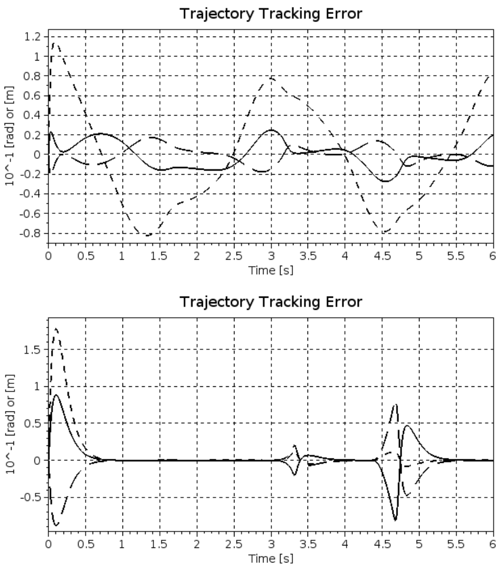

Figure 3: The tracking error under unknown disturbances: with modified tuning *without RFPT* (upper chart), and modified tuning *with RFPT* (lower chart)[q_1: solid, q_2: dashed, q_3: dense dash lines]

Figure 4 reveals that similar "abnormal" tuning-discrepancies occur in both cases, but the RFPT-based method well compensates the simultaneous consequences of the disturbances and improper parameter tuning.

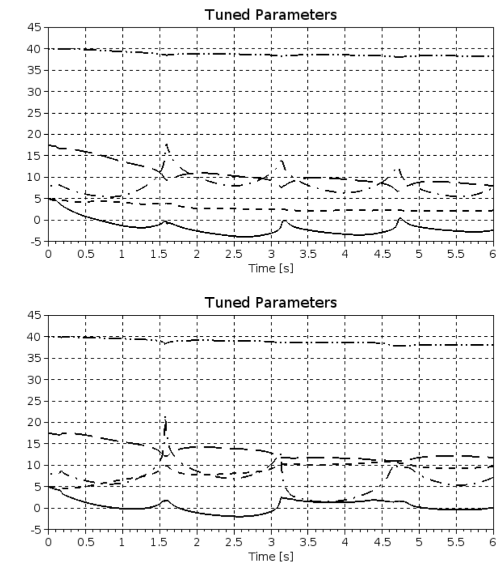

Figure 4: Tuning of the adaptive parameters under unknown disturbances: with modified tuning *without RFPT* (upper chart), and modified tuning *with RFPT* (lower chart)[Θ_1: solid, Θ_2: dashed, Θ_3: dense dash, Θ_4: dash-dot, and Θ_5: dash-dot-dot lines]

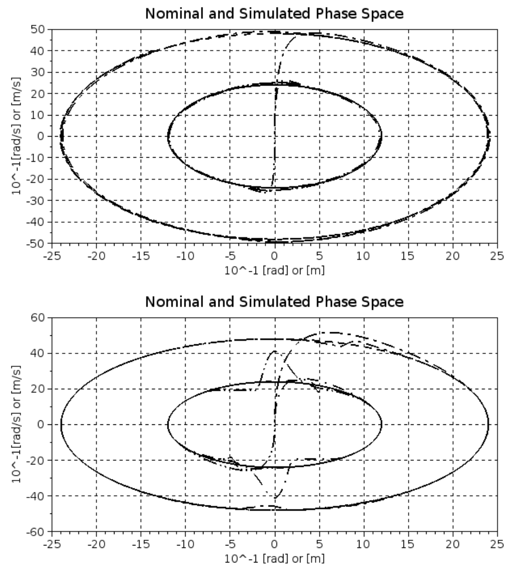

Figure 5: The phase trajectories under unknown disturbances: with modified tuning *without RFPT* (upper chart), and modified tuning *with RFPT* (lower chart)[q_1: solid, q_2: dashed, q_3: dense dash lines]

In this case important details are revealed by the phase trajectories (Fig. 5). Without using the RFPT-based adaptation more even and greater tracking errors are present. The RFPT reduces these errors in long sections, while in the problematic sections it generates significant changes in the phase space. Certain details can be also observed in the charts of trajectory tracking (Fig. 6). In Fig. 7 the operation of the RFPT-based method is illustrated: the realized (simulated) 2nd time-derivative is in the close vicinity of the kinematically computed "desired" value, i.e. the primary design intent, i.e. the realization of a kinemtically prescribed tracking error relaxation is successfully demosntrated.

4 Conclusions

In this paper the successful co-operation and symbiosis of a modified traditional adaptive controller, the "Slotine-Li Adaptive Robot Controller" and the "Robust Fixed Point Transformations"-based design was demonstrated. s a result, the combined controller can correctly learn the parameters of the controlled system if unknown external perturbations are absent while the RFPT-based part guarantees precise trajectory tracking even in the early phase of the adaptive learning.

It was also shown that though the unknown external disturbances cause "parameter learning", the RFPT-based part is able to keep precise trajectory tracking even in this case, too.

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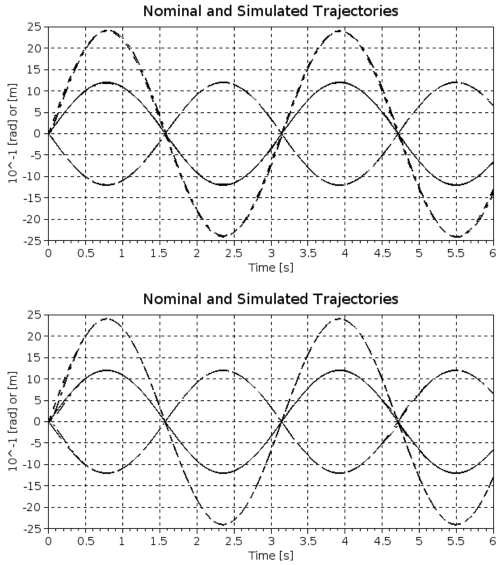

Figure 6: The trajectory tracking under unknown disturbances: with modified tuning *without RFPT* (upper chart), and modified tuning *with RFPT* (lower chart)[q_1: solid, q_2: dashed, q_3: dense dash lines]

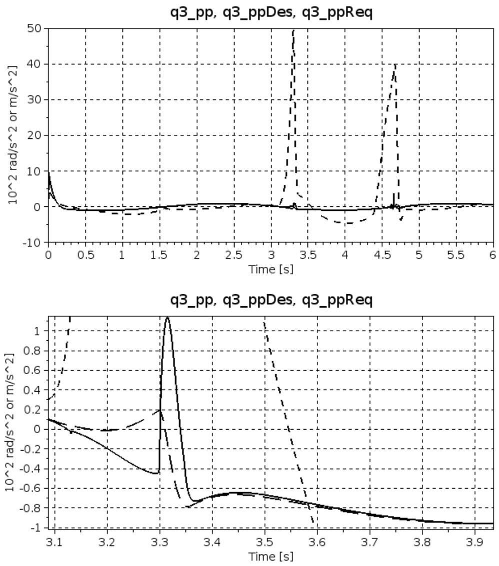

Figure 7: The second time-derivatives of generalized coordinate q_3 with modified tuning and RFPT-based adaptation (zoomed excerpt in the lower chart) [$\ddot{q}_3$ (realized): solid, $\ddot{q}_3^{Des}$ ("desired"): dashed, $\ddot{q}_3^{Req}$ (adaptively deformed): dense dash lines]

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