

# Mathematical Modeling and Analysis of an Electro-Hydrostatic Servo-Actuator with Brushless d.c. Motor

LIVIU DINCA

University of Craiova, Faculty of Electrical Engineering  
Blv.Decebal,No.107,Craiova,Dolj

ROMANIA

[ldinca@elth.ucv.ro](mailto:ldinca@elth.ucv.ro)

JENICA-ILEANA CORCAU

University of Craiova, Faculty of Electrical Engineering  
Blv.Decebal,No.107,Craiova,Dolj

ROMANIA

[jicorcau@yahoo.ro](mailto:jicorcau@yahoo.ro)

TEODOR LUCIAN GRIGORIE

University of Craiova, Faculty of Electrical Engineering  
Blv.Decebal,No.107,Craiova,Dolj

ROMANIA

[ltgrigorie@yahoo.com](mailto:ltgrigorie@yahoo.com)

NICOLAE JULA

Military Technical Academy

Bd. George Cosbuc, Nr. 81-83, sector 5, Bucharest

ROMANIA

[nicolae.jula@gmail.com](mailto:nicolae.jula@gmail.com)

FLORIN MINGIREANU

Romanian Space Agency (ROSA)

Str. Mendeleev, Nr. 21-25, sector 1, Bucharest

ROMANIA

[florin.mingireanu@rosa.ro](mailto:florin.mingireanu@rosa.ro)

## Abstract

In this paper one presents an electro-hydrostatic servo-actuator (EHA) driven by a brushless d.c. motor. The constructive solution studied here was chosen due to the current trends in making All-Electric Aircraft (AEA). EHA contains two subsystems: the electrical subsystem and the hydraulic subsystem. The hydraulic subsystem is composed by a local gear pump, a hydraulic cylinder with bilateral rod, a hydro-accumulator which has also the role of tank and discharge valves. The electric subsystem contains a three-phase inverter which feeds the pumps brushless motor and also commands and controls the motor. The advantages of the EHA actuators were already proven and experimentally validated and they are already used on aircraft serial. In other papers were presented studies on EHA with d.c. motor driven pump. The motor was fed by a d.c. to d.c. Buck converter. One considered the brushless motor due to its big starting torque contributing to a shorter time response for the servo-actuator and also due to its high reliability in the absence of the brushes.

**Keywords:** electro-hydrostatic, servo-actuator, brushless d.c. motor, mathematical modeling

## 1 Introduction

Aviation accidents caused by the lack of the command surfaces actuations and also the necessity to obtain more efficient actuations lead to search of new solutions for the actuation systems of the aircraft aerodynamic surfaces. The main idea is to use actuators as independent as possible for each command surface. In this aim, one tested electro-mechanic and electro-hydrostatic servo-actuators. Due to the low reliability of the electro-mechanic servo-actuators until now the electro-hydrostatic servo-actuators were preferred. In this situation, the energy transport for the servo-actuators feeding is performed now by the electrical power network, not by the hydraulic centralized network like before. So, one can say the airplane became “more electric”, and concepts of MEA (More Electric Aircraft) and AEA (All Electric Aircraft) [1]-[8] appeared.

Today, the aircraft built on MEA and AEA concepts use mostly the electrical energy in the actuation systems and sub systems rather than pneumatic and hydraulic energy, including the primary flight actuations, air conditioning system, defrosters systems and other small actuation systems.

The researches were focused on two main directions: electro-hydrostatic actuators (EHA) and electro-mechanical actuators (EMA). Usually, in an EMA the electric motor drives the output rod through a gearbox. In an EHA, the electric motor drives first a miniature hydraulic pump and the hydraulic pump circulates the hydraulic liquid to a cylinder. In the experimental researches one found an excessive heating and even jamming of the EMA due to the big stresses it is supposed.

These problems can be solved using additional components for the EMA as cooling system but it increases the complexity, weight and cost of the servo-actuator. On the aircraft the heat evacuation is very difficult due to the confined space. But the EMA is quite good for the secondary flight actuations, where the stresses are lower and the certification requirements are not so strict.

Due to the hydraulic subsystem, the EHA needs no gearbox. The hydraulic resistances present in this subsystem are very small, so the heat generation in this system is at a low level and permits the servo-actuator functioning. Heat exchange problems

appear in this case in the electronic power system, but these are present also in the electronic power system of the EMA.

One can consider the EHA composed from two different subsystems: one hydraulic subsystem and one electric subsystem. The electric subsystem contains the electrical bus, electronic power system and the electric motor. The hydraulic subsystem is composed mainly from a hydraulic pump, a cylinder and auxiliary hydraulic components (valves and filters). In the following we present the principles and some mathematical models for these subsystems.

## 2. Hydraulic Subsystem

In different papers [9]-[13] one proposed different principle schemes for the hydraulic subsystem of an EHA. These solutions aimed to obtain sufficient flows to the hydraulic cylinder, corresponding to the aerodynamic surfaces commands, with as simple as possible construction. One research direction concerned to use a variable displacement pump, driven by the electric motor and commanded by a second hydraulic circuit which contains a hydraulic servo-valve. The electric motor in this case is maintained at constant rotating speed. Other research direction proposed a constant displacement pump driven by a variable speed motor. The first solution has the advantage of a lower demand for the electric motor, its rotating speed being constant, but the hydraulic subsystem is more complex. The second solution simplifies as more as possible the hydraulic subsystem but the electric motor is hard demanded (large and frequent speed and torque variations). This variant implies a more sophisticated electronic power system, but was chosen in order to simplify the hydraulic subsystem, obtaining by this way less global weight and more simplicity for the entire system. The hydraulic scheme of the servo-actuator studied in this paper is shown in Fig. 1. As we mentioned before, it contains a motor, pump, hydraulic cylinder and some auxiliary components (hydro-accumulator, valves and filters) [9].

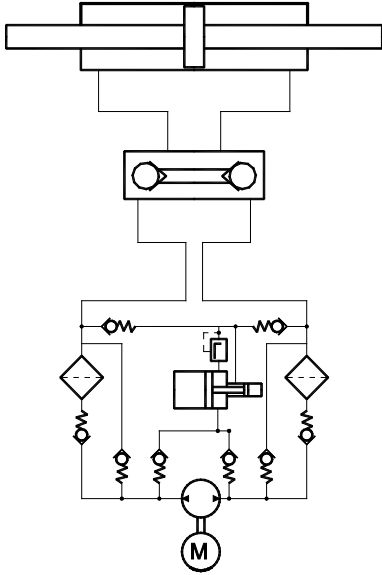


Fig. 1 Hydraulic scheme of the servo-actuator

In order to obtain the mathematical model of the servo-actuator one considered a simplified scheme, without valves, filters and hydro-accumulator. This scheme is shown in Fig. 2.

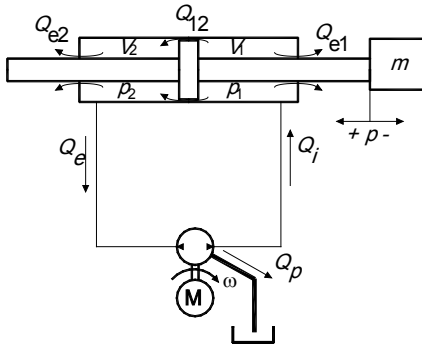


Fig. 2 Simplified hydraulic scheme

The linearized model of the hydraulic subsystem is one used for small displacements around the neutral point and was obtained in a research contract lead by INCAS Bucharest. One considered the load represented by the aerodynamic surface, which was described as a mass-spring-dumper group which simulates its behavior. During the studies one considered also the dry friction between the piston gaskets and the cylinder, but one observed from the numerical simulations their influence upon the servo-actuator dynamic is negligible.

The mathematical model of the hydraulic subsystem considers the liquid compressibility and the

pumps flow rate in order to calculate the pressures in the hydraulic cylinder. These pressures are used further to calculate the piston-command surface ensemble dynamic. As we mentioned before, this ensemble was modeled as a second order oscillator composed by a mass  $m$ , a spring with the constant  $k$  which represents the aerodynamic forces upon the command surface and a viscous dumper  $f$  which models the friction between the piston and cylinder. The model was developed the simulation scheme for the hydraulic subsystem in MATLAB/SIMULINK.

One considered the leakage flow of the pump composed from two flows: one flow between the high pressure chamber and low pressure chamber, and one flow from the high pressure chamber to outside. One can express the pressures in the hydraulic cylinder as it follows:

$$(\beta V_{01} + Sz) \frac{dp_1}{dt} = Q_1 - Q_{e1} - Q_{12} - Q_{p1} - S \frac{dz}{dt} \quad (1)$$

$$(\beta V_{02} - Sz) \frac{dp_2}{dt} = -Q_2 - Q_{e1} + Q_{12} - Q_{p2} + S \frac{dz}{dt} \quad (2)$$

$$Q_{12} = c_{12}(p_1 - p_2) \quad (3)$$

$$Q_{e1} = c_1 p_1 \quad (4)$$

$$Q_{e2} = c_1 p_2 \quad (5)$$

$$Q_{p1} = c_2 p_1 \quad (6)$$

$$Q_{p2} = c_2 p_2 \quad (7)$$

$$Q_{pompa} = D\omega \quad (8)$$

$$m \frac{d^2 z}{dt^2} = (p_2 - p_1)S - f \frac{dz}{dt} - kz - F_u \quad (9)$$

In Eq. (4) to Eq. (7) one considered linear dependence between the pressures and the leakage flows.

In the volumes  $V_{01}$  and  $V_{02}$  one included the cylinder chambers volume and the corresponding pipes volumes. Considering small variations around the neutral point one can obtain the following relations:

$$\frac{d\Delta p_1}{dt} = \frac{1}{\beta V_1} \left( -S \frac{dz}{dt} + D\omega - c_{12}(p_1 - p_2) - c_1 p_1 - c_2 p_1 \right), \quad (10)$$

$$\frac{d\Delta p_2}{dt} = \frac{1}{\beta V_2} \left( S \frac{dz}{dt} - D\omega + c_{12}(p_1 - p_2) - c_1 p_2 - c_2 p_2 \right) \cdot (11)$$

If the movement around the neutral point is taken account, volumes  $V_1$  and  $V_2$  are equal and one can obtain a single relation ( $c_{tp} = c_{12} + c_1 + c_2$ ):

The electrical subsystem contains several components: three-phase diode rectifier, three-phased feeding source (115V, 400Hz), d.c. to d.c. converter, three-phase inverter and

$$\frac{d(\Delta p_1 - \Delta p_2)}{dt} = \frac{1}{\beta V_2} \left[ -2S \frac{dz}{dt} + 2D\omega - c_{tp}(p_1 - p_2) \right] \cdot (12)$$

### 3. Electrical Subsystem

The electrical subsystem contains several components: three-phase diode rectifier, three-phased feeding source (115V, 400Hz), d.c. to d.c. converter, three-phase inverter and permanent magnet synchronous machine with trapezoidal back-emf. One considered included in the in permanent magnet synchronous machine the rotor position measuring system with Hall sensors.

Brushless DC Motor Drive (BLDC) are used in servo applications such as aerospace, automotive, actuation, medical instrumentation, robotics, industrial automation equipment and machine tools. BLDC motors have some advantages over induction motors and conventional brushed DC motors: higher speed ranges, high dynamic response, better speed versus torque characteristics, easy to control, reliable and long operating life [14]-[16].

The equivalent circuit of the BLDC (one phase) is presented in Fig. 3 and the functioning equations are the following [15]:

$$V_t = R_s I_s + L_s \frac{dI_s}{dt} + E_s \quad (13)$$

$$E_s = k_E \omega_r \quad (14)$$

$$T_e = k_T I_s \quad (15)$$

$$T_e = T_L + J \frac{d\omega_r}{dt} + B\omega_r \quad (16)$$

All these components are included in the Brushless DC motor drive (BLDC) block. One considered the existing model in Matlab/ Simulink library with small changes. The control loop was closed by a PI controller, also existing in Simulink. Putting together the two subsystems, the scheme in Fig. 4 resulted.

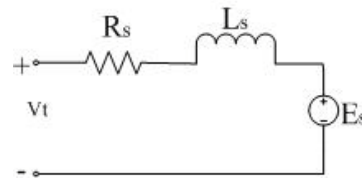


Fig. 3 The equivalent circuit of the BLDC

### 4. Simulation Results

Using scheme in Figure 4 one performed some numerical simulations for the servo-actuator functioning. The systems parameters are presented in [14]. One tested the EHA behavior at sinusoidal and step signals. The results are presented in Fig. 5 and Fig. 6.

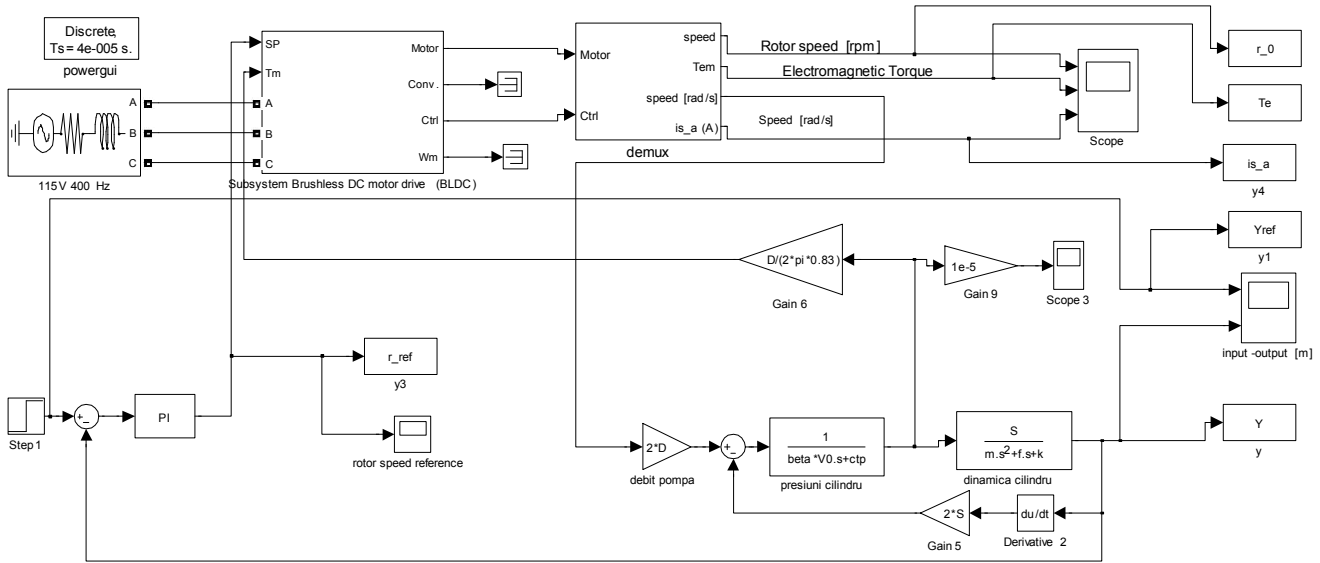
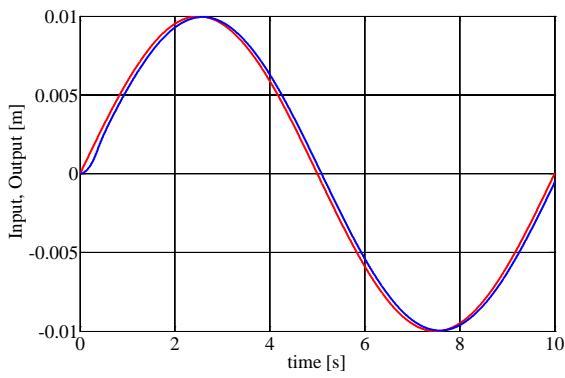
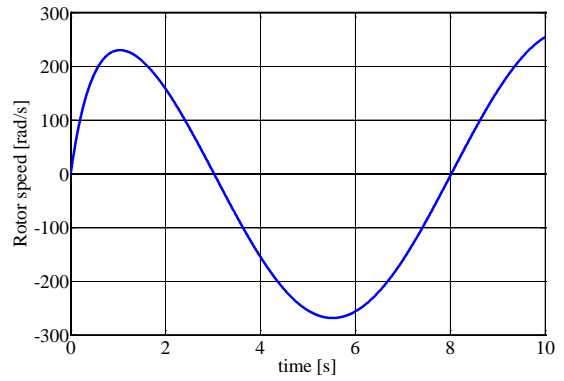


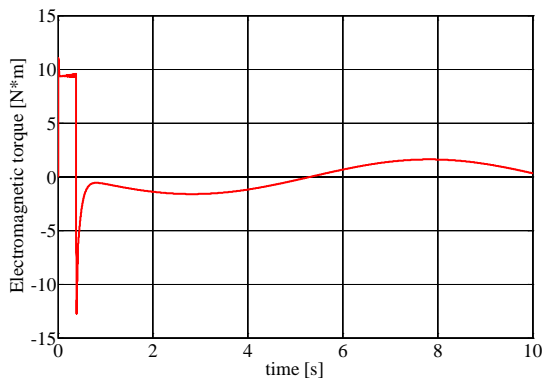
Fig. 4 MATLAB/SIMULINK model of the EHA



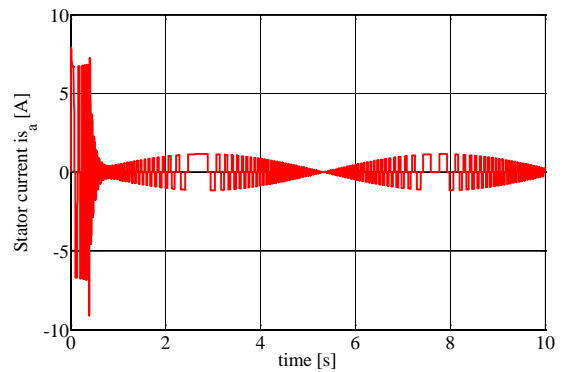
a.



c.



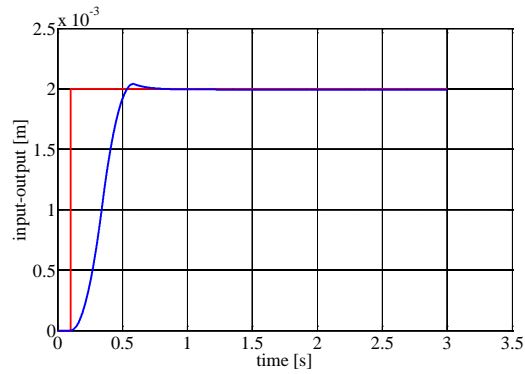
b.



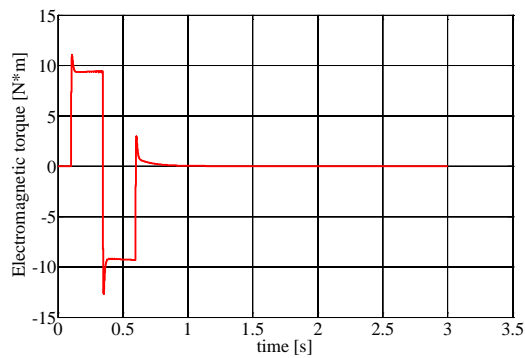
d.

Fig. 5 Response at sinusoidal signal

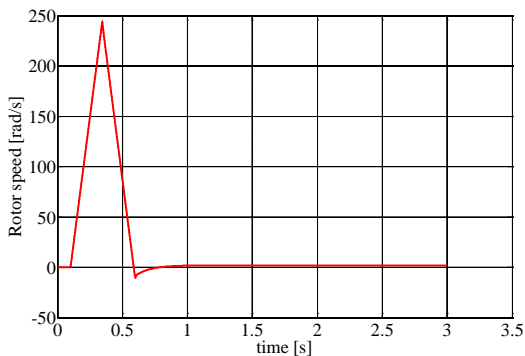
- a. Input, output
- b. Electromagnetic torque
- c. Rotor speed
- d. Stator current



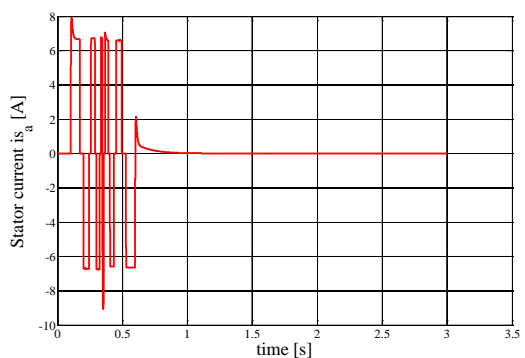
a.



b.



c.



d.

Fig. 6 Response at step signal

- a. Input, output
- b. Electromagnetic torque
- c. Rotor speed
- d. Stator current

## 5. Conclusions

In this paper is presented a mathematical model for an electro-hydrostatic servo-actuator driven by a BLDC motor. The hydraulic subsystem was modeled using the results obtained in other authors' papers and the electric and electronic subsystem was modeled using predefined blocks in Matlab/Simulink. One studied the possibility to use BLDC motor in the servo-actuator due to the advantages with respect the classical d.c. motors - higher speed ranges, high dynamic response, better speed versus torque characteristics, easy to control, reliable and long operating life. Based on simulations it was found a good behavior which permits to use this EHA in the command chain of an aircraft. Given the advantages of the brushless motor with respect the classical d.c. motor, one consider this solution promising. The presented solution can be improved both from the electric subsystem point of view and the controller point of view. In the works [9] – [13] an electro-hydrostatic servo-actuator driven by a classical d.c. motor was studied. An important advantage of the BLDC motor shown in the simulations is a smaller current with respect the classical d.c. motor. BLDC needs 8A/phase, while using a classical d.c. motor, 50 A peaks appear. Although the control scheme for a BLDC is more sophisticated than for a classical d.c. motor, smaller currents produce less heat, so an improved functioning is achieved both for the servo-actuator and for the entire system. Is well known an important problem of EHA and EMA using on aircraft is the heat evacuation from power electronics and also from the mechanical system of the EMA.

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