Optimal Movement Planning of Semi-independent Elements

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Summary. The result of a logical and mathematical analysis led to a reconsideration of the graph notion: a particular construction can be attached to a formalized space for only two orthogonal types of movements when initiated movements from one specific location runs in straight line until an obstacle interposes(and then the movement can continue following the same rules: two dimensions, automatically start, change of direction when an obstacle interposes).If the space is populated with more mobile and/or fixed elements – beyond its limits – the structure of the mentioned graph becomes variable.Thereby, the movement of a mobile element between the initial point of placement and the final destination becomes, using the route which involves a minimum number of steps, a problem which has a multi-spacial domain, the solution found in this paper relying on the transfer between stacked graphs in parallel planes.

Key-words. Orientated graphs, transfer between graphs, shortest paths, warehouses, mobile and fixed racks.

1 Warehouses with fixed racks versus warehouses with mobile racks

When a warehouse is being expertised , one of the most apparent disadvantage is the free space between the racks(the space which is unoccupied by the racks), the absolute space which is needed for the movement of the handling equipments.The main role of handling equipments is to carry and to store the goods in the predeterminated locations inside the warehouse, locations which previously were free.According to some studies, the allocated surface for these intern transports (not for storring itself) reaches almost to one third of the total surface of the warehouse- based on merchandise and handling equipment type.In this context, a way to reduce this wasted proportion suppose to adopt a cinematic procedure to realize the warehouse-exterior exchange : equipping the warehouse with mobile racks(instead of fixed racks) that should autonomously move the exchange point(door, gate, sales ramp etc.).Obviously, the free space reserved for the movements of the autonomous racks is lower than the space reserved for mobile racks itself (it is well known the toy that has for example 3*3 square locations of which only one free- all square pieces- which are movable only on two axes-fig.1- and that by any number of piece movements can permute till the marked piece reaches the initial empty location 3,3 :

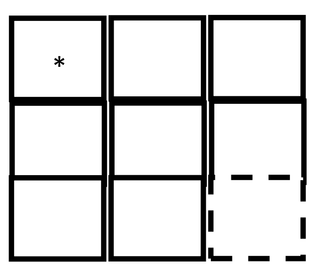

Fig.1 The “toy origin” for the procedure shown in this material

Similarly, in a warehouse, mobile racks can free space one to another (in other therms, in standby, the racks may occupy any location and without any distancing from each other, using the entire space of the warehouse up to values close tu 100%).This approach has at least two perspectives:

- One in which there is a constant exchange of information between the racks and running path: in this case the movement from the initial location of the rack to the exchange point is carried out on a route - consisting of orthogonal segments (racks are not allowed to "make" curves) impregnated in the electronic system which is embedded in the running path and can order to move and to stop in the target point.
- Another in which this exchange of information doesn't exist, because of the common running path, inert in transinformational terms: in which case the racking movement depends only on the mobile rack (that can move only on two axes of the running path plane); the consequence of the lack of running path guidance system is that rack will move following the ordered direction till an obstacle is met.

Intuitively:

- The first choice will lead to a truly substantial unused space saving, but will require a major investment in the floor embedded electronic system.
- The second will reduce the unused space in a lower measure, but the level of investment will be much lower (the reader is asked to note that expensive cars are made, but no investments have been realized in the road infrastructure yet - in terms of the automatic vehicles guidance system - although theoretically and practically this thing is possible).

However ,in any case, the transition from fixed racks to mobile racks requires specialized software for the coordination of movements. Softwares should be based on mathematical models in order to intercede the implementation of the movement idea from the location where the required merchandise is located to the exchange point. While for the first alternative the racks movements can be directed moment by moment (actually digital, location to location) in the second alternative, mobile racks can be set in motion, but their movements can only be limited by physical barriers (actually by the other racks and walls of the warehouse). Emphasized that the racks movements can be made through mechanisms which can't provide steering movements namely the movements can be made on the two axes of the running path (if the racks would have been equipped with steering mechanisms, the costs for these steering mechanisms might become prohibitive and the space for performing the movements will decrease the previous claimed economy). Two simple examples will clarify some aspects of the problem.

Example 1.The information: exchange point requires the A rack from 1,1 location to 3,2 location. Solution: 4 units of length.

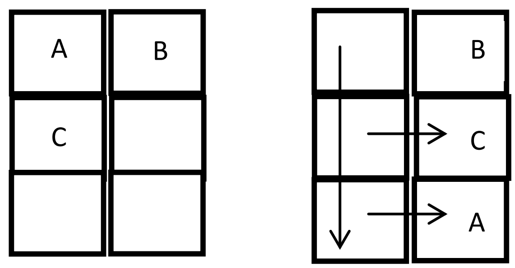

Fig.2 A simple solution for the movement problem of autonomous racks

But for a larger warehouse, if the information is the same,we have the following solution: 6 units of lenght (because if the 3,3 location would be unoccupied by the rack C, the rack A would move up to 3.3 location).

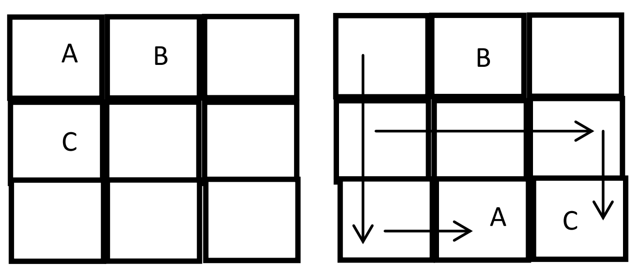

Fig.3 A complex solution for the movement problem of autonomous racks

2 The physical model

For analyzing this problem let it be the following situation - Fig. 4:

- The warehouse is spread over 5/6 locations (relative equal size like the racks)

- Circumstances include three fixed locations represented by *X*, two mobile racks represented by S, a target rack which is required to operate a goods exchange "1", exchange point is "0".

And for the formalization of the problem matrix addressing type is being used: from line 1, column 1 to line 6, column 5.

	1			
X			S	
			X	
S				
	X			
			0	

Fig.4 The situation chosen to be solved

Working graph for solving the problem is shown in Fig.5 . The requirements of the problem are:

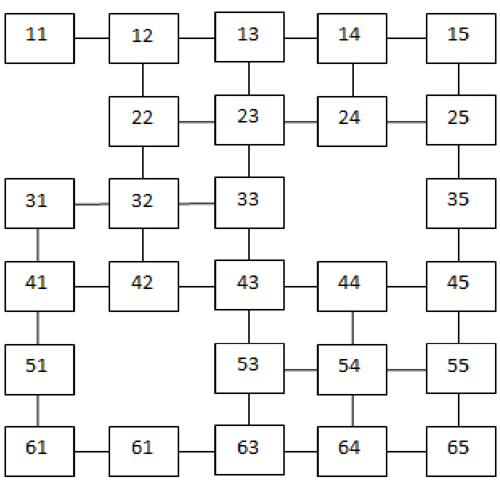

Fig.5 The previous situation graph

- to bring target rack (placed in location 12) to the exchange point (placed in location 64) crossing fewer locations,
- basically on the shortest path between origin and destination, because the requirement which provides minimal minimal waiting time for the target rack to be brought to the exchange point can be accepted – hypothesis which is perfectly consistent with the spirit of action to accelerate the transit of goods through the warehouse.

Taking into account that the movement of any rack can be made only on the two axes of the running path, and changing the direction can be made only after the linear movement of the rack is stuck by the warehouse walls or other rack (eg Route 43, 53, 63 is blocked by the wall of the warehouse, without introducing any organizational limitation the route goes through 64-65 passing over the exchange point).

An example of desired route is the overall is represented by all locations that provide in any rational solution (down and to the right movements) the shortest path between the location of the targe track and the exchange point. Let it be two types of the shortest path:
As is shown for the first option - fig. 6 - three "obstacles" placed on the route between origin and destination are necessary, two of them being provided by the structure of the warehouse (the name of these obstacles is Stops = structural stop point) and one obstacle provided by mobile racks (the name of these obstacles is STOPO =organisational stop point) and target rack performs seven steps.According to the image for the second version -fig. 7 - also four "obstacles" are needed but three of them are STOPO (and at the same number of steps required by the target rack coverage points procedure will involve a higher number of steps than for the additional steps that implicated by the first version): and target rack performs 7 steps.

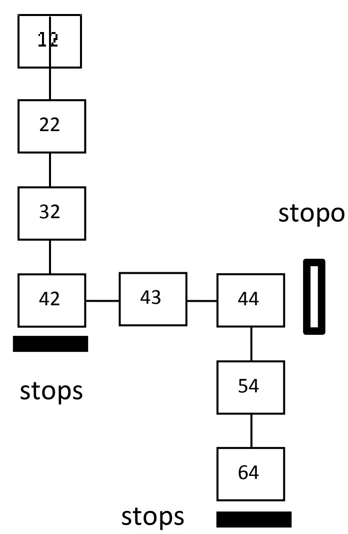

Fig.6 The shortest path involves one stop

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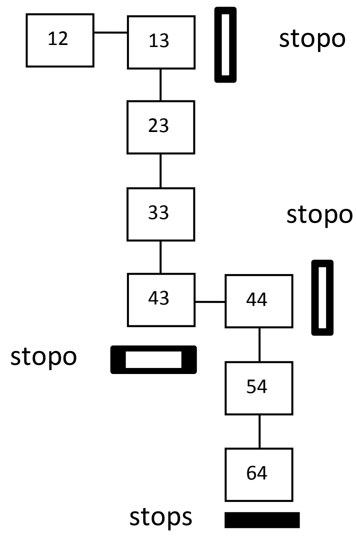

Fig.7 The shortest path involves 3 stopo

In the context of the first version the solution involves finding of a second shortest path between one of the mobile racks and the stopo point required by the target rack route.

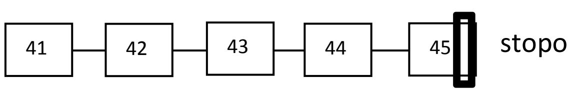

Fig.8 The shortest path for the helping rack

Summarising, the exemplified situation has a solution presented in the below figure:

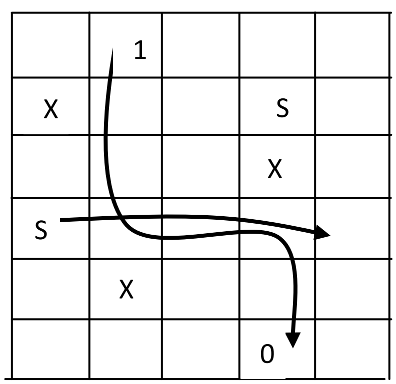

Fig.9 The solution for the chosen situation

3 The mathematical model

In the following figures movement possibilities ("own routes") are presented for each of the three mobile elements:

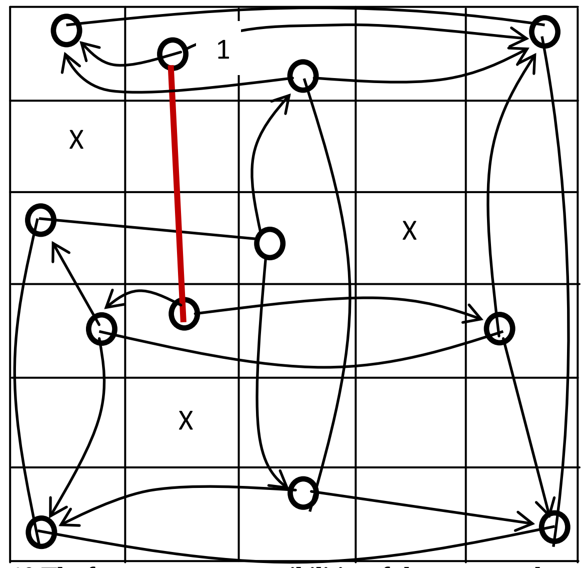

Fig.10 The free movement possibilities of the target rack

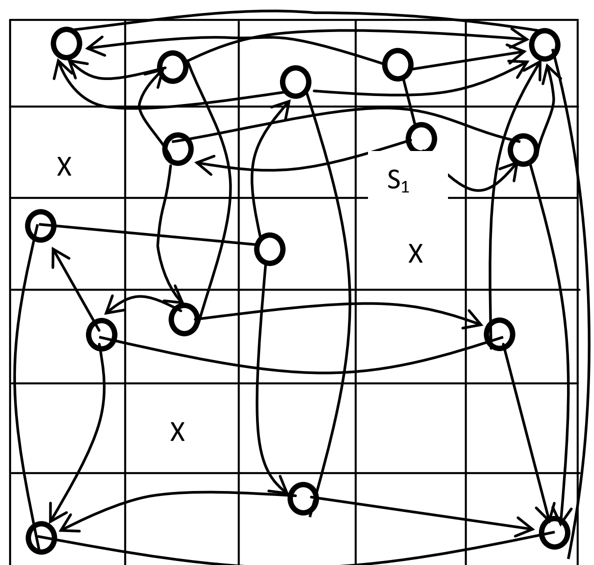

Fig.11 The free movement possibilities of the first mobile rack

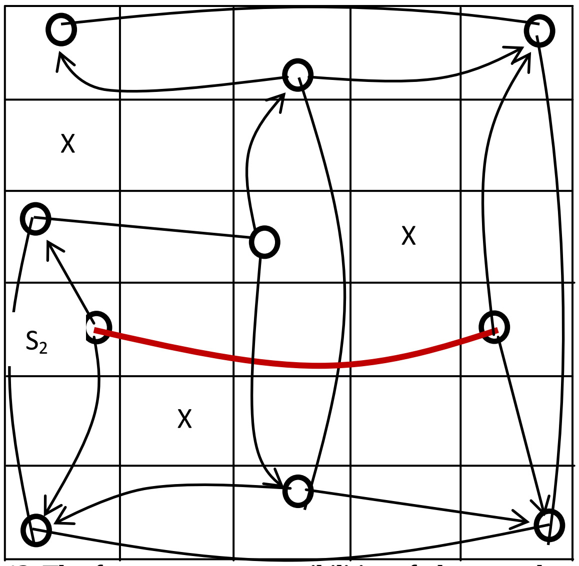

Fig.12 The free movement possibilities of the second mobile rack

The proposed heuristic procedure has been developed through a exhaustive study on a restraint dimensional case - because of the complicated involved graphs (but by the description of the procedure we will return to the wider situation which has been described by graphs from the Fig. 10, 11 and 12).

Let it be the next situation (1 = target rack, 0 = destination):

	1	
X		
	0	

Fig. 13 Test-situation

The graph that describes the movements of the target rack:

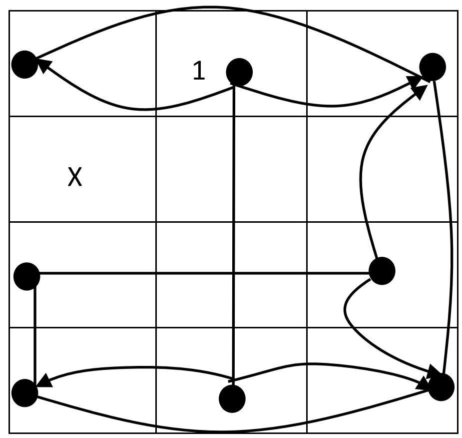

Fig. 14 Target rack graph

The graph that describes the movements of a fictive rack placed in point "0" :

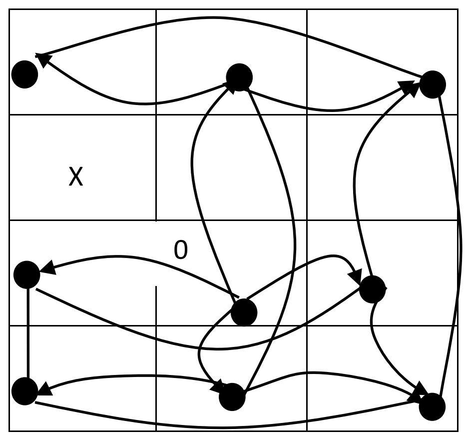

Fig.15 The graph of the fictive rack (in point "0")

The reversed graph of the point "0"-which describes the movement to the destination is-0:

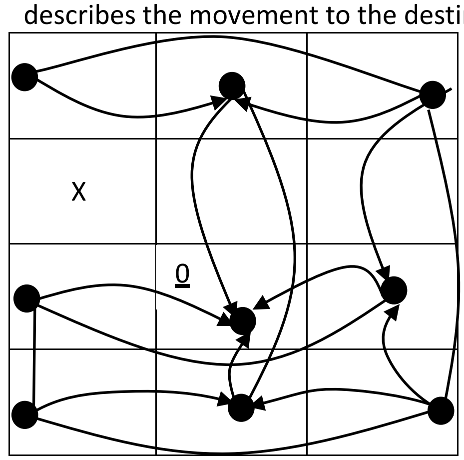

Fig.16 The "reversed" graph

Basically, for the rack 1 to reach the point 0 is necessary that on the shortest running path to go from graph 1 to graph 0. It is considered so all of the two graphs, plus a number of transfer points with orientated arcs from the graph 1 to graph 0 only from the nodes of the graph 1 to the homologous nodes of the graph 0 or between counterparts arcs - with the same orientation in both graphs - Fig. 17. If the shortest running path between point 1 and point 0 from the initial table of the situation to be solved is directly represented it can be seen that the problem has no solution because after passing to graph 0 reaching the destionation is conditioned by the existence of at least one stopo (target rack will not be brought to destination without any other mobile rack).

But if in the colored circle a mobile rack would be find the problem would have a solution (the picture above must be analyzed as two graphs each in his plane, but with transfer possibilities from the close to the distant plane).

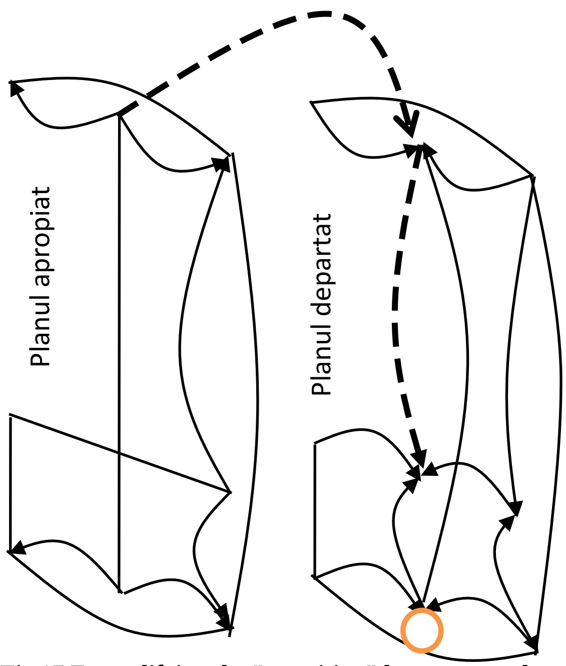

Fig.17 Exemplifying the "transition" between graphs

Let it be the situation where there is still a mobile rack S racking Help - location 23. Graph of movements rack S:

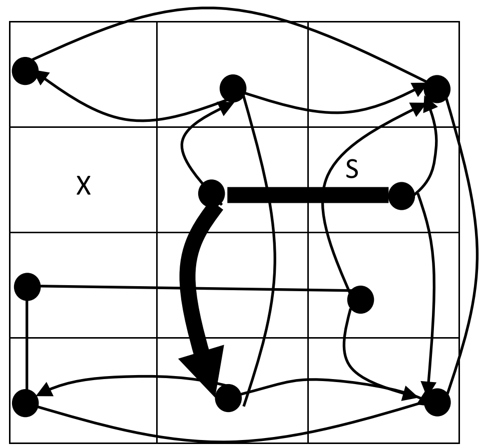

Fig. 18 Mobile rack graph

It is found that the helping rack can reach independently the stopo point which could prevent the target rack to override the destination (and his shortest path way is shown above with thick lines). The solution is (in 5 steps):

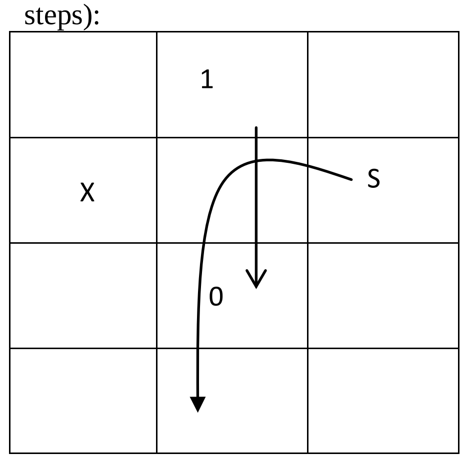

Fig.19 The solution of the test-situation

Below there are presented the cases that would artificially require that the arrival of the target rack to the destination point to be prevented by a stopo point which will be located - by turn - in each of the three remaining locations surrounding the destination.

Next case with stopo point at the left; Further is presented the shortest path of the rack 1 with transfer from one graph to another, but requiring 2 stopo points:

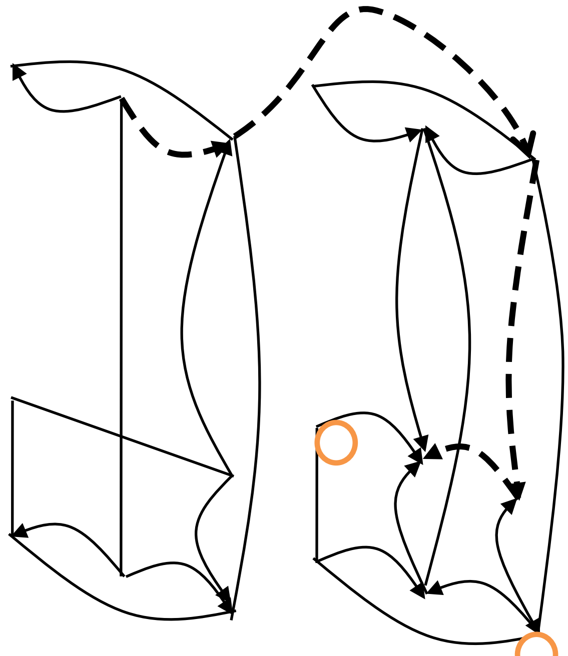

Fig.20 The case with stopo in location 3,1.

From the movements graph S results that the helping rack can reach by turn in both stopo points. The solution is the next - in 9 steps (of course that the helping rack will follow the shortest running path) - Fig. 21.

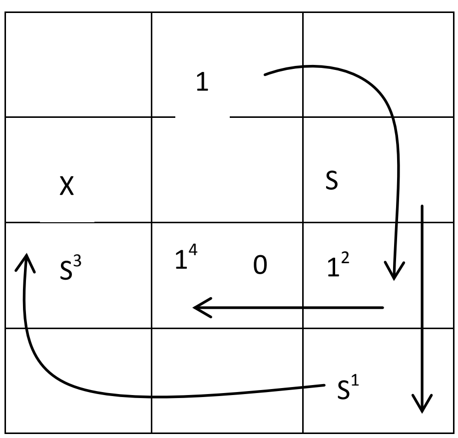

Fig.21 The solution with stopo in location 3,1 (the exponents show the order of movements)

The next case-stopo on the right – Fig.22.

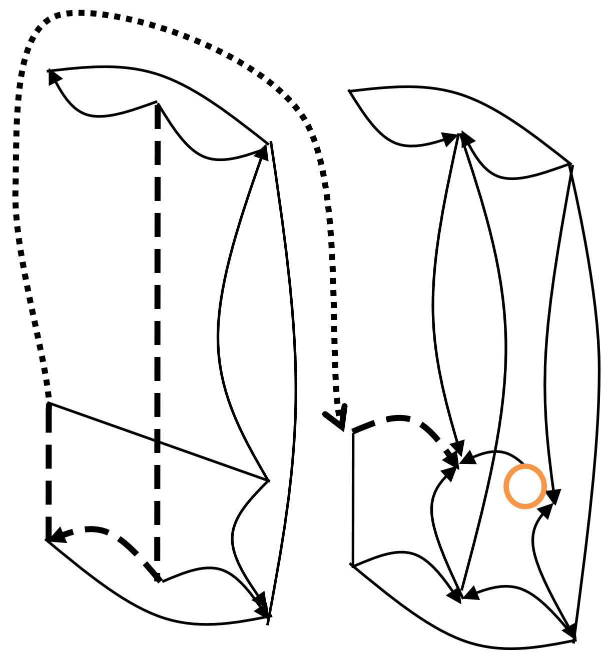

Fig. 22 The case with stopo in location 3,3

Also the graph S shows that the helping rack can reach independently the stopo point. The solution is the next - in 13 steps (of course that the helping rack will follow the shortest running path):

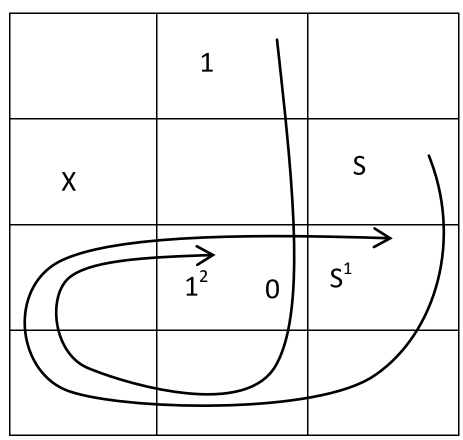

Fig.23 The solution with stopo in location 3,3

The last case-stopo point on the bottom.

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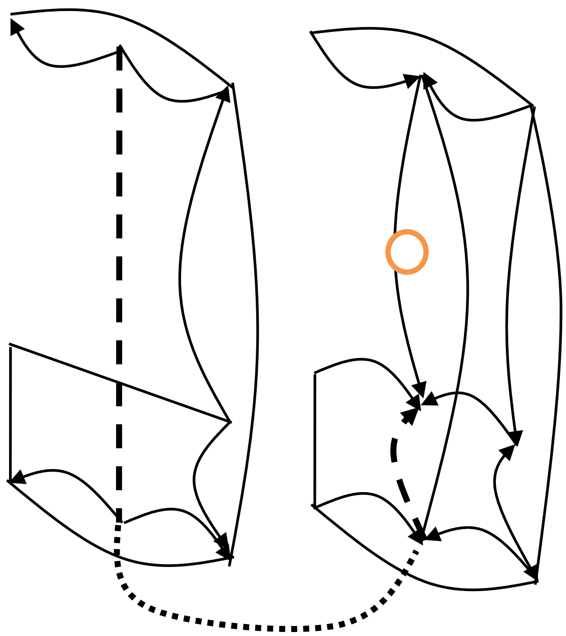

Fig.24 The case with stopo in location 2,2

Also the graph S shows that the helping rack can reach independently the stopo point.

The solution is the next - in 13 steps (of course that the helping rack will follow the shortest running path):

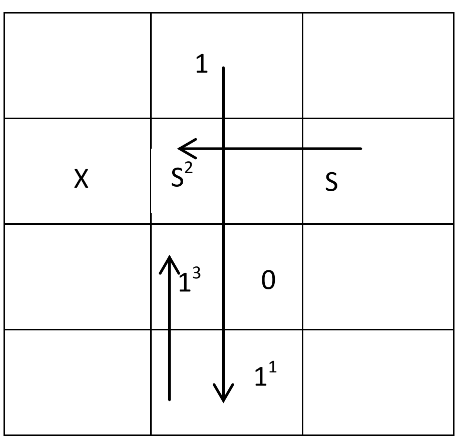

Fig.25 The solution for stopo point placed in location 2,2

Finally the procedure is applied on the main situation. In the following figures are represented the movement possibilities ("own routes") of a fictional rack from the destination point and the "reversed" of these movements."Movement" possibilities of zero point (location 64):

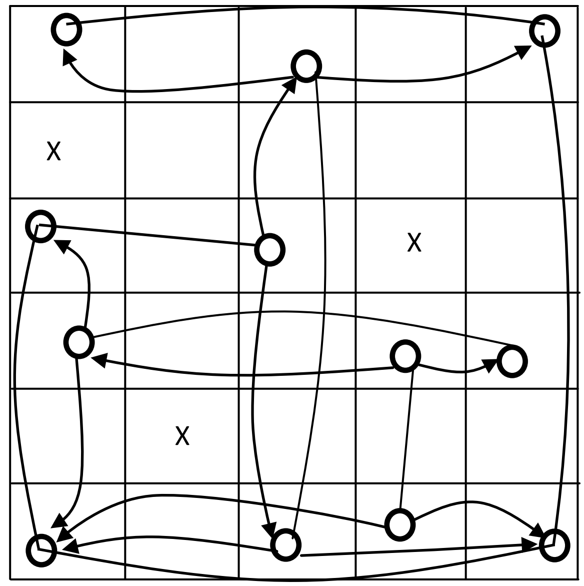

Fig.26 Graful miscarilor din punctul de schimb

"Movement" possibilities to the zero point (location 64):

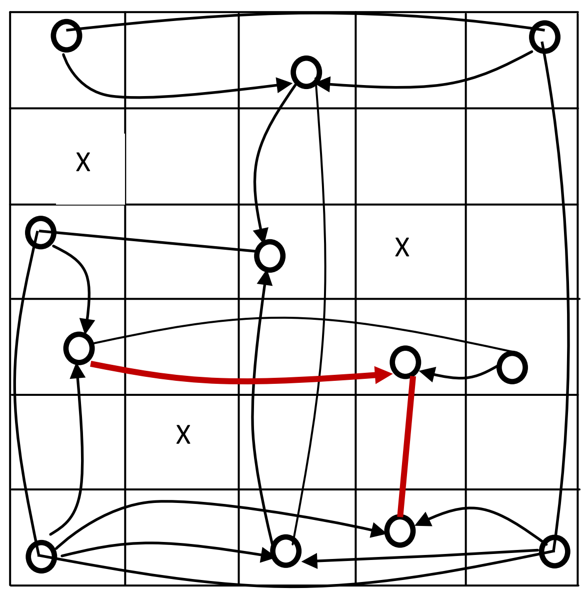

Fig.27 The graph of the reversed movements to the exchange point

Analizing the two essential graphs (target rack and the movements to the reversed destination) and passing on the shortest path from the first graph to the second graph - colored links Fig. 10 and Fig. 27) results that a stopo point is necessary in location 45.

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Counting the movement possibilities of the two mobile racks Fig. 11 and 12 it follows that both can reach the stopo point, but rack 2 requires fewer steps. So the solution (in 11 steps) is already advanced in Fig. 9.

4.Conclusions

The following structure procedure is foreseen:

- The possible paths for the target rack are determinated.
- The “reversed” paths are determinated from the perspective of destination.
- The stopo points that are necessary on the trajectory of the target rack from a start graph and a final graph are identified(the stopo points become destinations for the helping rack).
- The possible paths of the helping rack are determinated.
- The shortest paths of the helping racks until reaching the required points are determinated.

The authors are forced to admit: the problem analyzed is only reached but not in any case exhausted; directions for the future:

- It is possible that the target rack routes to be disrupted by the helping rack. What way should be followed ? (bypassing or removing obstacles are allowed
- Certainly there can arise situations in which a roundabout path of the mobile rack involves so shorter paths that all movements ensemble is inferior than in the situation in which target rack follows the shortest path.
- The movements of the target and mobile can be simultaneously performed ; in this case the most important thing is the time until the target rack reaches the exchange point(on the other hand simultaneous movements can lead to conflict situations which should be avoid through modelling).

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