Abstract: This paper deals with an embedded control system for an aircraft jet engine with afterburning. Engine’s mathematical model was built from VK-1F engine. The system uses also three controllers (for engine’s speed, for exhaust nozzle and for afterburner’s fuel injection), which were already modeled and presented in other papers and formally adapted to the studied engine. Some simulations were performed, based on the combined mathematical models and several useful conclusions were drawn, regarding system’s behavior. System’s model, method and its time behavior conclusions can be useful for similar further studies.

Key-Words: afterburning, fuel, control, exhaust nozzle, throttle, pressure ratio, temperature, thrust.

1. Introduction
Thrust augmentation for aircraft engines involves several methods. The most important of them is the afterburning, which involves both constructive and gas-dynamics measures, such as variable exhaust nozzle and supplementary fuel injection system(s).

Aircraft jet engines as controlled objects are studied in [10], [11] and [12], where the authors have identified, amongst a multitude of parameters, possible control parameters (inputs), possible controlled parameters (outputs), as well as theoretical command laws. As input parameters, one has identified only three: a) the combustor fuel flow rate $Q_c$ (for all engine types); b) the exhaust nozzle’s opening $A_5$ (for engines with variable-area nozzles); c) the afterburner fuel flow rate $Q_p$ (for engines with afterburning systems).

However, the aircraft’s pilot has at his disposal only the throttle (as unique possibility to control the engine). Consequently, the throttle has to generate somehow, by its displacement, the input signals forming, which means that engine input parameters should be determined as some other control sub-systems’ outputs.

This paper has as main purpose to identify the embedded system (single-spool jet-engine and afterburning system, also called EAS) as controlled object and to determine its simplified mathematical model, as well as its time behavior.

Fig. 1 presents such an EAS, its main parts being: a) the air inlet; b) the compressor; c) the engine’s combustor; d) the gas-turbine, its rotor being connected to the compressor’s rotor through the shaft, resulting the engine’s spool; e) the afterburner with fuel injectors and flame stabilizers; f) the adjustable exhaust nozzle; fuel systems and exhaust nozzle’s control systems are not presented.

One can affirm that EAS is an interconnection between two propulsion systems, the basic engine and the afterburning, both from gas-dynamic and control point of view.
2. System presentation

EAS control system must assure, in the same time, a correct operation of both of embedded propulsion systems, while the unique possible input is the unique throttle’s position. Block diagram in Fig. 2 presents such a technical possibility, which will be further discussed and studied.

From the automatic control system’s point of view, one can identify three control sub-systems:

a) basic engine’s fuel injection control system, as a speed control system;

b) exhaust nozzle control system, with respect to the basic engine’s turbine’s pressure ratio;

c) afterburning fuel injection control system.

Aircraft single-spool single-jet engine with variable exhaust nozzle was already studied, so this paper studies its operation working together with an afterburning system, similar with the studies realized in [15] and [16]. Consequently, only input parameter for the discussed scheme is the throttle’s position and as output parameters one can assume the afterburner’s gases temperature, as well as total thrust.

One has chosen an exhaust nozzle control sub-system, independent of throttle’s position, but with inner links, being conceived to correlate the nozzle’s opening with the turbine’s pressure ratio, which depends on the basic engine’s operation.

Fuel injection sub-systems (fuel pumps and controllers), although using same fuel, are relatively independent. Basic engine’s fuel pump is part of the speed control system, while afterburning fuel pump has no connection to the afterburning control system. Both of the pumps are driven by engine’s shaft, but afterburner’s fuel pump becomes active only when afterburning is switched on and is conditional on basic engine’s maximum regime achieving, because afterburning is a thrust augmentation method when basic engine has reached its maximum thrust. Consequently, one can assume that engine speed has no influence above afterburning fuel injection.

3. EAS mathematical model

As fig. 2 shows, one can identify five operational blocks, as follows:

a) jet engine with afterburning

b) speed controller;

c) exhaust nozzle controller;

d) afterburning fuel injection controller;

e) input signal formatting block (which can be very simple, as here is, or more complicate, as presented in [12]).

3.1. Engine’s mathematical model

For a single-jet engine with afterburning, the linearised non-dimensional matrix motion equation, as presented in [11] and [12], is

\[ A \times u = b, \]

where \( A \) is the engine’s matrix, \( u \) – controlled parameters vector and \( b \) – control parameters vector, as follows

\[ A = \begin{bmatrix}
\tau_1 s + p_1 & -k_{1T3} & -k_{1p2} & -k_{1p4} & 0 & 0 \\
\tau_2 & -k_{2T3} & \tau_3 & 0 & 0 & 0 \\
0 & -1 & -k_{3p2} & k_{3p4} & 0 & 1 \\
0 & k_{4T3} & k_{4p2} & k_{4p4} & -k_{4yp} & 0 \\
k_{5T} & k_{5T3} & k_{5p2} & 0 & 0 & 0 \\
0 & k_{6T3} & k_{6p2} & 0 & k_{6yp} & k_{6T4}
\end{bmatrix}, \]

\[ u^T = \begin{bmatrix}
\tau_1 & \tau_2 & \tau_3 & \tau_4 & \tau_5 & \tau_6
\end{bmatrix}, \]

\[ b^T = [0 \ 0 \ 0 \ k_{4A} \ A_b \ \bar{Q}_b \ k_{6Q_p} \bar{Q}_p], \]

where the involved coefficient forms are those presented in [12]. For the above presented mathematical description one has used the formal annotations, as follows: \( X \) – parameter’s current value, \( X_0 \) – steady state value, \( \Delta X = |X - X_0| \) - deviation or static error value, respectively
\[ X = \Delta X / X_0 \] — adimensional parameter value.

Using Cramer method, the above-presented system can be solved; most interesting output parameters are: basic engine speed \((\bar{n})\), which is useful for the speed controller, afterburner’s temperature \(T_{4p}^*\), as well as turbine pressures \(p_2^*\) and \(p_4^*\). This study is realized for a hypothetic EAS, consisting of an engine similar to VK-1F, which is equipped with brand new controllers.

In the particular situation of maximal operating regime, \(A\)-matrix for a VK-1F engine is

\[
A = \begin{bmatrix}
0.83s + & -0.5 & -2.25 & -1.2 & 0 & 0 \\
+3.15 & 3.1 & 0.5 & -1.9 & 0 & 0 \\
0 & -1 & 0.25 & -0.25 & 0 & 1 \\
-0.5 & 0.1 & 1 & -1 & -1 & 0 \\
3.1 & 1.8 & -1.17 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & -1 \\
\end{bmatrix}, \tag{5}
\]

and the input parameters' coefficients are

\[ k_{4A} = 1, k_Q = 1, k_Q = 1. \]

Consequently, one has obtained, for the above-mentioned main parameters, the formulas

\[ \bar{n} = \frac{3.402 \bar{T}_5 + 2.3837 \bar{Q}_c + 3.402 \bar{Q}_p}{2.9413s + 13.486}, \tag{6} \]

\[ \bar{p}_2^* = \frac{4.836 \bar{T}_5 + (0.5187s + 5.7633 \bar{Q}_c + 6.3182 \bar{Q}_p)}{2.9413s + 13.486}, \tag{7} \]

\[ \bar{p}_4^* = \frac{1}{2.9413s + 13.486} \left[-(2.353ls + 0.9942)\bar{T}_5 + (0.1452s + 7.5263)\bar{Q}_c - (2.353ls + 0.9942)\bar{Q}_p \right], \tag{8} \]

\[ \bar{T}_{4p}^* = \frac{1}{2.9413s + 13.486} \left[(0.5883s + 6.2936)\bar{T}_5 - (0.612ls + 5.3263)\bar{Q}_c + (2.353ls + 7.1188)\bar{Q}_p \right]. \tag{9} \]

### 3.2. Total thrust

Most important output parameter of EAS is its total thrust \(F_p\). Thrust calculation methods are presented in [6], [8] and [12]. Total thrust depends on air flow rate \(Q_a\) and on specific thrust \(F_{sp_p}\), which depends on afterburner’s temperature \(T_{4p}^*\). One can assume that between basic engine specific thrust and afterburning specific thrust there is a mathematical relation (being proportional to the square root of the temperature ratio). Consequently,

\[ F_p = Q_a F_{sp_p} = Q_a F_{sp} \sqrt{T_{4p}^* / T_{4p}^*}. \tag{10} \]

where \(F_{sp}\) is basic engine specific thrust.

According to [11], [12], \(Q_a = Q_a \left(p_2^* \right)\), respectively \(F_{sp} = F_{sp} \left(T_{3p}^* , p_2^* , T_{4p}^* \right)\); applying the same linearisation method for Eq. (10), one obtains

\[ F_p = k_{FT3} T_{3p}^* + k_{FP2} p_2^* + k_{FP4} p_4^* + k_{FT4} T_{4p}^* + k_{FTP} T_{4p}^* \tag{11} \]

Finally, one has to complete \(A\)-matrix with a seventh line, given by Eq. (11) and with a column given by vector (4), where the coefficients are keeping their expressions. It results, for the last line

\[ [0 \ -1.2 \ -0.13 \ -2.2 \ 0.5 \ 0.5 \ 0], \tag{12} \]

which completes \(A\)-matrix in (5). It results, for the total thrust

\[ F_p = \frac{1}{2.9413s + 13.486} \left[(4.4883s + 1.3816)\bar{T}_5 + (2.1196s + 24.7434)\bar{Q}_c + (6.0591s + 6.3248)\bar{Q}_p \right]. \tag{12} \]

Equations (6), (7), (8), (9) and (12) are the EAS linear mathematical model, which will be further used.

### 4. Controllers’ mathematical models

#### 4.1. Throttle

As fig. 2 shows, EAS has a single input, which is throttle’s position \(\theta\). One can affirm that throttle’s positioning has two operation intervals:

- from “idle” to “maximal” (or “full”), when it controls the basic engine’s speed, \(\theta\) being proportional to the speed reference \(n_{ref} \);  
- beyond “maximal”, into afterburning domain, when \(\theta\) is conceived to be proportional to \(F_p\) total thrust. In fact, it can be assumed as proportional to \(Q_p\) (consequently, to \(T_{4p}^*\)).

Such a throttle assisting system (input signal formatting block) is presented in [5]; a similar system is described in [12], but operating after a different command law, \(A_s = A_s(\theta)\).

#### 4.2. Speed controller

For this kind of engine, one has considered usable a simple speed controller, as described and studied in
[13] (or in [17] without flight regime and air flow correctors). Its equation is

\[
\overline{Q}_r = \frac{0.683 \theta}{0.078s^2 + 1.813s + 5.3068} + \frac{0.9065s + 2.4795}{1.6183s + 6.308},
\]

so, the fuel flow rate supplied by the engine’s main pump depends on the throttle’s position, as well as on the effective engine’s speed.

### 4.3. Exhaust nozzle’s controller

Exhaust nozzle’s effective area is an engine input parameter, but it is also a controlled parameter in order to keep constant the turbine pressure ratio; keeping constant this pressure ratio means keeping constant the spool’s speed (engine’s operating regime). This controller is essential, because it keeps working the basic engine at the maximum regime, no matter the afterburning pressure and thermal regime were.

According to [1] and [15], exhaust nozzle controller’s equation is

\[
\Delta A_5 = \frac{0.2524s^2 + 1.6634s + 1.5816}{(0.81s + 1)(0.187s + 1)(0.23s + 5.17)} - \frac{0.234}{0.187s + 1} \overline{p}_4 - \overline{p}_2^*.\]

### 4.4. Afterburning fuel injection controller

While the basic engine fuel injection (supplying) system is a part of the speed control system, the afterburning fuel system is essentially different. Therefore, afterburner’s fuel supplying system is a follower one (as presented in [18] or [14]), not a feed-back system; \( Q_p \) fuel flow rate level is established by the throttle’s position \( \theta \) and it’s correlated to the air/gases flow rate, in order to obtain appropriate \( T^*_4p \) - afterburner’s temperature an \( F_p \) - thrust values. Thus, a fuel system such as the one presented in [18] is enough for this EAS; its simplified mathematical model is

\[
\overline{Q}_p = \frac{0.8776s - 4.0144}{1.345s + 6.2311} \left( \frac{0.477}{0.586s + 1.432} \overline{\theta} - \frac{0.879}{1.5658s + 6.5822} \overline{p}_2^* \right).
\]

### 5. System’s quality

Above-presented EAS mathematical model, completed with controllers’ models, are building the embedded system model, which can be used for further studies.

System’s quality study consists of system’s step response analysis, for a step input (step throttle displacement). One has considered the maximal engine’s operating regime (full thrust regime), when afterburning may be switched on.

Most important output parameter, from the EAS point of view, is total thrust; although, one has also studied some other important input/output parameters behavior (such as speed, fuel flow rates, exhaust nozzle’s area, turbine’s pressures).

EAS effective inputs are exhaust nozzle’s area and fuel flow rates; main output is total thrust (as well as afterburner’s temperature) and secondary outputs are engine’s speed and turbine pressures, which are used as inputs or feed-back in controllers’ operating block diagrams.

Exhaust nozzle’s area parameter \( \Delta A_5 \) behavior (fig.4) shows that throttle’s step input induces nozzle’s opening (with a small override), in order to
compensate turbine’s pressure ratio modification (see pressures behavior, fig. 3) and keep it at the same imposed value. Engine’s speed tends to grow up, but stabilizes itself after one second with an insignificant static error (0.25%). Both of fuel flow rates behaviors are presented in fig.5; although both of them are growing, the afterburning fuel flow rate parameter has a negative but a little higher static error rate (3.8%) than the other (3.2%). Supplementary, one can observe for \( \bar{Q}_p \) an initial small positive override. Response time for both of fuel flow rates is around 2.5 s (a little higher for \( \bar{Q}_p \)), which is acceptable.

Main output parameter(s) time behavior is presented in fig. 6 and 7. One can observe nearly the same behavior, the same response time and static error rates. Small initial overrides are observed for both of parameters, but their further behavior indicates system’s stability.

4. Conclusion

This paper has studied an aircraft jet engine with afterburning as controlled object. Jet engine VK-1F was considered as basic engine and three controllers were theoretically adapted to it.

As control parameters (inputs) one has identified exhaust nozzle area, combustor’s fuel flow rate and afterburner’s fuel flow rate. Control laws were established in order to keep the embedded system (engine+afterburning) stable running, no matter its regime were. Afterburning must be switched on only when basic engine has reached its maximal regime and should operate without influence above the basic engine’s regime. Therefore, it is compulsory to use an exhaust nozzle with variable area, in order to keep the same turbine pressure ratio and, consequently, the same engine speed. Exhaust nozzle’s controller was chosen and designed for this purpose.

Basic engine speed controller was chosen simplest possible, in order to assure an appropriate correlation between throttle’s position and engine’s speed (operating regime).

Afterburning’s controller was chosen as a follower system, to realize a proportional dependence between thrust and throttle’s position.

Embedded system has a single input parameter (throttle’s position), which generates (directly or indirectly) EAS input parameters’ level. Most important output parameters are afterburner’s temperature and total thrust.

Some simulations were performed, using the mathematical model(s), for each system part, as well as for whole embedded system; one has established system’s quality as system step response. Studied embedded system has proved to be a stable one, with small static error rates and small response time.
However, fig. 7 shows an initial small thrust decrease, caused by the delay between exhaust nozzle opening and fuel flow rate(s) stabilization. This study was performed without considering the flight regime influence and without considering the possible inner feed-back caused by the afterburning fuel pump’s driving by the engine’s shaft (considering a constant engine’s speed, equal to its maximum value).

The paper subject and used method can be extended for multi-spool jet engines with afterburning, as well as for further improved studies, concerning other engines, with different coefficient values.

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