Heat Transfer with Full Boiling in System with Double Wall and Double Fins

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Abstract: - This paper considers a mathematical model describing and modelling steady state heat conduction in double-layered 2D body consisting of a wall and fin. We analyse the problem when heat transfer includes processes of boiling, and construct an approximate analytical solution using conservative averaging method for L-type domains and finite difference method.

Key-Words: - double wall, double fin, L-type domain, temperature fields, stationary problem, boiling boundary condition, conservative averaging method, finite difference scheme.

1 Introduction

In this article, we continue our research (see, [4], [9]) on stationary heat transfer in 2D *double wall with double fins* when boiling process is present.

As already mentioned in [4], [9], a double wall with double fins is an element that is made up of a flat surface which roughness is produced by adding densely distributed vertical nanowires, and then covered by some kind of coating, e.g., fluorine carbon. Such asperity structures are often developed to control and enhance boiling heat transfer (see, e.g., [1] - [3]).

Our mathematical models used in this study are slightly different than those conducted on relatively simple fin assemblies (see, e.g., [5] - [7], [9]). In this paper we focus on stationary process when there is boiling occurring on the fins' surface. The structure is 2D and it has constant properties. Just like in the publications [6], [7], and [9] conservative averaging method is exploited to construct an approximate solution for the given problem.

2 Formulation for 2D Problem

Since the given system can be divided into several symmetrical L-shaped parts (see Fig.1), it is sufficient to analyse only one of those parts [4], [9]. We are going to decompose this L-shaped domain

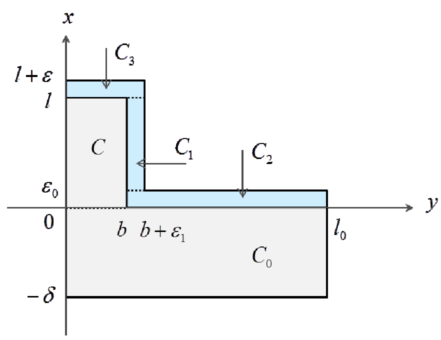

Fig.: Definition of geometrical parameters for the sample

as a union of five non intersecting rectangular sub domains C_i (see Fig.1) with appropriate conjugation conditions along the lines connecting two neighbour domains.

Let $V_i(x,y)$ denote the temperature in the domain C_i. And k_i and h_i represent thermal conductivity and heat transfer coefficient (for simplicity reasons we are going to assume that $k = k_0$ and $k_2 = k_3 = k_1$).

So, the equations governing stationary heat transfer are

$$\frac{\partial^2 V_i}{\partial x^2} + \frac{\partial^2 V_i}{\partial y^2} = 0,\ x, y \in C_i .$$

For this model there are several boundary conditions that must be met.

It is important to apply the symmetry boundary conditions along the lines $y = 0$ and $y = l_0$:

$$\frac{\partial V_i}{\partial n} = 0 .$$

In the formula above n denotes the exterior unit normal to the boundary of the domain C_i.

Let's define a heat flux boundary condition at the wall surface $x = -\delta$:

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=-\delta} = -Q_0(y) .$$

From now on, we shall assume that the function $Q_0(y)$ is constant, that is, $Q_0(y) = Q_0$.

With boiling occurring on the surface of the upper layer, it is required to apply non-linear boundary conditions at those sides of the sample (unlike in our previous work [9], here we have boiling conditions imposed on all sides of the fin):

$$\frac{\partial V_i}{\partial n} + \beta_1^1 V_i^m = 0 ,$$

where $\beta_1^1 = \frac{h_1}{k_1}$. But the value of the index m can take on any real number in a closed interval $\left[3, 3\,{}^{1}\!/_{3}\right]$ (see [9]).

At the interface between the connected parts the conditions of ideal thermal contact (the continuity of temperature and heat fluxes) are assumed:

$$V_i\big|_{x=\ldots} = V_j\big|_{x=\ldots} ,$$

$$\left.\frac{\partial V_i}{\partial x}\right|_{x=\ldots} = \frac{k_j}{k_i}\left.\frac{\partial V_j}{\partial x}\right|_{x=\ldots} ,$$

$$V_i\big|_{y=\ldots} = V_j\big|_{y=\ldots} ,$$

$$\left.\frac{\partial V_i}{\partial y}\right|_{y=\ldots} = \frac{k_j}{k_i}\left.\frac{\partial V_j}{\partial y}\right|_{y=\ldots} .$$

3 Approximate Solution of Problem

As the upper layer is quite thinner than the substrate, the changes in temperature are considerably smaller here. What this means is that we can safely assume that the temperature is uniform across the substrate thickness. Let's use conjugation conditions to write down approximate analytic expressions of the temperature distribution in the upper layer:

$$V_2(x,y) = v_2(y) = V_0(0,y) , \qquad (1)$$

$$V_1(x,y) = v_1(x) = V(x,b) , \qquad (2)$$

$$V_3(x,y) = v_3(y) = V(l,y) . \qquad (3)$$

This assumption simplifies the original problem so now we have to solve only those equations that are defined for the basic layer. We seek the functions $V(x,y)$, $V_0(x,y)$ as solutions of the following Laplace equations

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 , \qquad (4)$$

$$\frac{\partial^2 V_0}{\partial x^2} + \frac{\partial^2 V_0}{\partial y^2} = 0 \qquad (5)$$

together with a complete set of boundary conditions. As we know from Section 2,

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=-\delta} = -Q_0 , \qquad (6)$$

$$\left.\frac{\partial V_0}{\partial y}\right|_{y=0} = 0 ,\ \left.\frac{\partial V_0}{\partial y}\right|_{y=l_0} = 0 , \qquad (7)$$

$$\left.\frac{\partial V}{\partial y}\right|_{y=0} = 0 . \qquad (8)$$

To derive the boundary conditions at $x = 0$, $x = l$, and $y = b$, let's use appropriate conjugation conditions and formulae (1) – (3). So, at $x = 0$ we would have

$$\left.\left(\frac{\partial V_2}{\partial x} + \beta_1^1 V_2^m\right)\right|_{x=\varepsilon_0} = \frac{1}{k_1}\left.\left(k_0\frac{\partial V_0}{\partial x} + h_1 V_0^m\right)\right|_{x=0} = 0$$

which can be written as

$$\left.\left(\frac{\partial V_0}{\partial x} + \beta_0^1 V_0^m\right)\right|_{x=0} = 0 ,\ y \in (b, l_0) . \qquad (9)$$

and

$$\left.\left(\frac{\partial V}{\partial x} + \beta_0^1 V^m\right)\right|_{x=l} = 0 ,\ y \in (0, b) \qquad (10)$$

$$\left.\left(\frac{\partial V}{\partial y} + \beta_0^1 V^m\right)\right|_{y=b} = 0 ,\ x \in (0, l) \qquad (11)$$

at $x = l$ and $y = b$.
And for contact area at $x = 0$ perfect thermal contact conditions are assumed:

$$V_0\big|_{x=-0} = V\big|_{x=+0}, \qquad (12)$$

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=-0} = \left.\frac{\partial V}{\partial x}\right|_{x=+0}. \qquad (13)$$

Using conservative averaging method (see [7], [9] etc.), we are going to construct an approximate solution for the given problem.

3.1 Solution for Boiling on Fin

We propose to use a very simple approximation for the temperature in the fin where the temperature is assumed to be constant with respect to y. It gives

$$v_1(x) = v(x). \qquad (14)$$

It may seem to be a first rough approximation pending a more accurate estimate.
Let's define the average value of the function $V(x,y)$ over the interval $[0,b]$:

$$v(x) = \rho\int_0^b V(x,y)dy,\ \rho = b^{-1}. \qquad (15)$$

When integrating equation (4), we get:

$$\frac{d^2v}{dx^2} + \rho\left.\frac{\partial V}{\partial y}\right|_{y=0}^{y=b} = 0,\ x\in(0,l). \qquad (16)$$

Under boundary conditions (11) and (8) the differential equation (16) can be simplified to

$$\frac{d^2v}{dx^2} - \lambda^2 v^m(x) = 0,\ x\in(0,l), \qquad (17)$$

where

$$\lambda^2 = \rho\beta_0^1.$$

Rewriting the boundary condition (10) in the from

$$\left.\left(\frac{\partial V}{\partial x} + \beta_0^1 V^{m-1}V\right)\right|_{x=l} = 0,$$

and integrating the last term (see [10] Ch. 4.7.):

$$\int_0^b V^{m-1}(x,y)V(x,y)\big|_{x=l}dy =$$

$$= \int_0^b V^{m-1}(l,y)V(l,y)dy = (v(l))^{m-1}bv(l)$$

we get a non-linear boundary condition:

$$v'(l) + \overline{\beta}_0^1(v(l))^{m-1}v(l) = 0,\ \overline{\beta}_0^1 = \beta_0^1 b. \qquad (18)$$

3.1.1 Numerical Method for Problem in Fin

Before continuing on our analysis of the model, let us rewrite the ordinary differential equation (17) as follows:

$$\frac{d^2v}{dx^2} - \lambda^2 v v^{m-1} = 0. \qquad (19)$$

We are going to solve it using a finite difference scheme and iterations. If we replace the continuous problem by its discrete approximation, we get

$$\frac{v_{i+1}^n - 2v_i^n + v_{i-1}^n}{h_x^2} - \lambda^2\left(v_i^{n-1}\right)^{m-1}v_i^n = 0, \qquad (20)$$

where v_i is a difference approximation to the value of $v(x_i)$ at the mesh point x_i, and h_x is the size of the discrete step:

$$h_x = \frac{l}{N},\ x_i = l - ih_x,\ i = \overline{0,N}.$$

The superscript n denotes the n-th iteration.
Rearranging terms in (20) we obtain (see [8]):

$$A_i^{n-1}v_{i-1}^n - C_i^{n-1}v_i^n + B_i^{n-1}v_{i+1}^n = -F_i^{n-1}, \qquad (21)$$

where

$$F_i^{n-1} = 0,\ A_i^{n-1} = B_i^{n-1} = \frac{1}{h_x^2},$$

$$C_i^{n-1} = A_i^{n-1} + B_i^{n-1} + \lambda^2\left(v_i^{n-1}\right)^{m-1}.$$

To obtain a second order approximation for the boundary condition (18), the derivative in (18) is approximated using the differential equation (19) and it's approximation (20) (see [8], [9]). So, (18) is approximated by

$$\frac{v_1^n - v_0^n}{h_x} + \frac{h_x}{2}\lambda^2\left(v_0^{n-1}\right)^{m-1}v_0^n + \overline{\beta}_0^1\left(v_0^{n-1}\right)^{m-1}v_0^n = 0$$

or

$$v_0^n = \chi_1^{n-1}v_1^n. \qquad (22)$$

where the coefficient χ_1^{n-1} is defined as

$$\chi_1^{n-1} = \left[1 - h_x\left(\overline{\beta}_0^1 + \frac{h_x}{2}\lambda^2\right)\left(v_0^{n-1}\right)^{m-1}\right]^{-1}.$$

As the functions $V_0(x,y)$, $V(x,y)$ satisfy the conjugation condition (12), for other boundary condition we can use the following identity given at the point $(0,b)$

$$v_N^n = V_0(0,b). \qquad (23)$$

Taking into account (22), for any given value of n (21) constitutes a set of N linear equations:

$$v_i^n = \alpha_{i+1}^{n-1}v_{i+1}^n + \beta_{i+1}^{n-1},\ i = \overline{0,N-1} \qquad (24)$$

with

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$$\alpha_{i+1}^{n-1} = \frac{B_i^{n-1}}{C_i^{n-1} - \alpha_i^{n-1} A_i^{n-1}},$$
$$\beta_{i+1}^{n-1} = \frac{A_i^{n-1}\beta_i^{n-1} + F_i^{n-1}}{C_i^{n-1} - \alpha_i^{n-1} A_i^{n-1}}$$

for $i = \overline{1, N-1}$, and

$$\alpha_1^{n-1} = \chi_1^{n-1},\ \beta_1^{n-1} = 0.$$

You can easily check that $\beta_i^{n-1} = 0$ for all $i = \overline{1, N}$.

For the zero-th iteration we can chose a linear function that satisfies the boundary conditions (6), (10), for example, let's take the function:

$$v(x) = Q_0\left(l - x + \frac{1}{\beta_0^1}\right),\ x \in [0, l].$$

3.2 Solution for Wall

Using conservative averaging method (see [4], etc.), we are going to transform the given mathematical model into a more usable form.

To obtain the solution for the wall let's use the original method of conservative averaging (see [4], [7] etc.), where the temperature field $V_0(x,y)$ is reduced to the following form:

$$V_0(x,y) = \\ = g_0(y) + \left(e^{-dx} - 1\right)g_1(y) + \left(1 - e^{dx}\right)g_2(y). \quad (25)$$

This representation includes three unknown functions $g_i, i = 0,1,2$, that are found by considering appropriate boundary conditions and integral equality (for more detailed information, see [4]):

$$v_0(y) = d\int_{-\delta}^{0} V_0(x,y)dx,\ d = \delta^{-1}. \quad (26)$$

If we integrate (25) over the segment $(-\delta, 0)$, we get

$$v_0(y) = g_0(y) + (e-2)g_1(y) + e^{-1}g_2(y). \quad (27)$$

But applying the boundary condition (6) to the function (25),

$$eg_1(y) + e^{-1}g_2(y) = \delta Q_0(y). \quad (28)$$

Let's combine these results (27) and (28) to express the function $g_1(y)$:

$$g_1(y) = \frac{1}{2}\left(g_0(y) - v_0(y) + \delta Q_0(y)\right). \quad (29)$$

Finally, from (28) and (29) we conclude that formula (25) becomes

$$V_0(x,y) = \\ = g_0(y)\left(1 + \frac{1}{2}\left(e^{-dx} - 1\right) - e^2\frac{1}{2}\left(1 - e^{dx}\right)\right) \\ + v_0(y)\left(-\frac{1}{2}\left(e^{-dx} - 1\right) + e^2\frac{1}{2}\left(1 - e^{dx}\right)\right) \\ + Q_0(y)\begin{pmatrix}\frac{1}{2}\delta\left(e^{-dx} - 1\right) + \\ + \left(\delta e - e^2\frac{1}{2}\delta\right)\left(1 - e^{dx}\right)\end{pmatrix}. \quad (30)$$

Before we continue, let's divide the wall into two parts, where the right part of the wall occupies the domain $x \in (-\delta, 0)$, $y \in (b, l_0)$ but the left one is for $x \in (-\delta, 0)$, $y \in (0, b)$.

3.2.1 Solution for Right Part of Wall

Upon applying the boundary condition (9) to (30), we get

$$g_0(y)\left(-\frac{1}{2}d + \beta_0^1\left(g_0(y)\right)^{m-1} + \frac{1}{2}de^2\right) \\ + v_0(y)\left(\frac{1}{2}d - \frac{1}{2}de^2\right) \\ + Q_0\left(-\frac{1}{2} - e + \frac{1}{2}e^2\right) = 0.$$

We can rewrite this expression as

$$g_0(y) = v_0(y)a_0^{n-1} + Q_0 b_0^{n-1}, \quad (31)$$

where

$$a_0^{n-1} = \frac{d - de^2}{d - 2\beta_0^1\left(g_0^{\,n-1}(y)\right)^{m-1} - de^2},$$
$$b_0^{n-1} = \frac{-1 - 2e + e^2}{d - 2\beta_0^1\left(g_0^{\,n-1}(y)\right)^{m-1} - de^2}.$$

From this expression we see that we should use iteration process to solve the equation for the right part of the wall as well. Substituting (31) into the expression (30), reduces it to

$$V_0(x,y) = \\ = v_0(y)\begin{pmatrix}a_0^{n-1} + \frac{1}{2}\left(a_0^{n-1} - 1\right)\left(e^{-dx} - 1\right) + \\ + e^2\frac{1}{2}\left(1 - a_0^{n-1}\right)\left(1 - e^{dx}\right)\end{pmatrix} \\ + Q_0\begin{pmatrix}b_0^{n-1} + \frac{1}{2}\left(b_0^{n-1} + \delta\right)\left(e^{-dx} - 1\right) + \\ + \left(\delta e - e^2\frac{1}{2}\left(\delta + b_0^{n-1}\right)\right)\left(1 - e^{dx}\right)\end{pmatrix}. \quad (32)$$

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You can see that now the function $V_0(x,y)$ depends only on one unknown - the function $v_0(y)$. To find that the main differential equation (5) is integrated in the x direction using (26):

$$d\frac{\partial V_0}{\partial x}\bigg|_{x=-\delta}^{x=0}+\frac{d^2v_0}{dy^2}=0. \qquad (33)$$

The boundary condition (6) and the definition (32) allows us to rewrite this identity as an ordinary differential equation

$$\frac{d^2v_0}{dy^2}-\kappa^2v_0(y)=\gamma Q_0,\quad y\in(b,l_0) \qquad (34)$$

where

$$\kappa^{2\,n-1}=-\frac{1}{2}d^2\left(1-e^2\right)\left(1-a_0^{n-1}\right)=d\beta_0^1a_0^{n-1},$$

$$\gamma^{n-1}=-\frac{1}{2}d\left(b_0^{n-1}d\left(e^2-1\right)+(1-e)^2\right).$$

But integration of the boundary condition (7_2) gives

$$v_0'(l_0)=0. \qquad (35)$$

The solution of the problem (34), (35) is

$$v_0{}^n(y)=c_2\left(e^{\kappa^{n-1}y}+\mu_0e^{-\kappa^{n-1}y}\right)-\frac{\gamma^{n-1}Q_0}{\kappa^{2\,n-1}} \qquad (36)$$

with

$$\mu_0=e^{2\kappa^{n-1}l_0}$$

and c_2 as an unknown constant.

To ensure that the solution of the given problem is continuous in all the domain, it is necessary for the temperatures $V_0(x,y)$, $V(x,y)$ to coincide at the contact point $x=0$, $y=b$ between the fin and the right part of the wall. So, from the condition (23) and definition (32) we get:

$$v_N^n=V_0(0,b)=a_0^{n-1}v_0(b)+b_0^{n-1}Q_0(b) \qquad (37)$$

Previously we assumed that the solution in the fin is constant in the y direction. That means (37) is true for all $y\in[0,b]$.

3.2.2 Solution for Left Part of Wall

When deriving equation for the left part of the wall, it is important to remember that for $x=0$ the functions $V_0(x,y)$, $V(x,y)$ must fulfil the conjugation conditions (12), (13). So, from (13) and (24) we get that

$$d\frac{\partial V_0}{\partial x}\bigg|_{x=-0}=d\frac{dv}{dx}\bigg|_{x=+0}=$$

$$=d\left[\frac{v_{N-1}^n-v_N^n}{h_x}+\frac{h_x}{2}\lambda^2\left(v_N^{n-1}\right)^{m-1}v_N^n\right]=$$

$$=d\left[\frac{\alpha_N-1}{h_x}+\frac{h_x}{2}\lambda^2\left(v_N^{n-1}\right)^{m-1}\right]v_N^n. \qquad (38)$$

Now, using (37) formula (38) becomes

$$d\frac{\partial V_0}{\partial x}\bigg|_{x=-0}=$$

$$=d\left[\frac{\alpha_N-1}{h_x}+\frac{h_x}{2}\lambda^2\left(v_N^{n-1}\right)^{m-1}\right]\times$$

$$\left[a_0^{n-1}v_0(b)+b_0^{n-1}Q_0(b)\right].$$

Before we find the 1D equation, let's rewrite the last expression as

$$d\frac{\partial V_0}{\partial x}\bigg|_{x=-0}=k_0^{n-1}v_0{}^n+\gamma_0^{n-1}Q_0. \qquad (39)$$

Here

$$k_0^{n-1}=a_0^{n-1}d\left[\frac{\alpha_N-1}{h_x}+\frac{h_x}{2}\lambda^2\left(v_N^{n-1}\right)^{m-1}\right],$$

$$\gamma_0^{n-1}=b_0^{n-1}d\left[\frac{\alpha_N-1}{h_x}+\frac{h_x}{2}\lambda^2\left(v_N^{n-1}\right)^{m-1}\right].$$

If we now use (33) and conditions (39) and (6), then integrated equation becomes

$$\frac{d^2v_0}{dy^2}+k_0^{n-1}v_0^n=-\gamma_0^{n-1}Q_0-dQ_0. \qquad (40)$$

From (7_1) we get a boundary condition:

$$v_0'(0)=0. \qquad (41)$$

So, the solution of the problem (40), (41) is:

for $k_0^{n-1}<0$

$$v_0{}^n(y)=c_1\left(e^{\sqrt{-k_0^{n-1}}y}+e^{-\sqrt{-k_0^{n-1}}y}\right)-\frac{\left(\gamma_0^{n-1}+d\right)}{k_0^{n-1}}Q_0; \qquad (42)$$

if $k_0^{n-1}>0$ then

$$v_0{}^n(y)=c_1\cos\left(\sqrt{k_0^{n-1}}\,y\right)-\frac{\left(\gamma_0^{n-1}+d\right)}{k_0^{n-1}}Q_0. \qquad (43)$$

Another possibility when constructing a 1D equation for the left part of the wall is to use expression (32) and find the derivative in the direction of x. In that case

$$d\frac{\partial V_0}{\partial x}\bigg|_{x=-0}=-\frac{1}{2}d^2\left[A_0^nv_0(y)+B_0^nQ_0\right],$$

$$A_0^n=\left(a_0^n-1\right)\left(1-e^2\right),$$

$$B_0^n=\left(b_0^n+\delta\right)+2\left(\delta e-e^2\frac{1}{2}\left(\delta+b_0^n\right)\right).$$

Here we get a linear equation that is also dependent of iterations:

$$\frac{d^2 v_0{}^n}{dy^2} - \frac{1}{2} d^2 A_0^n v_0{}^n(y) = \frac{1}{2} d^2 B_0^n Q_0 - dQ_0 . \quad (44)$$

3.2.3 Conjugation of Solutions

We have just found solution to the given problem. But we are still left with the unknown constants c_1 and c_2 in formulae (36) and (42), (43). These are found from conditions:

$$v_0{}^n(b-0) = v_0{}^n(b+0),$$

$$\frac{d}{dy} v_0{}^n(b-0) = \frac{d}{dy} v_0{}^n(b+0).$$

Both constants are iteration-dependent. The iteration algorithm is organised like this: at first we perform the first iteration in the fin, finding α_i and constants c_1, c_2. After that we use (37) and (24) to get a value for $V_0(0,b)$ and calculate temperature in the fin and in the wall. Afterwards we proceed with the iteration process.

For that other case when 1D equation for the left part of the wall is found in a different way, equation (44) is also solved with iterations. Once again we find the unknown constants and the mean temperature in the wall - $v_0(y)$. Knowing this we can use representation (32) to calculate $V_0(0,y)$. According to (12) the latter gives v_N^n. So, the assumption that the temperature in the fin is constant in the y direction breaks down.

4 Conclusion

We have given a formulation of a problem for stationary heat conduction in 2D double wall with double fins when boiling is present on all sides of the fin. To solve this problem and construct an approximate analytical solution we have used conservative averaging method and finite difference scheme with iterations.

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