Methods of Operational Risk Economic Capital Estimation and Allocation in Russian Commercial Banks

OLEG NIKONOV, VLADIMIR VLASOV
Department of System Analysis and Decision Making
Ural Federal University
620000, Ekaterinburg, Mira str., 19
RUSSIA
vlasovve@inbox.ru, o.i.nikonov@ustu.ru

Abstract: Modern systems of risk management in financial institutions require a process of estimation of the amount of capital that is needed to cover losses arising from various types of risk and its allocation to business units in order to measure their risk-adjusted performance. In this paper we describe the structure of operational risk economic capital estimation model suitable for implementation of sophisticated methods of capital allocation to business units. We compare different methods of allocation and discuss their applicability for Russian banks and describe their practical implementation for a large Russian bank.

Key-words: operational risk, economic capital, Loss Distribution Approach, linear allocation, marginal risk contributions, Euler’s allocation.

1 Introduction
Economic capital (EC) is the amount of banking capital required to cushion losses arising from different types of risk. From a managerial perspective EC comprises in one number all the information about a bank’s total risk exposure and can be seen as a tool for solving several tasks:
1. Capital adequacy control;
2. Formalization of risk-appetite;
3. Risk-adjusted performance measurement and pricing.

Today the topic of EC is a matter of a great concern for large Russian financial institutions on their way to Basel III. Operational risk EC (OREC) model is the essential part of the bank’s EC aggregation and allocation framework. It provides information about the distribution of operational losses that is used in aggregation procedures (typically, via Monte Carlo simulations) together with distributions of other risks to estimate a bank’s total diversified EC. It is also used to allocate a bank’s total EC to business units in order to calculate risk-adjusted performance measures, such as risk-adjusted return on risk-adjusted capital and shareholder value added.

Detailed descriptions of OREC estimation methodologies provide [2], [4], [7]. The problem of EC allocation is described in [3], [5], [6], [8]. The central role in the majority of papers on the topic of allocation is given to credit risk and all approaches are examined with regard to a credit portfolio.

The goal of this article is to compare several approaches to OREC allocation to business units and to describe their main limitations with respect to OREC model design and data sufficiency for Russian banks. As far as the method of allocation is dependent on the OREC estimation methodology, we briefly describe a general framework for the OREC estimation model based on Loss Distribution Approach and discuss additional assumptions important for implementation of several allocation schemes. We will also demonstrate the practical usage of several methods of allocation for a large Russian bank.

2 Problem setting
Basel offers three methods for OREC estimation: Basic Indicator Approach (BIA), The Standardized Approach (TSA) and Advanced Measurement Approaches (AMA). Under the most sophisticated approach (AMA), bank’s losses that arise from different types of operational risk are seen as a random variable – \( \theta \). Denoting the target level of statistical confidence as \( \alpha \), OREC is determined by either:

\[
OREC(\alpha) = q_{\alpha} = \inf \{ x \mid P(\theta \leq x) \geq \alpha \}, \quad (1)
\]

1Those “types of risk” are formalized as 56 so called “cells” – intersections of 8 business lines and 7 types of risk, defined in Basel II.
OREC(\alpha) = E(\theta | \theta \geq q_\alpha) \quad (2)

where \(q_\alpha\) is \(\alpha\)-quantile, or \(VaR_\alpha(\theta)\) and \(P(\theta \leq x) = G_{\theta}(x)\) is a cumulative distribution function of \(\theta\).

Measures, used in (1) and (2) are called Value-at-risk and Expected Shortfall accordingly.

After the OREC has been determined it needs to be allocated to business units in order to measure their risk-adjusted performance. OREC allocation is a process of defining the contribution of each unit of segmentation (i.e. business unit, business line, type of risk) to the overall OREC of the bank, such that

\[
\rho(\theta) = \sum_{l=1}^{L} AC_l \quad (3)
\]

where \(\rho(\theta)\) is OREC (according to 1 or 2) and \(AC_l\) – is an amount of capital allocated to \(l\)-th business unit \((l = 1, 2, ..., L)\).

In other words, OREC allocation is used to determine how much capital (financial resources) a business unit demands to cover unexpected losses from operational risk. Generally, in the condition of stable profit, an increase of allocated EC to a business unit indicates an increase in its operational risk and should be penalized.

There are three broad categories of allocation methods, which can be divided into:

1. Linear allocations – a business unit (BU) is assigned the amount of capital, according to its share in a sum of shares of the specific measure for all BUs. BUs’ revenues, losses, assets or stand-alone ORECs can serve for this purpose.
2. Discrete marginal allocations, that are given by:

\[
AC_l = \frac{\rho^{\text{marg}}(\theta^l)}{\sum_{l=1}^{L} \rho^{\text{marg}}(\theta^l)} \rho(\theta) \quad (4)
\]

where
\[
\rho^{\text{marg}}(\theta^l) = \rho(\theta) - \rho(\theta - \theta^l)
\]
\(\theta^l\) - operational loss of the \(l\)-th BU.

3. Continuous marginal allocations (Euler’s method), given by either:

\[
AC_l = E[\theta^l | \theta = VaR_\alpha(\theta)] \quad (5)
\]

w.r.t. a chosen risk-measure (VaR or ES accordingly).

All those allocation methods, except for linear method based on balance sheet and P&L values (revenue, losses, assets), imply estimation of distribution functions of losses for each BU on a standalone basis in order to assess its \(\rho(\theta^l)\). Thus, the main problem with allocation of OREC is that its estimation methodology should consider somehow those distributions. This is not always attainable if data is insufficient for assessing the parameters of those distributions.

In section 3.1, we describe the general framework for the OREC estimation model based on LDA considering the distributions of losses of standalone BUs. Furthermore, several OREC allocation methods will be compared from a theoretical (section 3.2) and practitioner’s (section 3.3) perspective. We will discuss the key problem of applicability of those methods to large Russian financial institution.

3 Problem solution

3.1 OREC estimation model

In this article, the Loss Distribution Approach (LDA), one of the advanced approaches, will be used as a framework to estimate the OREC. Following [2], [4], describing LDA in great detail we will set its prerequisites for the large Russian bank.

Let us consider a bank, consisting of \(L\) BUs \((l = 1, 2, ..., L)\) exposed to \(I\) \((i = 1, 2, ..., I)\) types of risk. Then \(\theta^{l,i}\) is the loss variable and according to LDA it is given by:

\[
\theta^{l,i} = \sum_{n=0}^{N^{l,i}} \xi_{n}^{l,i} \quad (7)
\]

where \(N^{l,i}\) is the random variable of the number of losses of type \(i\) in \(l\)-th BU during a certain period (1 year) usually called frequency.

\(\xi_{n}^{l,i}\) – denotes the \(l\)-th BU type \(i\) loss severity.

For the purposes of allocation, OREC is estimated with the use of internal data of operational losses. This dataset should be sufficient to obtain
adequate estimation of distributions of \( N^{i,l} \) and \( \xi^{i,l} \). After parameters of these distributions have been estimated and a goodness-of-fit has been tested, the distribution of \( \theta^{i,l} \) is obtained via \( M \) Monte-Carlo simulations (\( m = 1, 2, ..., M \)). This procedure consists of the following steps:

1. Estimation of correlation matrices \( R_{l \times l}^{1} = \{ r_{l,j}^{1} \} = corr(N^{1,l}, N^{1,l}) \).
2. Generation of multivariate Poisson random numbers (frequencies) for each \( l \) assuming \( N^{i,l} \) is Poisson random variable with \( E(N^{i,l}) = \bar{\lambda}^{i,l} \). The method for generating multivariate Poisson random variables can be found in [9]. As a result of this step we have a \( M \) random frequencies for each BU and type of risk denoted by \( x_{m}^{i,l} \).
3. Generation of random severities. For each BU and type of risk we should generate \( x_{m}^{i,l} \) individual losses \( \xi_{m,p}^{i,l} \) from the predefined distributions \( F_{\xi_{m}}(x) \).

The \( m \)-th loss of a BU is given by:

\[
\bar{\theta}^{m,l} = \sum_{l=1}^{L} \sum_{p=0}^{\xi^{i,l}} \xi_{m,p}^{i,l} \quad (8)
\]

Empirical cdf of operational losses of the \( l \)-th BU is then given by:

\[
G^{l}(x) = \frac{1}{M} \sum_{m=1}^{M} 1_{[0,x]} \bar{\theta}^{m,l} \quad (9)
\]

Empirical cdf of operational losses of the entire bank is given by:

\[
G(x) = \frac{1}{M} \sum_{m=1}^{M} 1_{[0,x]} \bar{\theta}^{m} \quad (10)
\]

The number of simulations \( M \) should be high enough to obtain a stable OREC estimator (formula (1) or (2)) given the estimated parameters of frequency and severity distributions. Since VaR is not a coherent risk measure [1] for the purpose of allocation, we will choose ES as an OREC estimate.

The above mentioned methods of allocation of OREC (cf. section 2) fulfill the property of full allocation. Among other intuitively desirable properties [3], [8] are the following (coherence axioms):

1. Core compatibility:
   \[
   \forall l: \theta_{i} \in \partial \rho(\theta_{i}) \geq AC_{l} \quad (11)
   \]
2. Symmetry: If by joining two BUs \( i \) or \( j \) to the coalition of BUs \( M \) they both make the same contribution to the risk capital
   \[
   AC_{i} = AC_{j} \quad (12)
   \]
3. Riskless allocation: OREC allocated to a riskless BU equals 0.
4. RARORAC-compatibility: \( AC_{l} \) is RARORAC compatible if:
   \[
   RARORAC(\theta_{i}) > RARORAC(\theta) \Rightarrow RARORAC(\theta + h\theta_{i}) > RARORAC(\theta) \quad (13)
   \]

As shown in [5], these desirable properties determine uniquely a capital allocation scheme, which is a marginal capital allocation. Although in certain real-life conditions it is impossible to use Euler’s method: if the data set is insufficient for the purpose of estimation of parameters of distributions. In the next section we present the developed LDA model in work and compare the allocation methods from a practitioner’s point of view.

3.3 Russian bank’s OREC allocation to business units: a comparison from a practitioner’s perspective

Let us consider a large Russian Bank, consisting of three BUs (geographical segmentation) exposed to three types of operational risk: 1 - clients, products & business practice, 2 - fraud (internal and external), 3 - execution, delivery & process management. Target rating of the Bank is BBB+, which corresponds to the default probability of 0.189% or \( \alpha = 0.9981 \).

Based on the internal data of operational risk losses from 2008 to 2012, we estimated the parameters of frequency and severity distributions and conducted the goodness-of-fit tests. The results are the following: frequencies are modeled by Poisson distributions. The distributions used for modeling severities are Log-Pearson (type 3),
Generalized Pareto and Log-Normal (for the values of estimated parameters of the distributions cf. Appendix 1).

An estimated value of OREC is 3 041 mil. rub. (for ES with Monte-Carlo simulations). For comparison, the capital charge for OR with BIA is 4 888 mil. rub. which is 1.6 times higher than OREC estimated with LDA.

The next step is OREC allocation to BUs. The results of linear allocation according to BUs’ revenues are presented in Table 1.

The contribution of a BU to a total OREC is estimated by the ratio in a bank’s total revenue. Main disadvantages of this method are the following:
- the revenue is not always reflecting the real contribution to operational risk;
- the method doesn’t account for diversification effect.

The main advantage of this method is that it can be used by banks during the periods when datasets of operational losses are insufficient to estimate the standalone distributions of losses across all BUs, but sufficient to estimate OREC for the entire bank. Moreover, those allocations can be easily implemented and are clear for senior management of those BUs.

The results of the next method – incremental or discrete marginal allocation – are presented in Table 2.

Table 1. Linear OREC allocation

<table>
<thead>
<tr>
<th># BU</th>
<th>Revenue, mil. rub.</th>
<th>Ratio, %</th>
<th>Allocated OREC, mil. rub.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11 643</td>
<td>31.07</td>
<td>945</td>
</tr>
<tr>
<td>2</td>
<td>11 594</td>
<td>30.94</td>
<td>941</td>
</tr>
<tr>
<td>3</td>
<td>14 237</td>
<td>37.99</td>
<td>1 155</td>
</tr>
<tr>
<td>Bank</td>
<td>37 474</td>
<td>100.00</td>
<td>3 041</td>
</tr>
</tbody>
</table>

The residual OREC for the $l$-th BU is estimated as ES of a coalition of all BUs except $l$-th.

The difference between the OREC of all BUs and the OREC of this coalition equals the OREC allocated to the $l$-th BU. Those differences are then scaled to add up to the total OREC.

This method accounts for a contribution of a BU to the total risk for the situation when this BU is excluded from the entire bank. Discrete marginal contributions themselves are not additive and the method requires scaling.

The third method (Euler’s) results in the following allocation (Table 3):

Table 3. Euler’s OREC allocation

<table>
<thead>
<tr>
<th># BU</th>
<th>Mean value of a BU losses in scenarios when the total loss is more than VaR - Allocated OREC, mil. rub.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 506</td>
</tr>
<tr>
<td>2</td>
<td>855</td>
</tr>
<tr>
<td>3</td>
<td>680</td>
</tr>
<tr>
<td>Bank</td>
<td>3 041</td>
</tr>
</tbody>
</table>

The BU’s allocated OREC implied by this method equals the mean value of a BU’s losses in the scenarios when the total loss is more than VaR. By construction the allocations add up to the total OREC. Risk contributions of BUs are measured on a marginal basis, i.e. BUs are viewed as parts of the entire bank.

Last two methods strictly imply the estimation of parameters of loss distributions on the level of BUs and are impossible to implement with insufficient data. Figure 1 illustrates the structure of the allocated OREC w.r.t a chosen method.

4 Summary
Operational risk economic capital estimation and allocation process is an important part of a bank’s
integrated risk methodology. The concept of economic capital is used in to formalize and control of risk-appetite of a bank, providing a basis for risk-adjusted performance measurement, pricing, and decision taking.

This paper provides a theoretical and practical overview of the loss distribution approach to OREC estimation and various methods of OREC allocation applicable for large Russian financial institutions. We provide an estimation of OREC based on a model developed for a large Russian bank and check the properties of coherence for several methods of allocation.

We also demonstrated the practical usage of several methods of allocation for a large Russian bank. The key problem of OREC allocation on BUs and more granular levels of segmentation (such as business lines in BUs) for Russian banks is data sufficiency for estimation of parameters of severity distributions. For the period of data collection (next few years) we recommend to estimate OREC on a bank-wide level (without granularity) and allocate it via linear methods based on the parameters of BUs’ financial statements. This will increase the quality of an OREC estimate. After sufficient data is collected we recommend switching to more sophisticated methods of allocation, such as discrete or continuous marginal contributions as far as they fulfill all the requirements.

References:
Appendix

Table A1. Estimated parameters of distributions (frequencies)

<table>
<thead>
<tr>
<th>BU</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-type</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$E(N^i)$</td>
<td>67</td>
<td>25</td>
<td>10</td>
<td>63</td>
<td>42</td>
<td>14</td>
<td>49</td>
<td>27</td>
<td>7</td>
</tr>
</tbody>
</table>

Table A2. Estimated parameters of distributions and goodness-of-fit tests results (severities)

<table>
<thead>
<tr>
<th>Risk type/BU</th>
<th>Distributions</th>
<th>Parameters</th>
<th>Kolmogorov-Smirnov</th>
<th>Anderson-Darling</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{n}d_{n}$</td>
<td>$H_{1} - ?$</td>
<td>SL</td>
</tr>
<tr>
<td>1/1</td>
<td>LogP 3 ($\alpha, \beta, \gamma$)</td>
<td>2.60</td>
<td>0.68</td>
<td>4.01</td>
<td>0.10</td>
</tr>
<tr>
<td>2/1</td>
<td>GenPar(k, $\sigma, \mu$)</td>
<td>0.65</td>
<td>118</td>
<td>82.4</td>
<td>0.13</td>
</tr>
<tr>
<td>3/1</td>
<td>LogN ($\sigma, \mu$)</td>
<td>1.56</td>
<td>6.20</td>
<td>-</td>
<td>0.19</td>
</tr>
<tr>
<td>1/2</td>
<td>LogP 3 ($\alpha, \beta, \gamma$)</td>
<td>2.58</td>
<td>0.63</td>
<td>4.02</td>
<td>0.12</td>
</tr>
<tr>
<td>2/2</td>
<td>GenPar(k, $\sigma, \mu$)</td>
<td>0.44</td>
<td>133</td>
<td>89.2</td>
<td>0.07</td>
</tr>
<tr>
<td>3/2</td>
<td>LogN ($\sigma, \mu$)</td>
<td>2.00</td>
<td>5.74</td>
<td>100</td>
<td>0.06</td>
</tr>
<tr>
<td>1/3</td>
<td>LogP 3 ($\alpha, \beta, \gamma$)</td>
<td>2.57</td>
<td>0.64</td>
<td>3.94</td>
<td>0.15</td>
</tr>
<tr>
<td>2/3</td>
<td>GenPar(k, $\sigma, \mu$)</td>
<td>0.38</td>
<td>108</td>
<td>89.4</td>
<td>0.09</td>
</tr>
<tr>
<td>3/3</td>
<td>LogN ($\sigma, \mu$)</td>
<td>1.12</td>
<td>5.80</td>
<td>78.0</td>
<td>0.06</td>
</tr>
</tbody>
</table>