Abstract: - In this paper we propose a new portfolio selection model to invest in the fixed income market without taking the default risk. The solution of the portfolio choice problem (without default risk) is useful to address better the investors’ choices in particular in the recent credit risk crisis during which several firms went bankrupt. Generally investors require a higher rate of return when they invest in bonds of a firm that can easily go bankrupt. For this reason we need to consider CDSs contracts to hedge the default risk of investments in the fixed income market. In this paper, we first describe the credit risk market and we summarize the valuation of classic financial instruments used to hedge the default risk. Secondly, we deal with the problem to hedge the risk in a dynamic framework using credit default swaps (CDSs). Thus we propose a new portfolio selection analysis where we evaluate the ex-post impact of portfolio investments hedged by the default risk.

Key-Words: - portfolio selection, default risk, credit default swap, fixed income market, immunization problem.

1 Introduction

In the last decade the financial international system has been characterized by a deep process of transformation and innovation that has carried to the birth and the development of new financial instruments and markets. Among new financial contracts we recall the credit default swaps (CDSs) that allow a subject, that has a credit exposure towards a counterpart considered unreliable, to transfer the risk of insolvency to another operator willing to assume this risk.

In this paper we discuss how to avoid the default risk, investing on the fixed income market. First of all we describe the credit risk market and how credit default swaps are generally used to hedge the default risk. Secondly we propose a portfolio empirical analysis where we evaluate the ex post investors’ wealth evolution using the classic immunization theory (see Fisher and Weil (1971), Redington (1952), De Felice (1995)). In particular we discuss and compare the choices of investors who immunize their portfolio by the price risk and by the default risk. In this framework we propose the construction of a portfolio by bonds and CDSs that hedge the default risk. Our aim is to observe how the final wealth held by the investor changes in relation to the decision to consider whether or not the default risk of the companies considered. The final objective is always represented by the maximization of the investor’s future wealth.

We elaborate an ex-post comparison based on portfolio selection among 88 bonds issued by 20 U.S. companies included in the Dow Jones Industrial Average Index in February 2012. The portfolio selection considers also a portfolio of credit default swaps that hedges the total default risk. The empirical analysis covers the credit risk crisis period (from January 2008 till February 2012). While in the fixed income portfolio selection only the price risk is considered, in this paper we analyze the influence of credit risk, measured through the estimation of the implicit probability of default extrapolated from the credit default swaps quotations. The construction of optimal portfolios is made on the basis of the financial semi deterministic immunization proposed by Fisher and Weil (1971). Moreover we also take into account a portfolio of CDSs that hedges the default risk. The chosen portfolio of bonds and CDSs will be therefore immunized from random additive shifts of the interest rates curve and from the possible default of a firm considered in the portfolio.

In section 2 we examine the credit risk market. Section 3 introduces the optimization problem and
describes the methodology used to select optimal portfolios hedged by the default risk. In section 4 we briefly summarize the results.

2 Credit risk and Credit Default Swaps market
The economic crisis of the financial international system has carried to a generalized worsening of the creditworthiness of sovereign and private issuers. Therefore the management of credit risk has become a determining factor in the risk management activity. In order to value the possible default loss of a given portfolio of \( n \) securities, we account the correlation \( \rho \) between the default events of the \( n \) securities in the portfolio that can be defined the tendency of two societies to fail more or less at the same time. Then we also need to consider for each security these definitions:

1) the Exposure at default (EAD) is the estimated value of the loan in the event of default;
2) the Loss given default (LGD) is an estimate of the total loss of the lender in the event of default by the counterpart and it is summarized by the formula:
   \[ \text{LGD} = 1 - \text{Recovery Rate (RR)}. \]
3) the Probability of default (PD) is the probability of bankrupt of the underlying firm. In this study the probabilities of default among the American companies included in the data sample have been considered independent and this has carried to the underestimation of the credit risk.

Therefore, if we assume that the Exposure at Default and the Loss Given Default are constant and independent over the entire range of evaluation, then the portfolio credit risk is given by:

\[
\text{unexpected loss}_\rho = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} EAD_i EAD_j LGD_i LGD_j \rho \sqrt{PD_i (1 - PD_j) PD_j (1 - PD_j)}}
\]

The generalized worsening of the creditworthiness has forced the institutional investors to transfer such risk to another subject without selling the underlying asset through the credit default swaps negotiations. CDSs are financial instruments traded over the counter (OTC) markets and characterized by an elevated efficiency, flexibility and customization in terms of risk, reward and maturity that allow to transfer, to limit, to cover or to assume the credit risk.

The credit default swap allows the holder of the bond to transfer to a third part the default risk. Originally they were born as insurance contracts but subsequently they have played a more speculative role in order to allow large financial institutions to carry out trading activities on credit risk.

Since the mid-nineties, these instruments have had an exponential growth in terms of volumes traded and in February 2012 they have reached the amount of 32 000 billion euros. Technically, the CDSs are bilateral derivative contracts negotiated between the protection buyer (the one who intends to hedge the default risk of a reference entity toward which he/she has a credit exposure), and the protection seller (who is the buyer of the risk). The reference entity is the issuer of the bonds and in this analysis it is represented by each of the American companies included in the DJ Index. The credit event is characterized by the default of the issuer over a period of time established which involves the obligation of the protection seller to pay the notional established.

A credit default swap allows the protection buyer, through a periodic premium payment expressed in basis points, to transfer the default risk of reference entity to the protection seller, unloading on the counterpart the risk of incurring in a loss caused by the non-repayment of obligation at maturity.

3 An ex-post empirical analysis with credit default swaps
In this section we investigate how to invest in a portfolio of bonds with total coverage of the default risk of the reference entities through the use of Credit Default Swaps (see ABI 2011). Each bond negotiation involves the negotiation of CDS associated with that reference entity.

The final goal is the maximization of future wealth for the investor that should avoid the default risk. In addition we want also to verify whether the credit risk trading is a way to amplify the profits for the investor.
Credit default swaps refer to 20 U.S. companies (belonging to the Dow Jones Industrial index in February 2012) and for each of them has been used the daily premium prices (expressed in basis points) paid by the protection buyer of 10 CDS. The deadlines of CDS are common to all companies (6 months, 1 year, 2 years, 3 years, 4 years, 5 years, 7 years 10 years 20 years and 30 years) in order to have a common basis on which to make the comparison between the different probabilities of default of companies. As suggested by Bertn et al. 2006 and Collender 2008, we further assume that:

1) the investor possesses a kind of "premiums fund" that allows him to make all payments associated to credit default swaps. So those payments are excluded from the function of maximization of wealth.

2) The determination of the probability of default is based on the absence of a credit curve, i.e. the curve that associates each deadline with a different probability of default.

3) The premium payment is made annually and postponed with respect to the signing of the contract of credit derivative.

4) The time of payment of the premium by the protection seller and the time to default of the reference entity coincide.

5) The CDS premium is calculated as the arithmetic mean of the CDS bid premium and the CDS ask premium.

6) The CDS premiums incorporate information related to the bankruptcy of the reference entity, the contracts were all determined according to the clause "no restructuring" proposed the International Swaps and Derivative Association (ISDA).

7) The recovery rate (RR) is constant for the entire CDS life and it is equal to 40% for all maturities and for all reference entities.

8) The risk-free rate is identified in the annual Treasury bill.

9) There is no counterpart risk since it is assumed that the protection seller is always solvent.

Thus, we have to determine the implied probability of default daily. This probability is associated with each reference entity for any different maturity of CDS contracts. Assuming a CDS with maturity \( t_n \) and notional equal to 1 in which the payment is annual and postponed, the premium leg (PL) can be described as the sum of the payments discounted paid by the protection buyer:

\[
PL = \sum_{i=1}^{n} [K \cdot S(t_i) \cdot v(t_i)],
\]

where:

- \( S(t_i) \) represents the unconditional probability of survival calculated between 0 and \( t_i \). Thus, \( 1-S(t_i) \) is the probability of default;

- \( K \) is the premium payable by the protection buyer and it is considered constant each year;

- \( v(t_i) \) is the discount factor.

Clearly, when no arbitrage opportunities are admissible, the CDS contract value is also equal to PL. The default payment leg, i.e., the payment due by the protection seller, is given by:

\[
DP = \sum_{i=0}^{n-1} S(t_i) \lambda(t_i, t_{i+1}) (1 - RR) v(t_{i+1}),
\]

where \( \lambda(t_i, t_{i+1}) \) is the hazard rate, that is the conditional probability of default.

The CDS value, at the time of conclusion of the contract, has always zero value i.e. the premium leg must be equal to the default payment leg so that the contract is fair. Therefore, by applying the equality:

\[
PL=DP
\]

we are able to determine every day the annual unconditional probability of default of each company from 01/01/2008 to 02/02/2012.

These probabilities of default have been used to determine the value of the CDS contract at any moment. We calculate these probabilities by solving the equation PL=DP where we use the 3 months Treasury bill as risk-free rate.

Thus the definition of the contract value provides an estimate of the evolution of the probability of
default and of interest rate risk. In order to analyze the portfolio problem in this framework, we suggest to optimize monthly three different functionals (namely, $F_1$, $F_2$, $F_3$) subject to the classic constraints of the Fisher and Weil theorem. The functionals we optimize are listed here below:

1) the first functional $F_{1,t}$ is the estimated bond portfolio future value at time $t$ plus the average of the CDS portfolio hedging (at time $t$), i.e.,

$$F_{1,t} = \left\{V_{Bonds(t)} \cdot \left(1 + tres_{Bonds(t)}\right)\right\} + mean(PR_{CDS(t)})$$

where \(V_{Bonds(t)} = [V_{1,t} \ldots V_{n,t}]\) is the vector of wealths invested in each bond at time $t$, the vector \(tres_{Bonds(t)} = [tres_{1,t} \ldots tres_{n,t}]^T\) is the vector of the yields to maturity of all bonds at time $t$, and $PR_{CDS(t)} = \text{CDS portfolio hedging}$. The mean and standard deviation are calculated according to the model of exponentially weighted moving average (EWMA). This function reflects the behavior of a risk-averse person, who intends to invest its available wealth in a portfolio characterized by low volatility of returns in order to limit losses, but above all, to have the maximum possible certainty about the future evolution of the bond values.

2) the second functional $F_{2,t}$ is the estimated bond portfolio future value at time $t$ plus the average of the CDS and minus the standard deviation of the CDS portfolio computed at time $t$, i.e.:

$$F_2 = Max \left\{V_{Bonds(t)} \cdot \left(1 + tres_{Bonds(t)}\right) + \text{mean}(PR_{CDS(t)})\right\}$$

3) the first functional $F_{3,t}$ is the estimated future value of the bond portfolio at time $t$ plus the average of the CDS portfolio hedging minus the Conditional Value at Risk (CVaR) (see Artzner et al. 1999 for a discussion on its properties) calculated at the $5^{th}$ percentile (computed at time $t$), i.e.:

$$F_{3,t} = \left\{V_{Bonds(t)} \cdot \left(1 + tres_{Bonds(t)}\right) + mean(PR_{CDS(t)}) - CVaR_{0.05}\left(PR_{CDS(t)}\right)\right\}$$

Therefore, we maximize monthly the above functionals $F_1$, $F_2$, $F_3$ with the following constraints (which are common for all portfolio problems):

1. fixed Macaulay modified duration ($MD_{prtf}$) of the portfolio of bonds, that is: $MD_{prtf} = k$ for different modified duration $k$ from 6.4 years till 13.05 years with step 0.35 years (see Macalauy and Durand (1951) and Weil (1973)).

2. fixed initial wealth ($W_t$) at the beginning of each recalibration time $t$ that should be equivalent to the total amount invested in bond and CDS portfolio at time $t$, i.e.:

$$W_t = V_{Bonds(t)} \cdot 1 + P_{CDS} \cdot 1$$

where

$$P_{CDS(t)} = [P_{1,t} \ldots P_{n,t}] = \text{values of CDS investments}$$

This initial wealth can be also expressed as:

$$W_t = \frac{prices_{Bonds,t} \cdot \text{weights}_{Bonds,t} + prices_{CDS,t} \cdot \text{weights}_{CDS,t}}{\text{weights}_{CDS,t}}$$

Clearly the weights of CDS depend on how much has been invested in bonds of the $k^{th}$ reference entity. The total hedge assumes that each bond issued by the $k^{th}$ reference entity is hedged by the CDS associated with the same reference entity. So the following equality holds:

$$\sum_{j \in k^{th} \text{company}} \text{weights}_{Bonds,j} = \sum_{i \in k^{th} \text{company}} \text{weights}_{CDS,i}$$

We assume a starting initial wealth $W_0 = 100000$ euro and then we maximize the functionals monthly from January 2008 till January 2012. The methodology to recalibrate wealth periodically is the same discussed by Angelelli and Ortobelli (2009). The results of this ex post analysis are reported in the following Figures 1, 2, 3 corresponding to functionals $F_1$, $F_2$, $F_3$. 
The goal of this study is to maximize the future wealth invested in bonds taking into account the default risk of the reference entities issuer of the bonds. In particular, we deal the selection problem of a portfolio of bonds hedged by the default risk. According to this aim we suggest to consider in the portfolio optimization a proportional number of CDSs to hedge the default of risk. The empirical analysis has been made in the years following the sub-prime mortgage crisis, characterized by a climate of pessimism and uncertainty about the future course of the securities. We examine the performance of three different strategies which evaluate in different way the reward and the risk of the portfolio of bonds and CDSs. Moreover the ex-post wealth is promising for all the strategies and for almost all the modified durations. We observe the worst ex-post final wealth when high levels of modified duration hold in the portfolio optimization problem since the price risk has deeper impact in the decisional process.

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