Time Scale Analysis and Synthesis of Wind Energy Conversion Systems

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Abstract: - In the last decade, the need to exploit renewable energy resources have been provided an impetus, by many governing bodies through favorable policy making and investment. This paper presents design of an optimal control scheme for Wind Energy Conversion Systems based on time scale analysis. The slow and fast dynamics governing the mechanical components of a variable speed wind turbine are modeled. The decoupled dynamics facilitate model order reduction, thereby making the control scheme computationally more efficient. Linear Quadratic Regulators are designed for the reduced order slow and fast subsystems. The simulations compare the proposed method to a full order system. The results manifest the effectiveness of the proposed method, which provides comparable control while reducing model order and computational efficiency.

Key-Words: - Time Scales, Wind Energy Conversion Systems, Linear Quadratic Regulator.

1 Introduction

The need to exploit renewable energy resources has been due to increasing concerns over global warming. One of the renewable energy sources which has grown significantly over the last few decades and is still growing in leaps and bounds is wind energy [1], [2]. The wind turbine capacity worldwide has grown exponentially from 1996 to 2011 with the global cumulative wind turbine capacity reaching 250,000MW by the end of 2011 [3]. This rapid growth has not only been stimulated by financial support from various governments, but also from private investors. As wind turbines increase in size and power, the control mechanisms associated with them become more complex.

Control systems help drive down operating costs and improve performance. In order to achieve very accurate predictions of loading conditions, its effect on system dynamics and performance, high order mathematical models are required. Higher order models, in turn, result in controller designs being computationally intensive and oftentimes time-consuming due to numerical complexities. One of the ways to streamline controller design is to investigate methods which would facilitate model order reduction while retaining all the inherent system dynamics. A Wind Energy Conversion System (WECS) is an example of a physical system, with slow and fast dynamics arising from mechanical and electrical interactions respectively. Furthermore, there can be slow and fast subsystems within the mechanical interactions of a WECS. Such systems which evolve on different time scales are called singularly perturbed or time scale systems. The applications of singular perturbation and time scale theory spans diverse fields of engineering such as aerospace, electrical, chemical and biological systems [4], [5].

Conventional modeling methods reported in literature neglect the fast dynamics, for WECS models characterized with time scale behaviour. There are numerous such instances where this approach is adopted. Rawn et. al. [6] consider a two mass model of a WECS and equate the fast dynamics to zero under the assumption that the faster states of the system are stable and settle to steady-state values. A similar assumption to obtain quasi steady state solutions, by neglecting the fast dynamics is seen in [7]. On the same lines [8], [9] and [10] neglect the fast states of the system to reduce a higher order WECS model. Even though neglecting the fast dynamics facilitates ease of controller design, the solutions obtained from such a reduced order model do not satisfy all the boundary conditions of the original system [11].

This paper investigates the time scale method to enable model order reduction and design a computationally inexpensive controller for WECS, by separating the original system into slow and fast subsystems. This method preserves the system...
dynamics in the process. Nguyen et. al. in [12] investigated the time scale method for WECS where the dynamics of the original system was decoupled into a ‘slow’ mechanical subsystem and a ‘fast’ electrical subsystem. Here, the mechanical interactions within a WECS are analyzed which are further separated into slow and fast subsystems depending on the moment of inertia of the turbine rotor and generator.

This paper is organised as follows: Section 2 presents the dynamic model of a WECS. In Section 3, the time scale method is described in which the full order WECS model is decoupled into reduced order slow and fast subsystems. In Section 4, Linear Quadratic Regulators (LQR) are designed for the reduced order subsystems. The LQR control is also applied to the nonlinear WECS. In Section 5, the simulation results for the full order and reduced order optimal control are compared to indicate the closeness of the reduced order and full order controller designs.

2 WECS Dynamics
WECS transforms the kinetic energy of the wind into electrical energy. The wind turbine rotor serves as the transducer which harvests this wind energy. In this paper, the main focus is on the aerodynamics and the drive train dynamics of the wind energy system. This research confines itself to the time scale behavior within the mechanical interactions of the WECS. The schematic of a variable speed wind turbine is shown in Fig. 1 [13].

![Schematic of the WECS](image)

Fig.1: Schematic of the WECS [13]

The moment of inertias of the turbine rotor and generator are represented by $J_r$ and $J_s$, respectively. The two masses in the model are connected by a flexible shaft characterized by stiffness $K_s$ and damping coefficient $D_s$ [14]. The flexible shaft is considered as a torsion spring connected between the masses. An ideal gear box is assumed with a gear ratio $N_s$ that relates the speed of the turbine rotor to that of the generator.

2.1 Aerodynamics
The kinetic energy of the wind stream is converted to mechanical energy by the turbine rotor blades which provide the aerodynamic torque,

$$T_r = \frac{P_r}{w_r},$$

where $w_r$ is the angular velocity of the rotor and $P_r$ is the aerodynamic power given by,

$$P_r = \frac{1}{2} \rho \pi R^2 v C_p(\lambda),$$

where $\rho$ is the air density, $R$ is the blade wing radius, $v$ is the wind speed and $C_p(\lambda)$ is the power coefficient which is a function of the tip speed ratio $\lambda$. It is defined as the ratio of the wind speed to the blade tip speed [13], [15], [16], [17],

$$\lambda = \frac{v}{Rw_r},$$

where $v$ is the wind speed, $R$ is the blade wing radius and $w_r$ is the angular velocity of the rotor. The power coefficient $C_p(\lambda)$ is defined as [13],

$$C_p(\lambda) = 0.22 \left( \frac{116}{\lambda} - 5 \right) e^{-\frac{116}{\lambda}},$$

$$\frac{1}{\lambda_0} = \lambda - 0.035$$

2.2 Drive Train Dynamics
The drive train system is approximated by a two-mass spring and damper model [7], [13], [18], [19]. This model yields a more accurate response of the wind turbine’s dynamic behavior during fluctuating wind conditions and results in a more accurate prediction of the impact on the power system [20], [21].

The mechanical model is driven by two torques, one from the turbine blades $T_r$ and the other from the electromagnetic torque $T_e$ exerted by the interacting fields of the generator. These torques cause the rotor and the generator to move with angular velocities $w_r$ and $w_g$ respectively. The equations of motion for the drive train system are obtained by summing the torques acting on each of the masses $J_r$ and $J_s$ [22],

$$\dot{w}_r = \frac{1}{J_r} \left( \frac{P_r(w_r, v(t))}{w_r} - K_s \dot{\theta}_{\text{diff}} - D_s (w_r - \frac{w_g}{N_s}) \right),$$

$$\dot{\theta}_{\text{diff}} = w_r - \frac{w_g}{N_s},$$

$$\dot{w}_g = \frac{1}{J_s} \left( \frac{K_s \dot{\theta}_{\text{diff}} + D_s (w_r - \frac{w_g}{N_s})}{N_s} - T_g \right),$$

where $\theta_{\text{diff}} = \theta_r - \theta_s$ is the difference between the
angular displacements of turbine rotor and generator respectively.

2.3 Non-linear Model of WECS
The nonlinear state space model of WECS is obtained by combining Equations (1-6). Comparing the state-space model to a nonlinear system representation \( \dot{x} = f(x,u) \), the state vector \( x \), input vector \( u \) and output vector \( y \) are defined as,

\[
x = \begin{bmatrix} w \ 
\theta_w \ 
w_j \end{bmatrix}^T,
\]

\[
u = \begin{bmatrix} v 
T_j \end{bmatrix}^T,
\]

\[
y = \begin{bmatrix} w \end{bmatrix}^T.
\]

In the WECS model, wind is a natural input to the system and since it cannot be controlled, for controller design purposes, only one control input \( (T_j) \) is considered. Wind turbine data [13] of a Vestas-v29 225KW wind turbine was used for simulations.

2.4 Eigenvalues of WECS
To understand system behavior, the nonlinear model (1-6) was linearized about an operating point which was at the maximum power conversion efficiency. A wind speed of 11m/s was chosen for linearization. The eigenvalues obtained were: \(-0.090002, -7.0573 + 36.892i \) and \(-7.0573 - 36.892i \). By comparing the eigenvalues, it is evident that the real eigenvalue (-0.090002) is much smaller than the real part of the complex eigenvalues (-7.0573 ± 36.892i). Systems characterized by widely separated groups of eigenvalues exhibit time scale phenomena [11]. Thus the presence of one slowly varying state and two fast states can be inferred. This separation in the ‘speed’ of the mechanical variables makes WECS a prime candidate for Time Scale Analysis [11].

The slow dynamics in the system are attributed to the large inertia of the turbine rotor, while the fast dynamics to the relatively small inertia of the generator and poorly damped drive train dynamics. Since the nonlinear model is dependent on wind speed, linearization is carried out at different wind speeds and the eigenvalues at every wind speed is provided in this section [11]. The general form of a linear singularly perturbed system is provided in Equation 8.

\[
\begin{bmatrix} \dot{x}_1 \\
\epsilon \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\
B_1 & B_2 \end{bmatrix} \begin{bmatrix} x_1 \\
x_2 \end{bmatrix} + \begin{bmatrix} A_3 & A_4 \\
B_3 & B_4 \end{bmatrix} u,
\]

where \( x_1 \) and \( x_2 \) are the \( m \)- and \( n \)- dimensional state vectors, \( u \) is an \( r \)-dimensional control vector, matrices \( A_i \) and \( B_i \) are of appropriate dimensions and \( \epsilon \) is the small parameter which can be generally small time constants, masses, moments of inertias, resistances, inductances or capacitances which are responsible for increasing the order of the system [11].

3.1 Decomposition of System Dynamics
A two-stage linear transformation [11], given by

\[
x_i = x_j - M x_j,
\]

\[
x_j = x_j + L x_j,
\]

is applied on the system in Equation (8) to decouple it into independent slow and fast subsystems,

\[
\begin{bmatrix} \dot{x}_1 \\
\epsilon \dot{x}_2 \end{bmatrix} (t) = \begin{bmatrix} A_1 & A_2 \\
B_1 & B_2 \end{bmatrix} \begin{bmatrix} x_1 \\
x_2 \end{bmatrix} (t) + \begin{bmatrix} A_3 & A_4 \\
B_3 & B_4 \end{bmatrix} u(t),
\]

where,

\[
A_i = A_i - A_i L,
\]

\[
A_i = A_i + A_i L,
\]

\[
B_i = B_i - M L B_i - M B_i,
\]

\[
B_i = B_i + L B_i.
\]

The subscripts ‘s’ and ‘f’ denote slow and fast states respectively. The matrices \( A_1 \) to \( A_4 \) and \( B_1 \) to \( B_2 \) are obtained from the equations in (8) as,

\[
A_i = \frac{A_i}{\epsilon}, \ A_i = \frac{A_i}{\epsilon}, B_i = \frac{B_i}{\epsilon}, (12)
\]

The variables \( L \) \((n \times m)\) and \( M \) \((m \times n)\) are solutions of the nonlinear Lyapunov-type equations,

\[
LA_i + A_i - LA_i - LA_i = 0,
\]

\[
LA_i - A_i L - M \epsilon (A_i + LA_i) + A_i = 0. (13)
\]

It is evident from (10) that the state variables \( x_s \) and \( x_f \) can be solved independently of each other. The \( L \) and \( M \) matrices are iteratively calculated using the high accuracy Newton method [24]. Newton’s algorithm converges quadratically in the neighborhood of the sought solution, at the rate of \( O(\epsilon^2) \) where \( i = 1, 2..., m_{\text{max}} \).

3.2 Application of Time Scale Method to WECS
The nonlinear WECS model (Equations 1-6) was transformed into a linear singularly perturbed form as shown in Equation (8). The small parameter was identified first, through a series of operations; such as scaling of differential equations and time scale transformations. The time scale method was then applied to the WECS model, which was linearized.
about an operating point (as discussed in Section 2.4). The small parameter identified, was the ratio of the moment of inertia of the generator to the moment of inertia of the turbine rotor [23].

A 3\textsuperscript{rd} order WECS model was reduced to two separate 1\textsuperscript{st} order and 2\textsuperscript{nd} order models. The reduced order slow and fast subsystems obtained are:

\[
\begin{bmatrix}
\dot{x}_s \\
\dot{\theta}_{as}
\end{bmatrix}
= 
\begin{bmatrix}
A_s & 0 & 0 \\
0 & B_s
\end{bmatrix}
\begin{bmatrix}
w_r \\
\theta_{as} + 0 \\
w_f
\end{bmatrix}
\begin{bmatrix}
B_f
\end{bmatrix}
\]

To ensure that the decomposed systems retain the slow and fast dynamics, the eigenvalues of the original system and the decomposed system were compared. Table 1 lists the eigenvalues of full order and reduced order systems. The results confirm that the time scale method decouples the system dynamics perfectly (the values obtained were same to up to six decimal places).

| Table 1: Comparison of full order and reduced order eigenvalues |
|----------------------|------------------|
| Full Order | Eigenvalues |
| \( A \) & \( \text{eig}(A) = -0.063321 \)  
\(-2.1103 + 37.714i\)  
\(-2.1103 - 37.714i\) |
| Reduced Order | Eigenvalues |
| \( A_s \) (Slow-subsystem) & \( \text{eig}(A_s) = -0.063321 \) |
| \( A_f \) (Fast-subsystem) & \( \text{eig}(A_f) = -2.1103 + 37.141i \)  
\(-2.1103 - 37.141i\) |

4 Optimal Control of WECS using Time Scale Analysis

This section discusses, LQR design for the reduced order WECS. LQR control laws are designed separately for each of the slow and fast subsystems. The performance index is chosen to minimize the error between the perturbed state and the desired state (which is zero) for infinite time period. The slow subsystem \( x_s \), which was defined in Equation (10), has a performance index,

\[
J_s = \frac{1}{2} \int \left[ x_s^T(t)Q_s x_s(t) + u_s^T(t)R_s u_s(t) \right] dt.
\]

where \( Q_s \) and \( R_s \) are the weighting matrices for the slow subsystem. The control signal \( u_s(t) \) for the slow subsystem is calculated as:

\[
u_s(t) = -K_s x_s(t) = -R_s^{-1}B_s^T P_s x_s(t).
\]

where \( K_s \) is the regulator gain of the slow subsystem and \( P_s \) is the solution of the slow algebraic Riccati equation (16),

\[
P_s A_s + A_s^T P_s + Q_s - P_s B_s R_s^{-1} B_s^T P_s = 0.
\]

Similarly for the fast subsystem, the LQR control is calculated as,

\[
u_f(t) = -K_f x_f(t) = -R_f^{-1}B_f^T P_f x_f(t).
\]

where \( P_f \) is the solution of the fast algebraic Riccati equation,

\[
P_f A_f + A_f^T P_f + Q_f - P_f B_f R_f^{-1} B_f^T P_f = 0.
\]

A block diagram describing LQR control design for the reduced order WECS is presented in Fig. 2. The feedback control is now a composite control \( u(t) \) i.e. sum of slow control \( u_s(t) \) and fast control \( u_f(t) \).

The control action of the LQR was further investigated, where the composite control was studied for the original nonlinear WECS model. (Previously discussed scheme was implemented on the linear WECS model). The control scheme is depicted in Fig.3. The states of the nonlinear WECS were simulated at nominal conditions of wind speed and control input (generator torque, \( T_g \)). At the point of linearization, the nominal states \( x(t) \) were perturbed by a small amount \( \delta x(t) \) and performance of the designed composite controller was observed.
5 Simulation Results

The LQR controllers were designed in MATLAB® and implemented in Simulink®. Matrices $A_s$, $B_s$, $A_f$, and $B_f$ are given in Section 3.2. The weighting matrices $Q_s$, $R_s$, $Q_f$, and $R_f$ were chosen such that they minimize the time taken to attain the nominal value. These matrices were chosen from multiple iterations. A comparison between the full order and reduced order control of the linear WECS is provided in Fig. 4. It can be seen that the controller regulates all the states to zero, for both full order and reduced order cases. A detailed view of the states near the origin is shown in Fig. 5. The very close matching between the full order and reduced order LQR controllers manifests the effectiveness of the time scale method. Thus the proposed method provides almost the very same control action with less computational effort.

![Fig.4: LQR control – Comparison of the full order system and reduced order system plots](image1)

![Fig.5: Detailed view of the LQR comparison plots near the origin](image2)

6 Discussion

A time scale analysis of the mechanical interactions of WECS was performed in this research. The prime objective was to develop a computationally less intensive controller scheme, which was brought about by the application of the time-scales method to the complex wind energy system. The method helped bring a 3rd order WECS system down to two 1st order and 2nd order subsystems facilitating simpler and efficient optimal control designs. The simulation results of linear WECS, for the full order system and the reduced order system, indicate that the performances closely match one another. This delay can be overcome by modifying the weighting matrices $Q$ and $R$.

![Fig.6: States and control of nonlinear WECS with composite LQR control.](image3)

7 Conclusion

A time scale analysis of the WECS was made which led to decoupling a full order wind energy system into independent slow and fast subsystems. The simulation results, for the full order system and the reduced order system, indicate that the performances closely match one another. This
research has important implications, especially when large model orders are used to describe a complete wind energy system (mechanical and electrical components). For real time applications, the decoupling of the full order system would bring about a reduction in on-line and off-line computational requirements as a result of simpler controller design, parallel and distributed processing of information with corresponding sampling rates (slow with slow sampling rate, fast with fast sampling rate). Also a wind energy system designed with two controllers, for each of the slow and fast subsystems, is more reliable than a system with one controller.

References: