Numerical Study of Friction Induced Vibrations of a Rail

IGNAT IGNATOV
Department of Mechanics
Technical University - Sofia
Kliment Ohridski blv. 8
BULGARIA
i_ignatov@tu-sofia.bg

Abstract: A FEM model of a rail, subjected on friction induced vibration is studied in this work. The model is presented as continuous system and the boundary conditions, mass and elastic properties of a real object are taken into account. The vibrations are induced by lateral friction force caused by the wheel/rail slipping. The friction forces are nonlinear functions of the relative velocity during slipping. The system matrices are built in Ansys, by the function substructuring and the equations are solved using Matlab Simulink. It is observed that the friction-induced vibrations occur with the frequency equal to the first natural frequency of the system.

Key Words: Self-excited vibration, continuous system, FEM, railway rail.

1. Introduction
The self-excited friction vibrations usually induce higher dynamic loads in machines. Such vibrations are the brake squealing, clutch chatter, the squealing of the rail bounded vehicles in curves etc. The cause for that kind of vibrations is the dependence of the friction force $F_f$ on the relative velocity of sliding $[1, 2, 8, 12, 13, 16, 17, 18]$.

The self-excited vibrations in rail-bounded vehicles (fig. 1) are caused by the interaction between the wheel and the rail $[7, 8]$. The vibrations are induced by friction forces acting lateral to the wheel axle, which occur during the motion of rail-bounded vehicle in curve. There is a difference between the self-excited vibrations caused by lateral forces, and the ones that are induced by the friction of the ledge and the rail head (fig.1).

![Fig.1 Scheme, showing the axles of a bogie in a curve.](image)

According to dependence between the curve radius and the self-excited vibrations, the magnitude of the self-excited vibrations is growing by the decrease of the curve radius. According to the Rudds’ rule $[16]$ noise from friction induced vibrations in rail-bounded vehicles will not occur if the curves have radius $R > 100b$ (where $b$ is the wheelbase). As an example a bogie having base $2.5m$ will emit noise passing through curves having radius $R < 250m$, while $10m$ bogie base or in other words railway wagon with two axes will induce vibrations passing through a curve with radius greater than $1000m$.

As general, the phenomenon does not occur in curves of radius greater than $500m$, while in curves less than $200m$ it is typical $[16]$.

It is known that, the wheels of the rail bounded vehicles slip in curve lateral to the rails $[4, 5, 7, 16]$. As a result of the slippage, vibrations are induced both in the wheel and the rail. It is assumed that the wheel is the energy source and the rail is the vibrating system. A number of mechanical systems with applied friction inducers are considered and studied as simple discrete systems. This is not always suitable for description of a real mechanical system $[1, 2, 8, 17, 18]$. The goal of this paper is to study the friction-induced vibrations in a rail from a railway, modeled as a continuous system.

2. Dynamic Model
The dynamic model is shown on fig. 2. A part of a rail is considered, and it is modeled as a supported beam. The distance $L$ between the supports, and the support length are picked according to the standards in the railway transport. The rail cross section is
selected from standard UIC60 rails [15], but the cross section is geometrically simplified, which does not influence the natural frequencies. The natural frequencies are considered as main criteria in the self-excited friction vibrations [6, 8]. From the forces acting in the contact, significant for the phenomenon study is the projection of the friction force caused by the lateral sliding of the wheels in curve. In this study it is assumed that the projection $F_f$ is constant in magnitude and its direction is horizontal, perpendicular to the rail ($F_f \parallel y$). To simplify the model (fig. 2) the rest of the forces and the moments in the contact are included in the model.

The function of the friction force in respect of the sliding velocity is shown on fig. 3, as in the drop-down section is described with a cubic function (equation) [6, 8]:

$$F_f = b_0 - b_1 v_r + b_2 v_r^2 + b_3 v_r^3,$$

where $v_r = v_0 - \frac{d}{dt}[y(L/2, t)]$ is the relative velocity of sliding, $v_0$ is the velocity of sliding of the wheel in axial direction, and $b_0, b_1, b_2, b_3$ are positive constants. In order to reduce the computing time, the stick effect is neglected, i.e. $v_0 \approx \frac{d}{dt}[y(L/2, t)]$.

Fig.2. Dynamic model of the rail in this study.

The solution of the problem is made by finite elements method (FEM), [3, 9,10,11,12]. The model is meshed with brick elements type SOLID185, having nodes with three degrees of freedom (DOF) – translations in the nodal $x$, $y$, and $z$ directions as shown on fig. 4.

After modeling the system with elements type SOLID185 in Ansys (fig. 4a), and applying the boundary conditions the equations of motion could be written as follows:

$$\begin{bmatrix} M^* \end{bmatrix}\ddot{u} + \begin{bmatrix} K^* \end{bmatrix}u = \{F^*\},$$

where $[\hat{M}^*]$ and $[\hat{K}^*]$ are the mass and elasticity matrices from equation (2), and to apply the boundary conditions in them.

The assembled matrices are adopted for implementation into Matlab, as the reordered numbering of the nodes is taken into account and the transition to general coordinates is considered as well. The friction force is implemented in the vector of the nodal loads according to fig. 4b. The differential equation after obtaining the mass and stiffness matrices could be written as follows

$$\begin{bmatrix} \hat{M}^* \end{bmatrix}\ddot{u} + \begin{bmatrix} \hat{C}^* \end{bmatrix}u + \begin{bmatrix} \hat{K}^* \end{bmatrix}u = \{F^*\},$$

where $[\hat{M}^*]$ and $[\hat{K}^*]$ are the mass and elasticity matrices, while $\{u\}$ and $\{F^*\}$ are labeled accordingly the vector of the nodal displacement and the nodal forces. $[\hat{C}^*] = \alpha [\hat{M}^*] + \beta [\hat{K}^*]$ - is the damping matrix according to the type of the damping accepted (Rayleigh damping [3,19]), the coefficients $\alpha$ and $\beta$ are determined according to the current type of the construction.

Fig.3. Friction force as a function of the relative velocity of sliding.

3. FEM Modeling

The assembled of the global matrices is made by Ansys [19]. The option “Superstructuring” is used to assemble the mass and elasticity matrices from equation (2), and to apply the boundary conditions in them.
4. Numerical Solution

The following parameters are used in the numerical solution [3, 6, 14]: 

\( L = 0.58 \, m \), Elasticity module - 
\( E = 2.1 \times 10^{11} \, Pa \), Density of the material - 
\( \rho = 7850 \, kg / m^3 \), \( v_0 = 6 \, m / s \), Cross section area - 
\( A = 78.87 \times 10^{-4} \, m^2 \), \( b_h = 1500 \, N \), \( b_i = 75 \, N s / m \), 
\( b_2 = 0 \), \( b_3 = 1 \, N s^3 / m^3 \), \( \alpha = 0 \), 
\( \beta = 3.4589 \times 10^{-7} \, s / rad \).

The natural frequencies of the system are calculated with Matlab and Ansys, in order to verify the reduced matrices from Ansys. The difference in the natural frequencies could be result from the different methods used for eigenvalue extraction. The values are presented in table 1.

Table 1. Natural Frequencies

<table>
<thead>
<tr>
<th>Natural frequency number</th>
<th>Matlab Hz</th>
<th>Ansys Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>345</td>
<td>348</td>
</tr>
<tr>
<td>2</td>
<td>612</td>
<td>612</td>
</tr>
<tr>
<td>3</td>
<td>1062</td>
<td>1033</td>
</tr>
<tr>
<td>4</td>
<td>1365</td>
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</tr>
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<td>1410</td>
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<td>2020</td>
</tr>
<tr>
<td>8</td>
<td>2826</td>
<td>2832</td>
</tr>
<tr>
<td>9</td>
<td>3098</td>
<td>3082</td>
</tr>
</tbody>
</table>

Equations (3) are solved in Matlab-Simulink, and the results are presented graphically on figs. 5-9. The results correspond to the results after the vibrations are established on the limit cycle.

The displacement in the time domain of the node (\( u_{644} \)), where the friction force is applied, is presented on fig. 5. Fig. 6 shows the limit cycle on which the vibrations of node (\( u_{644} \)) are established, as \( v_{644} \) is the velocity of the node. Fig. 7 shows the frequency spectrum analysis of the displacement \( u_{644} \), as from the figure could be stated that the system is vibrating with frequency close to its first natural (see Table 1). Fig. 8 presents the velocity of the node with general coordinate \( u_{644} \). The vibration mode, corresponding to the first natural frequency of the system could be observed on fig. 9.
5. Conclusion

In this paper friction induced vibrations of a railway rail modeled by FEM are studied. The vibrations are induced by lateral friction force caused by the wheel/rail slipping. Stationary process is studied and the internal damping is taken into account (Rayleigh damping). Analyzing the numerical solution of the dynamic model, it could be stated that the friction-induced vibrations occur with the first natural frequency of the system. In a non-stationary process the vibrations have the same frequency, but different amplitude [1]. It is important to note that, the real process is non-stationary, as in this study, the shifting of the friction force according to the number of the wheel axels of the rail bounded vehicle it is not modeled.

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References: