Numerical Simulations in Jerk Circuit and It's Application in a Secure Communication System

A. SAMBAS[1], M. SANJAYA WS[1], M. MAMAT[2], N. V. KARADIMAS[3], O. TACHA[4]

[1]Bolabot Techno Robotic School,
Sanjaya Star Group
Jl. A.H. Nasution No. 105, Bandung
West Java
INDONESIA

[3] Informatics LAB
Faculty of Mathematics & Engineering Sciences
Department of Military Science
Hellenic Army Academy
Athens, GR-16673
GREECE

[2]Department of Mathematics
Universiti Malaysia Terengganu
21030 Kuala Terengganu
MALAYSIA

[4]Department of Mechanical Engineering
Aristotle University of Thessaloniki
Thessaloniki, GR-54124
GREECE

acenx.bts@gmail.com, nkaradimas@ilabsse.gr, rtacha@auth.gr

Abstract: - In this work, the case of synchronization between two mutually coupled identical chaotic circuits, for use in a secure communication scheme, is studied. The chosen circuit is the well-known Jerk circuit with a simple nonlinear function. By using various tools of nonlinear theory, such as phase portraits, Poincaré maps, bifurcation diagrams and Lyapunov exponents, the chaotic behavior of this system is confirmed. The comparison of the results between the numerical simulation using the MATLAB and circuit's simulation with the MultiSIM verifies that the Jerk circuit presents the expected behavior. Finally, the effectiveness of the mutually coupling scheme between two identical Jerk circuits in a secure communication system is presented in details.

Key-Words: - Jerk circuit, chaotic synchronization, bidirectional coupling, Poincaré map, bifurcation analysis, Lyapunov exponents, secure communication system.

1 Introduction

Chaos is generally defined as a state that exists between definite and "random" state. It is a very interesting phenomenon and has been intensively studied during the last four decades. The characteristics of chaotic systems can be used in many practical applications, such as secure communications [1-2], chemistry [3], biology [4,5], robotic [6], bits generators [7], psychology [8], ecology [9-10], economy [11,12], and cryptography [13].

Pecora and Carroll first demonstrated how chaotic systems could be synchronized, using an electronic circuit coupled unidirectionally to a subsystem made up of components of the parent systems [14]. This innovation provided a new perspective on chaotic dynamics and inspired many studies on synchronizing chaotic systems. Cuomo and Oppenheim further expanded the area by demonstrating how synchronized chaotic systems could be used in a scheme for secure communication [15].

The plan of the paper is as follows. In section 2, the details of the proposed autonomous Jerk circuit's simulation using MATLAB 2010 and MultiSIM 10.0, are presented. In Section 3, the bidirectional coupling method is applied in order to synchronize two autonomous Jerk circuits. The chaotic masking communication scheme by using the above mentioned synchronization technique are presented in Section 4. Finally, in Section 5, the concluding remarks are given.

2 Jerk Circuit

Sprott found the functional form of three-dimensional dynamical systems which exhibit chaos. Jerk equation has a simple nonlinear function, which can be implemented with an autonomous electronic circuit. In this work, the Jerk circuit, which was firstly presented by Sprott in 2000 [16], is used. This is a three-dimensional autonomous nonlinear system that is described by the following system of ordinary differential equations:

$$\left.\begin{aligned}\dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -az - by + |x| - 1\end{aligned}\right\} \qquad (1)$$

This equation has only one nonlinear term in the form of absolute value of the variable x. The parameters and initial conditions of the Jerk system (1) are chosen as: (a, b) = (0.6, 1) and (x_0, y_0, z_0) = (0, 0, 0), so that the system shows the expected chaotic behavior.

2.1 Numerical Simulations Using MATLAB

In this Section the numerical simulations are carried out using MATLAB 2010. The four-order Runge-Kutta method is used to solve the differential equations of system (1). Figs.1(a)-(c) show the projections of the phase space orbit on to the *x–y* plane, the *y–z* plane, the *x–z* plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, the Jerk system presents chaotic attractors of Rossler type.

Also, it is known from the nonlinear theory, that the spectrum of Lyapunov exponents provides additional useful information about system's behavior. In a three dimensional system, like this, there has been three Lyapunov exponents (λ_1, λ_2, λ_3). In more details, for a 3D continuous dissipative system the values of the Lyapunov exponents are useful for distinguishing among the various types of orbits. So, the possible spectra of attractors, of this class of dynamical systems, can be classified in four groups, based on Lyapunov exponents [17].

- (λ_1, λ_2, λ_3) → (–, –, –): a fixed point
- (λ_1, λ_2, λ_3) → (0, –, –): a limit point
- (λ_1, λ_2, λ_3) → (0, 0, –): a two-torus
- (λ_1, λ_2, λ_3) → (+, 0, –): a strange attractor (Fig.1).

So, from the diagram of Lyapunov exponents of Jerk's system of Fig.2, the expected chaotic behavior, from the same set of parameters and initial conditions, is confirmed.

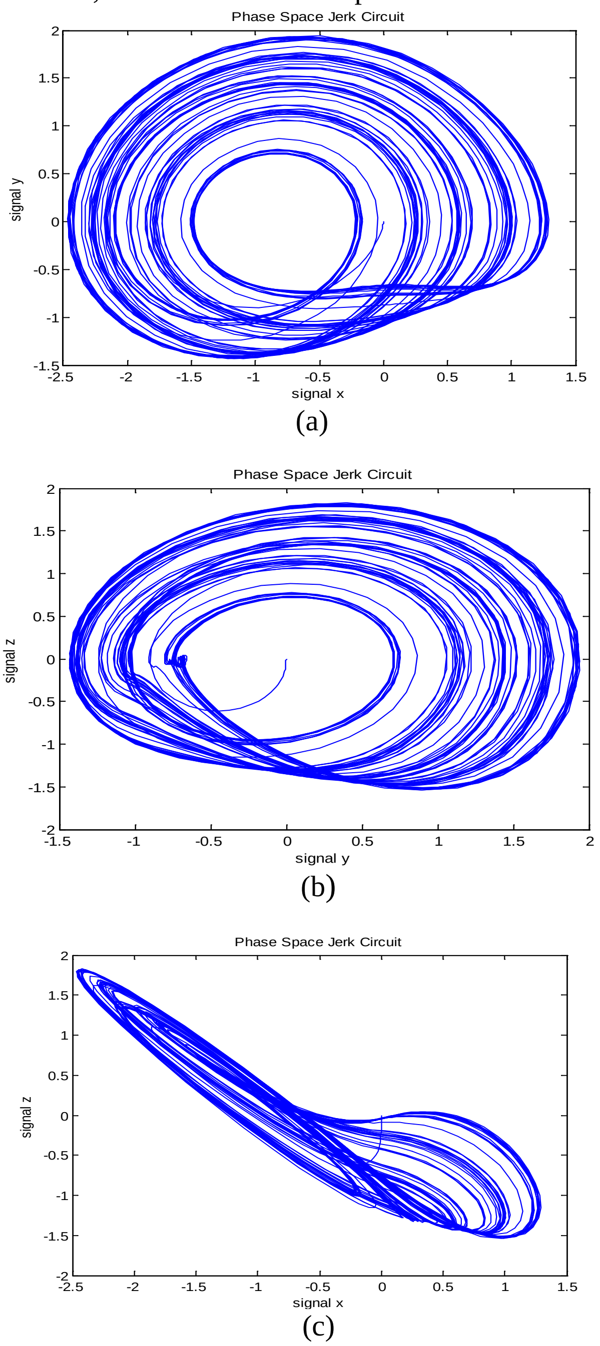


Fig.1. Numerical simulation results using MATLAB 2010, for a = 0.6, b = 1: (a) x-y plane, (b) y-z plane, (c) x-z plane.

Bifurcation theory was originally developed by Poincaré. It is used to indicate the qualitative change in a system's behavior, in terms of the number and the type of solutions, under the variation of one or more parameters on which the system depends [18].

To observe the system dynamics under all the above possible bifurcations, a bifurcation diagram may be

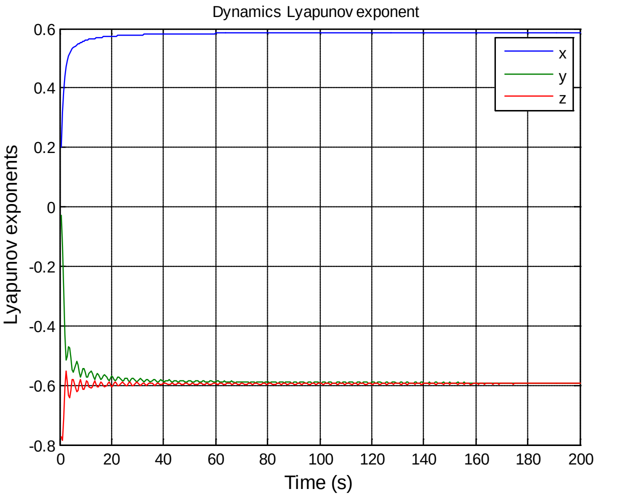


Fig.2. Diagram of Lyapunov exponents of the Jerk system, for (a, b) = (0.6, 1).

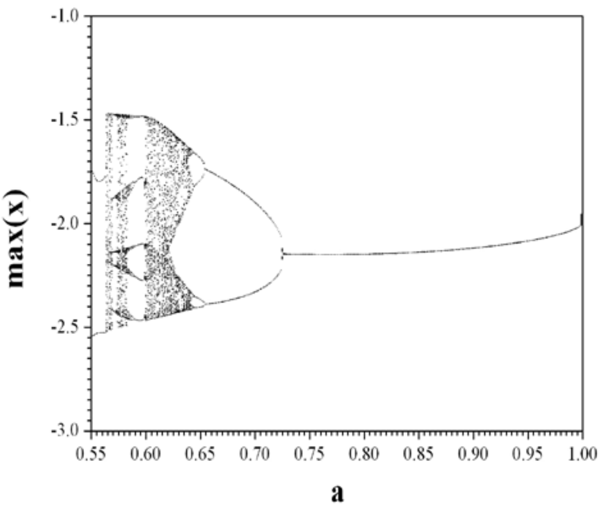


Fig.3. Bifurcation diagram of x vs. the control parameter a, for b = 1, with MATLAB 2010.

constructed, which shows the variation of one of the state variables with one of the control parameters. A MATLAB program was written to obtain the bifurcation diagram for the Jerk circuit of Fig.3. So, in this diagram a possible bifurcation diagram for system (1), in the range of $0.55 \le a \le 1$, is shown. For the chosen value of a = 0.6 the system displays the expected chaotic behavior. For $0.6 < a \le 0.76$, a period-2 behavior system and finally for $a > 0.76$ a period-1 behavior system is shown.

Another useful tool for analyzing the dynamical characteristics of a nonlinear system is the Poincaré map. In the chaotic state the phase portrait is very dense with the traces of the motion. It can be only

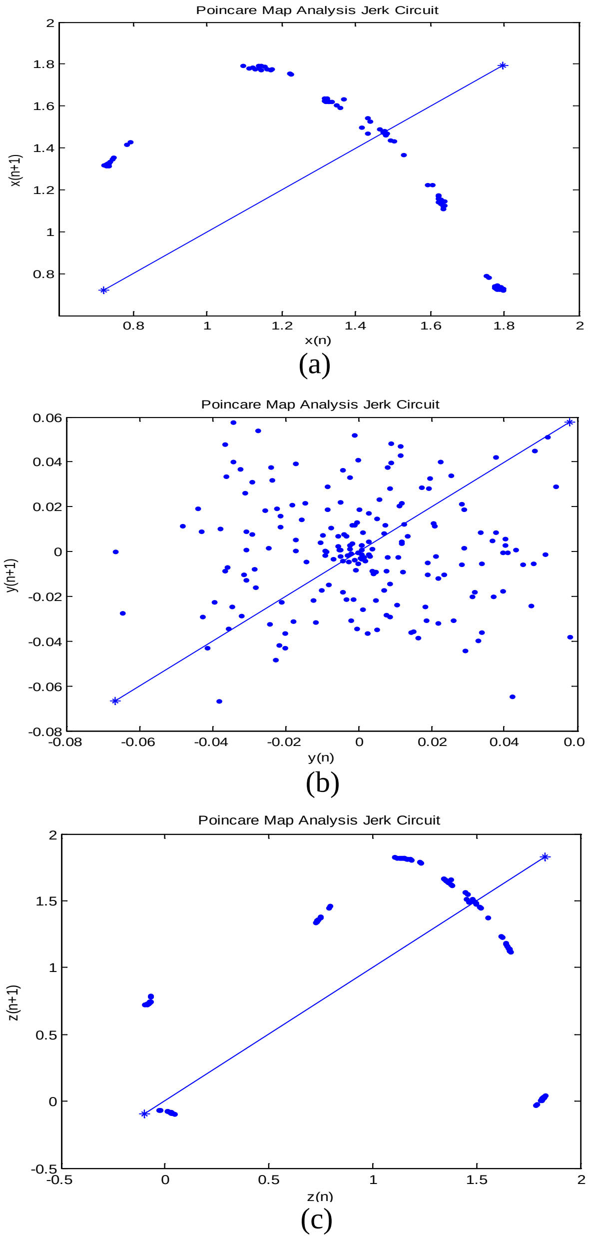


Fig.4. A gallery of Poincare maps for system (1) for a = 0.6, b = 1, with MATLAB 2010: (a) the maxima of x(n + 1) against those of x(n), (b) the maxima of y(n + 1) against those of y(n), (c) the maxima of z(n + 1) against those of z(n).

indicative of the minima and maxima of the motion. Any other characterization of the motion is difficult to interpret. One way to capture the qualitative features of the strange attractor is to obtain the Poincaré map [19]. Figs.4(a)-(c) shows the Poincaré section map using MATLAB, for a = 0.6, b = 1.

ISBN: 978-960-474-310-0

2.2 Analog Simulation Using MultiSIM

The designed circuitry, realizing system (1), is shown in Fig.5. Also the circuit has a basin of attraction outside of which the dynamics are unbounded, which manifests itself in the saturation of the op-amps. If the op-amps saturate, it is necessary to restart the circuit. The relationship among the resistors R, R_A used in the circuit and the parameter 'a' is given below.

$$R_A = \frac{R}{a} \qquad (2)$$

The occurrence of the chaotic attractor can be clearly seen in Figs.6(a)-(c). By comparing Figs.1(a)-(c) and Figs.6(a)-(c) a good qualitative agreement between the numerical integration of (1) by using MATLAB 2010, and the circuit's simulation by using MultiSIM 10.0, can be concluded.

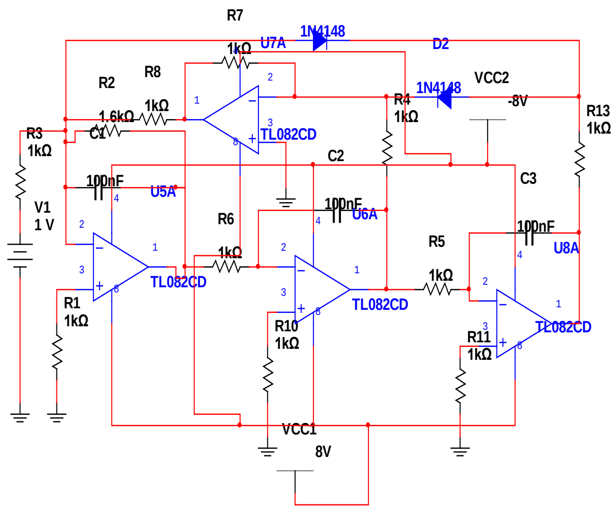
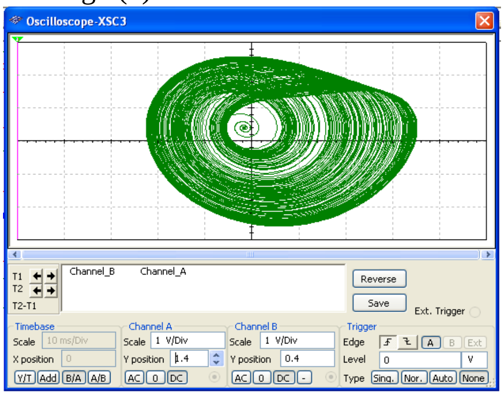


Fig.5. Schematic of the proposed Jerk circuit using MultiSIM 10.0.

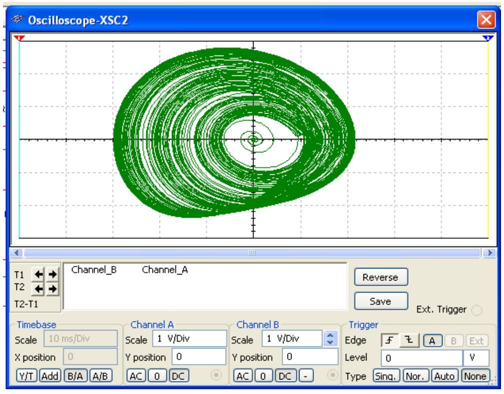
3 Bidirectional Coupling Between Two Identical Jerk Circuits

For the bidirectional coupling (also called mutual or two-way), both drive and response systems are connected in such a way that they mutually influence each other's behavior.

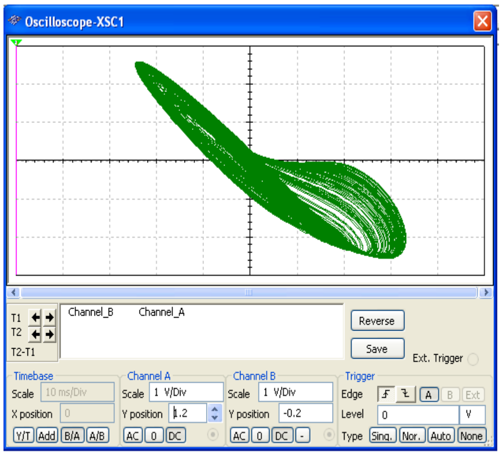
We used TL082CD op-amps, appropriate valued resistors, two diode and capacitors for MultiSIM simulations. The system of two identical Jerk circuits bidirectional or two-way coupled via a linear resistor Rc is shown in Fig.7. Chaotic synchronization appears for a coupling strength Rc ≤ 100 mΩ, as shown in Fig.8(a). For different initial conditions or resistance coupling strength Rc > 100 mΩ, the synchronization cannot occur as shown in Fig.8(b).



(a)



(b)



(c)

Fig.6. Various projections of the chaotic attractor using MultiSIM 10.0: (a) x-y plane, (b) y-z plane, (c) x-z plane.

4 Secure Communication Scheme

In chaos-based secure communication scheme, chaos synchronization is the critical issue, because two identical chaos generators, in the transmitter and the receiver end, need to be synchronized. Information signal is added to the chaotic signal at transmitter and at receiver the masking signal is regenerated and subtracted from the receiver

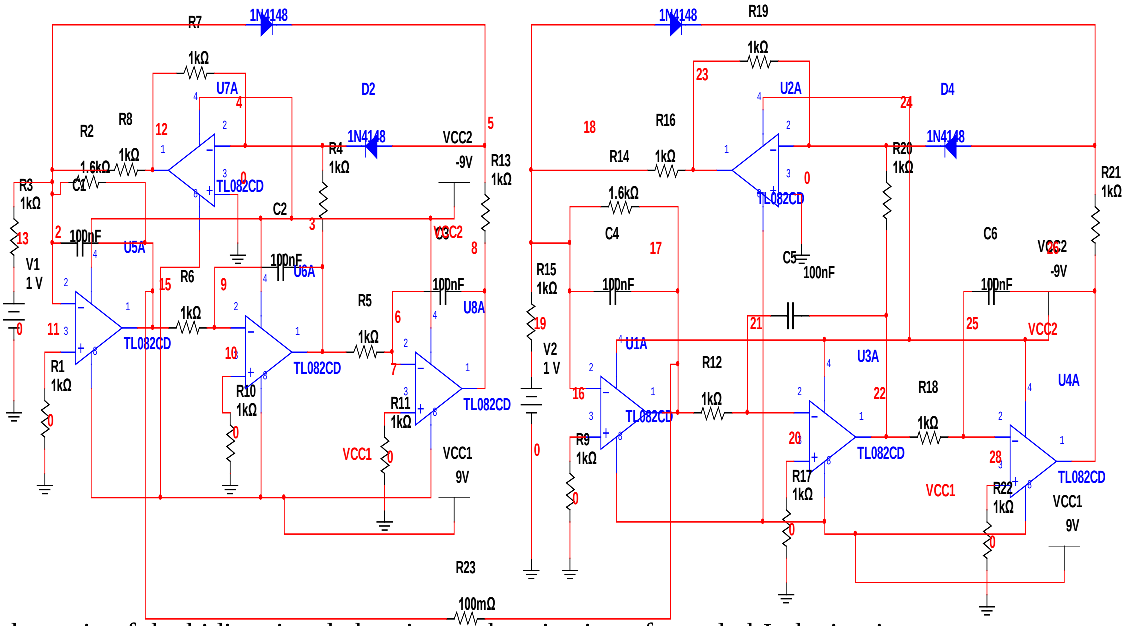
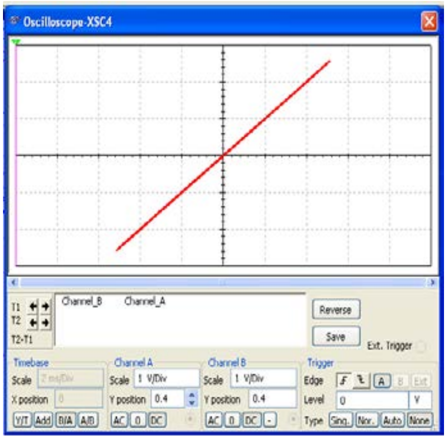
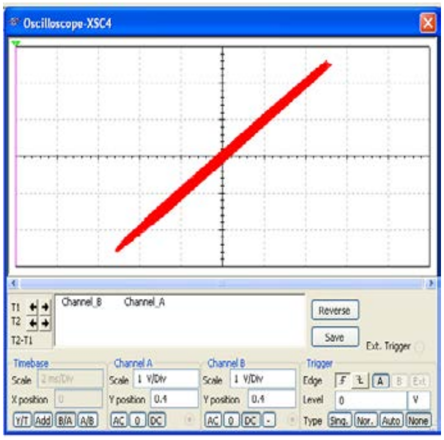


Fig. 7 Schematic of the bidirectional chaotic synchronization of coupled Jerk circuits



(a)



(b)

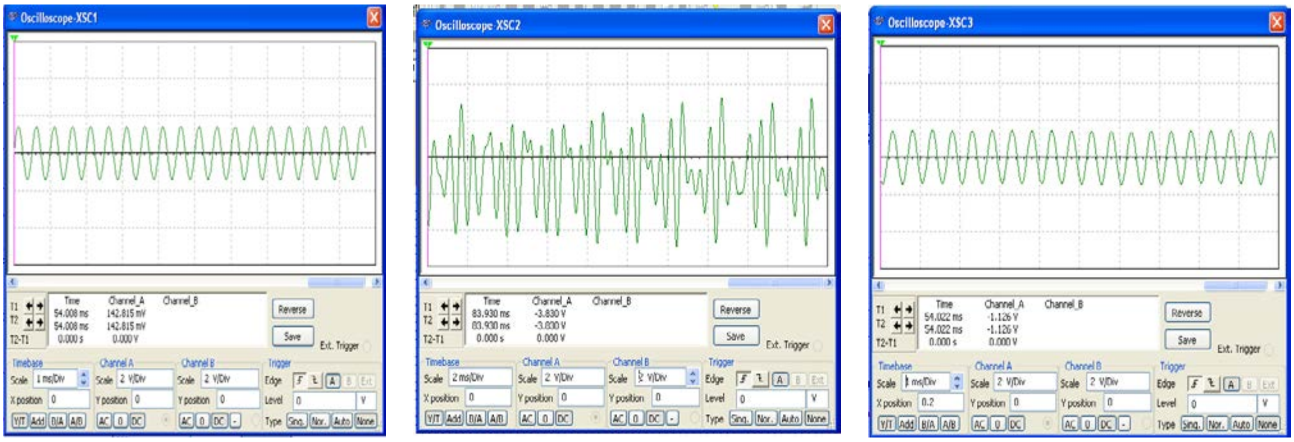
Fig. 8 Synchronization phase portrait of y_2 versus y_1, for (a) R_c = 100 mΩ and (b) R_c = 1Ω with MultiSIM 10.0.

signal. For synchronization of transmitter and receiver, bidirectional synchronization method of identical coupled Jerk circuits, is used.

The sinusoidal wave signal of amplitude 1 V and frequency 2 kHz is added to the generated chaotic signal x and the S(t) = x + i(t) is fed into the receiver. The chaotic signal x is regenerated allowing a single subtraction to retrieve the transmitted signal, $[x + i(t)] - x_r = i(t)$, if $x = x_r$. Fig.9 shows the MultiSIM 10.0 simulation results for the proposed masking signal communication scheme. Fig. 10 shows the circuit schematic for implementing the Jerk's circuit chaotic masking communication system.

5 Conclusion

In this paper, the chaotic synchronization in coupled identical Jerk circuits has been investigated by implementing bidirectional synchronization technique. We have demonstrated with simulations that chaotic circuits can be synchronized and used in a secure communication scheme. Chaos synchronization and chaos masking were realized using MATLAB 2010 and MultiSIM 10.0 programs. Furthermore, some comparisons are made with some existing results. Finally, the simulation results demonstrate the effectiveness of the proposed scheme.



(a) (b) (c)

Fig.9. MultiSIM 10.0 outputs of Jerk's circuit masking communication system, for a sinusoidal wave signal of amplitude 1 V and frequency 2KHz: (a) Information signal, (b) Chaotic masking transmitted signal, (c) Retrieved signal.

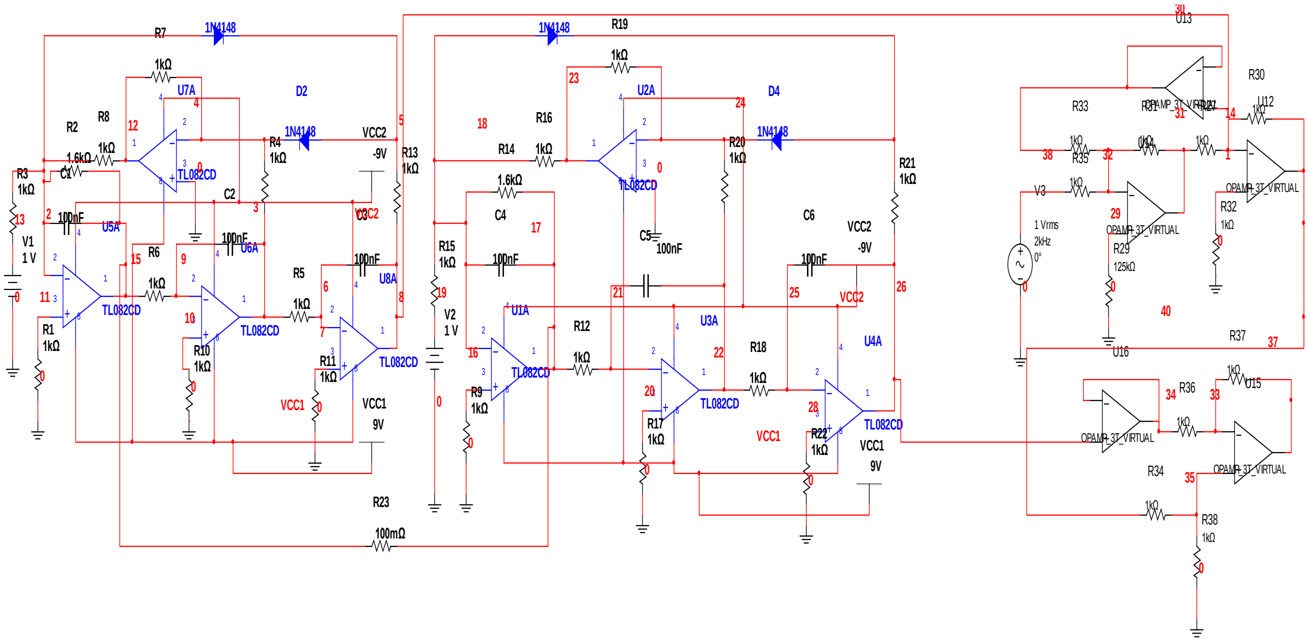


Fig. 10 Jerk circuit masking communication circuit.

References:

[1] A. Sambas, M. Sanjaya W.S and Halimatussadiyah, Unidirectional Chaotic Synchronization of Rossler Circuit and Its Application for Secure Communication, *WSEAS Trans. Syst.*, Vol. 11(9), 2012, pp. 506-515.

[2] A. Sambas, M. Sanjaya W.S., M. Mamat and Halimatussadiyah, Design and Analysis Bidirectional Chaotic Synchronization of Rossler Circuit and Its Application for Secure Communication, *Applied Mathematical Sciences*, Vol. 7(1), 2013, pp.11–21.

[3] K. Nakajima and Y. Sawada, Experimental Studies on the Weak Coupling of Oscillatory Chemical Reaction Systems. *J. Chem. Phys.*, Vol. 72(4), 1979, pp. 2231–2234.

[4] M. Sanjaya W. S, M. Mamat, Z. Salleh, and I. Mohd, Bidirectional Chaotic Synchronization of Hindmarsh-Rose Neuron Model, *Applied Mathematical Sciences*, Vol. 5(54), 2011, pp. 2685–2695.

[5] I. M. Kyprianidis, V. Papachristou, I. N. Stouboulos and Ch. K. Volos, Dynamics of Coupled Chaotic Bonhoeffer – van der Pol Oscillators, *WSEAS Trans. Syst.*, Vol. 11(9), 2012, pp. 516 I–526.

[6] Ch. K. Volos, N. G. Bardis, I. M. Kyprianidis and I. N. Stouboulos, Implementation of Mobile Robot by Using Double-Scroll Chaotic Attractors, *WSEAS Recent Researches in*

Applications of Electrical and Computer Engineering, Vouliagmeni Beach, Athens, Greece. 2012, pp. 119–124.

[7] Ch. K. Volos, I. M. Kyprianidis and I. N. Stouboulos, Motion Control of Robots Using a Chaotic Truly Random Bits Generator, *Journal of Engineering Science and Technology Review*, Vol. 5(2), 2012, pp. 6–11.

[8] J. C. Sprott, Dynamical Models of Love, *Nonlinear Dyn. Psych. Life Sci*., Vol. 8, 2004, pp. 303–314.

[9] M. Sanjaya W. S., I. Mohd, M. Mamat and Z. Salleh, Mathematical Model of Three Species Food Chain Interaction with Mixed Functional Response, *International Journal of Modern Physics*: *Conference Series*, Vol. 9, 2012, pp. 334–340.

[10] M. Sanjaya W. S, M. Mamat, Z. Salleh., I. Mohd and N. M. N. Noor, Numerical Simulation Dynamical Model of Three Species Food Chain with Holling Type-II Functional Response, *Malaysian Journal of Mathematical Sciences*, Vol. 5(1), 2004, pp. 1–12.

[11] Ch. K. Volos, I. M. Kyprianidis, and I. N. Stouboulos, Synchronization Phenomena in Coupled Nonlinear Systems Applied in Economic Cycles, *WSEAS Trans. Syst.*, Vol. 11(12), 2012, pp. 681–690.

[12] Ch. K. Volos, I. M. Kyprianidis, S. G. Stavrinides, I. N. Stouboulos, I. Magafas, anA. N. Anagnostopoulos, Nonlinear Dynamics of a Financial System from an Engineer's Point of View, *Journal of Engineering Science and Technology Review*, Vol. 4(3), 2001, pp.281–285.

[13] Ch. K. Volos, I. M. Kyprianidis, and I. N. Stouboulos, Fingerprint Images Encryption Process Based on a Chaotic True Bits Generator, *International Journal of Multimedia Intelligence and Security*, Vol. 1(4), 2011, pp. 320–335

[14] L. M. Pecora and T. L. Carroll, Synchronization in Chaotic Systems, *Physical Review Letters*, Vol. 64, 1990, pp. 821–825.

[15] K. M. Cuomo and A. V. Oppenheim, Circuit Implementation of Synchronized Chaos with Application to Communication, *Phys. Rev. Lett.*, Vol. 71(1), 1993, pp. 65–68.

[16] J. C. Sprott, Simple Chaotic Systems and Circuits, *Am. J. Phys*., Vol. 68, 2000, pp. 758–763.

[17] Q. H. Alsafasfeh and M. S. Al-Arni, A New Chaotic Behavior from Lorenz and Rossler Systems and Its Electronic Circuit Implementation, *Circuits and Systems,* Vol. 2, 2011, pp. 101–105.

[18] F. Han, Multi-Scroll Chaos Generation Via Linear Systems and Hysteresis Function Series, *PhD thesis*, Royal Melbourne Institute of Technology University, Australia, 2004.

[19] A. L. Fradkov, and A. Y Pogromsky, *Introduction to Control of Oscillations and Chaos*, World Scientific, Singapore, 1998.