

# OFDM-BPSK Wireless Communication System Performance under Employment of Generalized Receiver

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**Abstract:** - In this paper, we discuss the performance of the orthogonal frequency division multiplexing (OFDM) wireless communication systems using the binary phase shift keying (BPSK) modulation over Rayleigh fading channel implementing the generalized receiver (GR) constructed in accordance with the generalized approach to signal processing in noise. We compare OFDM-BPSK wireless communication systems based on the Neyman-Pearson receiver and GR. The relied analysis and simulation results demonstrate a superiority of GR implementation in OFDM-BPSK wireless communication systems in comparison with the Neyman-Pearson one.

**Key-Words:** - Generalized receiver; orthogonal frequency division multiplexing (OFDM); binary phase shift keying (BPSK); Neyman-Pearson receiver; signal-to-noise ratio (SNR); probability of error.

## 1 Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation technique. This approach is widely used in wireless communication systems. With growth in the number of mobile users, the requirements for high data rate in wireless communication systems have been sharply increased. Since OFDM technique ensures high data transmission rate and high bandwidth efficiency, it has become a potential research technology.

In general, there are several data modulation schemes that are used with OFDM, such as the binary phase shift keying (BPSK), the quadrature phase shift keying (QPSK), and the quadrature amplitude modulation (QAM). Different modulation schemes have different code rate. Additive white Gaussian noise (AWGN) channel, Rayleigh fading channel, and Rician channel are usually considered under analysis of wireless communication systems. There are a large number of journal papers devoted to investigation of OFDM technique in wireless communication systems using the above mentioned channel models [1-10].

Owing to high performance, the OFDM wireless communication systems are investigated intensively. Various channel estimation and diversity techniques are proposed to enhance the performance of OFDM wireless communication systems [11-13]. The

channel estimation for OFDM systems based on block-type pilot arrangement is a very useful and efficient method [14]. This technique is based on the least square (LS) method or minimum mean-square error (MMSE) method.

In this paper, we investigate the performance of OFDM wireless communication systems applying the generalized receiver (GR) designed based on the generalized approach to signal processing in noise. GR is a linear combination of the Neyman-Pearson (NP) receiver that is optimal for detection of signals with known parameters and the energy receiver that is optimal for detection of signals with unknown parameters [15-17]. This combination allows us to make a decision a “yes” signal or a “no” signal in the input process based on analysis of the jointly sufficient statistics of the mean and variance of the likelihood function. The Neyman-Pearson (NP) receiver is based on definition of sufficient statistics of the likelihood mean only. The energy receiver is based on definition of sufficient statistics of the likelihood function variance only. Thus, the GR delivers us more information about the likelihood function in the form of the jointly sufficient statistics of the mean and variance.

This paper is organized as follows. The basic principles of OFDM wireless communication system model are discussed in Section 2. We recall briefly

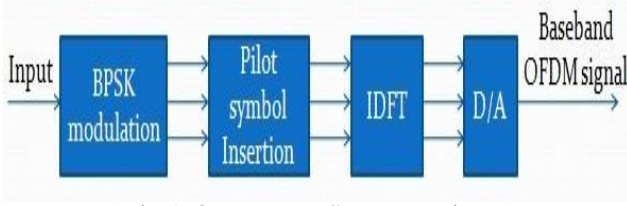


Fig.1 OFDM-BPSK transmitter

the main functioning principle of the GR in Section 3. The performance of the OFDM-BPSK wireless communication system employed GR is investigated in the Section 4. Simulation results are discussed in the Section 5. Finally, some conclusions are made in Section 6.

## 2 System Model

### 2.1 Transmitter

The transmitter block diagram of OFDM-BPSK wireless communication system is shown in Fig.1 [18]. In order to maintain the orthogonality characteristic of the carriers, the input data should be modulated firstly. Pilot symbol insertion is used to estimate the channel. At the transmitter, the signal is defined in the frequency domain. The complement of inverse discrete Fourier transform (IDFT) allows us to convert the frequency domain into the time domain.

The baseband BPSK-OFDM transmitted signal can be presented in the following form:

$$x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi nt/T} \quad 0 \leq t \leq T, \quad (1)$$

where  $N$  denotes the number of narrowband channels,  $T$  is a pulse duration, and  $X_n$  is the BPSK message signal transmitted over the  $n$ th subchannel.

### 2.2 Channel Model

The baseband signal at the receiver input in OFDM-BPSK wireless communication system can be presented in the following form:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}, \quad (2)$$

where  $\mathbf{X}$  is the baseband transmitted signal matrix,  $\mathbf{H}$  is the channel matrix, elements of which are the channel response, and  $\mathbf{N}$  is the noise matrix, elements of which are AWGN with zero mean and the variance  $\sigma_n^2$ .

Rayleigh fading channel model is applicable when there is no line of sight communication channel. Owing to Rayleigh fading, the elements of the channel matrix  $\mathbf{H}$  are distributed in accordance

with Rayleigh law. In addition, applying the central limit theorem, each path of Rayleigh fading can be modeled as the circularly symmetric complex Gaussian random variable [19]. In this case, the elements of channel matrix can be presented in the following form:

$$H_{mn} = x_1 + ix_2 = r_1 e^{i\phi_1}, \quad (3)$$

where  $r_1$  and  $\phi_1$  are the amplitude and phase of the channel matrix elements  $H_{mn}$ ,  $r_1$  is Rayleigh distributed;  $\phi_1$  is uniformly distributed within the limits of the interval  $[-\pi, \pi]$ ; and  $x_1$  and  $x_2$  are the zero mean independent and identically distributed (i.i.d.) Gaussian random variables with variance  $\sigma_1^2$ .

### 2.3 Generalized Receiver (GR)

According to the generalized approach to signal processing (GASP) in noise, GR has two input linear systems that can be presented in the form, for example, of the bandpass filters, namely, the preliminary filter (PF) and the additional filter (AF). PF is detuned with respect to AF by the central or resonant frequency on the value 4~5 times the signal bandwidth. Owing to this condition, there are additive mixture of signal and noise at the PF output and only the noise at the AF output. The noise forming at the AF output is called the reference noise. The model signal generator (MSG) is used to generate the reference signal or model signal in the GR. The GR flowchart is presented in the Fig. 2.

At GR input, there is only noise  $w[n]$  under  $H_0$  and noise and signal  $s[n] + w[n]$  under  $H_1$ . Because the AF and the PF are the linear systems, we can observe the Gaussian noise at the AF and the PF outputs if the noise is the Gaussian noise at the AF and the PF inputs. The output noise at the PF and the AF take the following form:

$$\xi_{PF}[n] = \sum_{n=0}^{N-1} h_{PF}[n] w[n - n_0] \quad (4)$$

$$\xi_{AF}[n] = \sum_{n=0}^{N-1} h_{AF}[n] w[n - n_0] \quad (5)$$

If the GR input noise is AWGN with zero mean and two-sided power spectral density  $N_0/2$ , the noise at the AF and PF outputs is also Gaussian with zero mean and variance given by [20]:

$$\sigma_n^2 = \frac{N_0 \omega_0^2}{8\Delta F}, \quad (6)$$

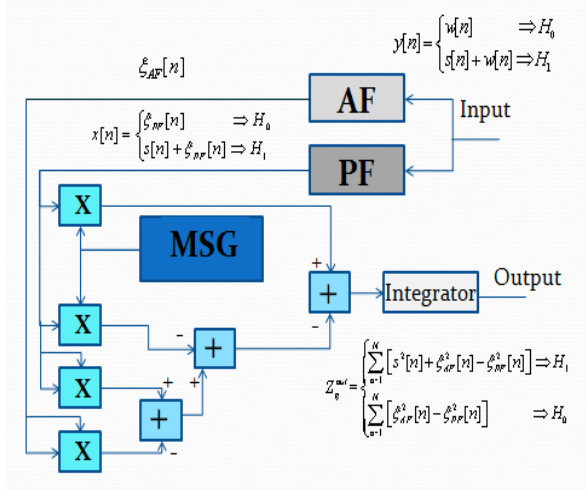


Fig.2 Generalized Receiver (GR)

where  $\omega_0$  is the resonance frequency, in the case of PF and AF are the RLC oscillatory circuits with the resonance amplitude-frequency response,  $\omega_0$  is equal to  $1/\sqrt{LC}$ , and  $\Delta F$  is the bandwidth of PF that can be presented in the following form:

$$\Delta F = \pi\beta, \quad \beta = \frac{R}{2L}. \quad (7)$$

### 3 Channel Estimation

In OFDM wireless communication system, the estimation of channel matrix is necessary because the radio channel is time-varying multiple fading and selective fading is presented in the time domain and frequency [21].

We can estimate the channel using pilot symbols. This pilot channel estimation has been developed under assumption about slow fading channel in wireless communication system.

The channel estimation for this pilot arrangement can use the least square (LS) or minimum mean square error (MMSE) techniques to obtain the channel estimation at the pilot locations. Employment of MMSE technique to obtain the channel information allows us to obtain the better performance in comparison with using the LS procedure [11].

Using the pilot channel estimation technology based on MMSE, we can present the estimation of channel gain matrix in the following from [23]

$$\hat{\mathbf{H}}_{MMSE} = \mathbf{R}_{HH} [\mathbf{R}_{HH} + \sigma_n^2 (\mathbf{X}\mathbf{X}^H)^{-1}]^{-1} \hat{\mathbf{H}}_{LS}, \quad (8)$$

where  $\mathbf{R}_{HH} = E[\mathbf{H}\mathbf{H}^H]$  is the autocorrelation matrix of the channel, and  $\hat{\mathbf{H}}_{LS}$  is the channel estimation based on LS method can be presented in the following form [23]:

$$\hat{\mathbf{H}}_{LS} = \mathbf{H} + \mathbf{X}^{-1}\mathbf{N}. \quad (9)$$

According to the [22], the channel estimation can be modeled as the true channel. Thus, it can be modeled as the bandpass stationary complex Gaussian process with zero mean. Hence, the elements of the estimated channel matrix can be presented in the following form:

$$\hat{H}_{mn} = x_3 + ix_4 = r_2 e^{i\phi_2}, \quad (10)$$

where  $r_2$  and  $\phi_2$  are the amplitude and phase of the elements of the estimated channel matrix  $\hat{\mathbf{H}}$ ,  $r_2$  is Rayleigh distributed,  $\phi_2$  is uniformly distributed within the limits of the interval  $[-\pi, \pi]$ , and  $x_3$  and  $x_4$  are the i.i.d. Gaussian random variables with zero mean and variance  $\sigma_2^2$  [22].

As follows from (3) and (6), we can define the variances of channel matrix elements and the elements of the estimated channel matrix in the following form:

$$\sigma_1^2 = \frac{1}{2} E[|H_{mn}|^2] = E[x_1^2] = E[x_2^2], \quad (11)$$

$$\sigma_2^2 = \frac{1}{2} E[|\hat{H}_{mn}|^2] = E[x_3^2] = E[x_4^2]. \quad (12)$$

It is easy to see that  $E[x_1 x_2] = 0$  and  $E[x_3 x_4] = 0$ .

The cross-correlations between  $x_1, x_3$  and  $x_2, x_4$  can be presented in the following form:

$$\mu_1 = E[x_1 x_3] = E[x_2 x_4] = \frac{1}{2} \text{Re}\{E[\hat{H}_{mn} H_{mn}^*]\}, \quad (13)$$

Also, the cross-correlations between  $x_1, x_4$  and  $x_2, x_3$  can be presented in the following form:

$$\mu_2 = E[x_1 x_4] = -E[x_2 x_3] = \frac{1}{2} \text{Im}\{E[\hat{H}_{mn} H_{mn}^*]\}. \quad (14)$$

Hence, we can define the correlation coefficients in the following form [22]:

$$\rho_1 = \frac{\mu_1}{\sigma_1 \sigma_2}, \quad \rho_2 = \frac{\mu_2}{\sigma_1 \sigma_2}. \quad (15)$$

The joint probability density function (pdf) of  $(x_1, x_2, x_3, x_4)$  can be presented in the following form:

$$p(x_1, x_2, x_3, x_4) = \frac{1}{(2\pi)^2 \sqrt{\Sigma}} \exp\left[-\frac{1}{2} \mathbf{x}^T \Sigma^{-1} \mathbf{x}\right], \quad (16)$$

where vector  $\mathbf{x} = (x_1, x_2, x_3, x_4)$ ,  $\Sigma$  is the cross-correlation matrix can be presented in the following form

$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 & \mu_1 & \mu_2 \\ 0 & \sigma_1^2 & \mu_2 & \mu_1 \\ \mu_1 & \mu_2 & \sigma_2^2 & 0 \\ \mu_2 & \mu_1 & 0 & \sigma_2^2 \end{pmatrix}. \quad (17)$$

Hence, the joint probability density function (pdf) of polar coordinate  $(r_1, r_2, \psi)$  can be presented in the following form:

$$p(r_1, r_2, \psi) = \frac{r_1 r_2}{2\pi\sigma_1^2\sigma_2^2(1-\rho_1^2-\rho_2^2)} \times \exp\left\{-\frac{1}{2(1-\rho_1^2-\rho_2^2)}\left[\frac{r_1^2}{\sigma_1^2} + \frac{r_2^2}{\sigma_2^2} - 2\frac{r_1 r_2}{\sigma_1\sigma_2}(\rho_1 \cos\psi - \rho_2 \sin\psi)\right]\right\} \quad (18)$$

where  $\psi = \phi_1 - \phi_2$ .

#### 4 Performance Analyses

The average SNR at the GR output can be determined in the following form [16]:

$$q_b = \frac{2\sigma_1^2}{\sigma_n^2}. \quad (19)$$

The error probability for wireless communication system in AWGN channels with BPSK modulation at GR output when  $N \rightarrow \infty$  is given by [24]:

$$P_b = Q\left(\sqrt{\frac{E_b}{2\sigma_n^2}} \times \log_2(M)\right), \quad (20)$$

$Q(\cdots)$  is the Gaussian  $Q$ -function and  $M = 2$ .

Hence, the error probability at GR output in AWGN channels with BPSK modulation can be rewritten as the form of

$$P_b(r_1, \psi) = Q\left(\frac{r_1}{\sqrt{2}\sigma_n} \cos\psi\right), \quad (21)$$

In the case of Rayleigh fading channel, the probability of error is defined in the following form [25]:

$$P_b = \int_0^\infty \int_0^\infty \int_{-\pi}^\pi p(r_1, r_2, \psi) P_b(r_1, \psi) d\psi dr_1 dr_2. \quad (22)$$

Substituting the (18) and (21) into (22), we can obtain the probability of error under employment of GR in wireless communication systems with Rayleigh fading channel and BPSK modulation in the following form

$$P_b^{GD} = \frac{1}{2} \left[ 1 - \frac{\rho_1 \sqrt{q_b}}{\sqrt{1 + q_b - q_b \rho_2^2}} \right]. \quad (23)$$

#### 5 Simulation Results

In this section, we present simulation results for the OFDM-BPSK wireless communication system employing the GR and compare it with the OFDM-BPSK wireless communication system using the

Table 1  
Simulation Parameters

Parameters	Specifications
Signal Constellation	BPSK
Channel Model	Rayleigh Fading channel
Average SNR $q_b$	0 ÷ 30 dB
$\rho_1$ (the real part of the correlation coefficient between channel matrix and estimated channel matrix)	0.95 ÷ 1
$\rho_2$ (the imaginary part of the correlation coefficient between channel matrix and estimated channel matrix)	0 ÷ 0.3

Neyman-Pearson receiver under the same conditions. The simulation is based on IEEE 802.11a wireless local area network (WLAN) standard [26]. For simplicity sake, we consider only BPSK modulation. The system parameters used in the simulation are presented in Table 1.

Simulation results for the case of single-carrier narrowband channel  $\rho_2 = 0$  are presented in the Fig.3. From Fig.3, we see that the probability of error is a function of the correlation coefficient  $\rho_1$ . When  $\rho_1$  is equal to 1, the OFDM-BPSK wireless communication system either under employment of the Neyman-Pearson receiver or GR presents the best performance. When the probability of error is approximately equal to  $10^{-3}$ , the SNR gain in favor of GR employment is more than 6 dB in comparison with the Neyman-Pearson receiver. With decreasing

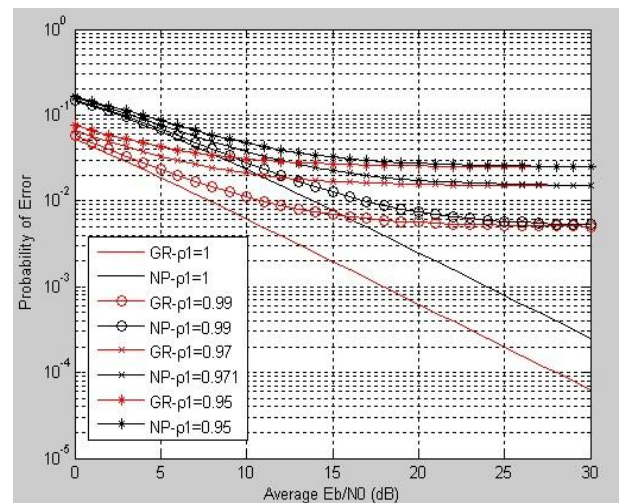


Fig.3 Performance of OFDM-BPSK system  $\rho_2 = 0$



$\rho_1$  from 1 to 0.95, a superiority of GR employment in OFDM-BPSK wireless communication system is kept, but the probability of error is increased. For example, at  $\rho_1 = 0.99$ , we observe that the SNR gain in favor of GR in comparison with the Neyman-Pearson receiver is approximately equal to 7dB, but the probability of error is equal to  $10^{-2}$ . At  $\rho_1 = 0.95$ , compared with the Neyman-Pearson receiver, the SNR gain is approximately equal to 5 dB, but the probability of error is equal to  $10^{-1.8}$  only.

Fig.4 presents the error probability of OFDM-BPSK wireless communication system as a function of average SNR when  $\rho_1$  is fixed, for example  $\rho_1 = 1$ , and  $\rho_2$  is the variable parameter. With increasing in  $\rho_2$ , the error probability decreases. The superiority of GR employment in OFDM-BPSK wireless communication systems in comparison with the Neyman-Pearson receiver is obvious. At  $\rho_2 = 0.3$ , when the error probability is equal to  $10^{-2}$ , we observe that the SNR gain is approximately equal to 6dB in comparison with Neyman-Pearson receiver. The OFDM-BPSK wireless communication system as a function of correlation coefficient  $\rho_2$  at the fixed  $\rho_1$  has the same character when  $\rho_2 = 0$ ,  $\rho_1$  is the variable. With decreasing in  $\rho_2$  from 0.3 to 0, the probability of error is increased. For example, at  $\rho_2 = 0$ , we see that the SNR gain in favor of GR employment in comparison with the Neyman-Pearson receiver is more than 5 dB, but the probability of error is equal to  $10^{-1.8}$ .

## 6 Conclusions

In this paper, we study the error probability performance of OFDM-BPSK wireless communication system employing both the Neyman-Pearson receiver and GR. We define the error probability of OFDM-BPSK wireless communication system as a function of correlation coefficients  $\rho_1$  and  $\rho_2$ , which are the correlation coefficient between the elements of the channel matrix and estimated channel matrix.

Under employment of the GR in OFDM-BPSK wireless communication system there is a superiority in comparison with implementation of the Neyman-Pearson receiver. SNR gain is for about 6 dB at the error probability equal to  $10^{-3}$  when  $\rho_2 = 0$ , and

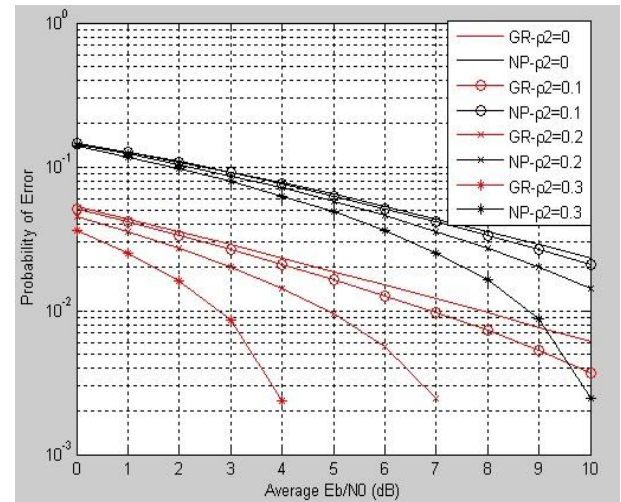


Fig.4 Performance of OFDM-BPSK system  $\rho_1 = 1$

for about 6 dB at the error probability equal to  $10^{-2}$  when  $\rho_1 = 1$ . With decreasing in the correlation coefficients  $\rho_1$  and  $\rho_2$ , we observe a superiority of GR implementation in OFDM-BPSK wireless communication system in comparison with the Neyman-Pearson receiver, but the probability of error is increased. The correlation coefficients  $\rho_1$  and  $\rho_2$  play a very important role to define the OFDM-BPSK wireless communication system performance under implementation of both the GR and Neyman-Pearson receiver.

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