Derivation of BER Expressions and Simulation of a Chip Interleaved System for WSNs Application

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Abstract: - This paper presents the theoretical analysis and simulation results of a communication system for application in physical layer for wireless sensor networks. The chip interleaving technique is investigated as the mean to mitigate fading in the channel. By theoretical derivations of closed form expressions for the bit error rate (BER), it is proved that the BER can be significantly improved using the interleaver/deinterleaver technique. Theoretical system analysis and derivations of the closed form BER expressions are based on discrete time domain representation of all signals in the system. The system is simulated using the spreading sequences and the modulation scheme proposed for wireless networks, in particular for wireless sensor networks. Simulation results confirmed the derived theoretical expressions.

Key-Words: - spread spectrum system, fading channel, interleavers, BER curves, wireless sensor networks.

1 Introduction

Due to limited power of sensor nodes, an energy efficient transmission is the key requirement in wireless sensor networks design and development. It was shown that the fading in direct sequence spread spectrum and CDMA systems [1] can be mitigated by using interleaving techniques [2, 3, 4, 5, 6]. Wireless networks based on IEEE 802.15.4 Standard [7] are novel wireless technology with a wide range of possible applications. The power efficient physical layer in these networks is of a particular interest. One design of modulator and demodulator blocks for this network is presented in [8] which included theoretical modeling, simulation and design in FPGA and implementation in CMOS technology. The transceiver design is presented in [9].

In this paper, the chip interleaving technique is investigated for the physical layer design in wireless sensor networks. The contributions of this paper are as follows. Firstly, the mathematical model of communication system representing Physical Layer is based on the discrete time domain signal representation and processing. In the existing theory these signals and their processing are represented in continuous time form, which is not suitable for direct implementation in digital technology. Secondly, the expressions for probability of bit error are derived for the case of noise, fading and interleaver/deinterleaver presence inside the communication system. Thirdly, simulators are developed in MATLAB that confirmed theoretical results.

2 Theoretical model of a system in presence of Gaussian noise and fading

A block scheme of a direct spread spectrum communication system using the Offset Quadrature Phase Shift Keying (OQPSK) modulation is presented in Fig. 1. Additive White Gaussian noise and fading are present in the channel. Blocks of interleaver (IL) and deinterleaver (DI) are included into block schematic in the figure. These blocks will be excluded from the scheme for the theoretical analysis conducted in this section and will be included in the next section.

Suppose the source generates message bits that are spread by the spreading sequence \(c_m(k)\) to obtain chip sequence \(m(k) = b_{ij}(k)c_{ij}(k)\), which is applied to the input of a multiplexer. The chip sequence \(m(k)\) is split into in-phase and quadrature sequences using the demultiplexer block (DEMUX) in such a way that the even-indexed chip sequence \(m_I(k)\) modulates the in-phase carrier \(\sqrt{2E_c/M}\cdot \cos \omega_k\) and odd-indexed chip sequence \(m_Q(k)\) modulates the quadrature carrier \(\sqrt{2E_c/M}\cdot \sin \omega_k\), as specified by the Standard for wireless sensor networks [7], where \(E_c\) the energy per chip and \(M\) is the number of interpolated samples contained in one chip interval.

All signals in this paper will be presented in discrete time domain specified with the discrete
time variable $k$, instead of continuous time variable $t$ which is usually used in similar systems analysis. The discrete time domain is intentionally used in this paper because the presented signal processing blocks and formal expressions can be directly used to implement the communication system in digital technology, if necessary.

The transmitted QPSK signal can be expressed as

$$
s(k) = m_i(k) \sqrt{2E_c / M} \cos \Omega_k
+ m_q(k) \sqrt{2E_c / M} \sin \Omega_k
$$

(1)

where $m_i(k)$ and $m_q(k)$ are in-phase and quadrature chip sequences in discrete time domain respectively and $\Omega$ is the normalized frequency of the carrier.

The band-limited pass-band noise samples are also generated in discrete time instants defined by $k$ as

$$
n(k) = n_i(k) \sqrt{2E_n / M} \cos \Omega_k
- n_q(k) \sqrt{2E_n / M} \sin \Omega_k
$$

(2)

where the energy of the noise samples inside a chip interval is $E_n$. Noise components $n_i(k)$ and $n_q(k)$ are in-phase and quadrature noise samples of zero mean and unit variance. The band-pass noise is intentionally expressed in this way to comply with the received signal and noise processing at receiver side, because the power of noise in respect to the power of signals during the simulation needs to be controlled at the transmitter side.

When fading and noise are present in the channel the fading coefficient $\alpha_i$ in Fig. 1 is different from one, i.e., $\alpha_i = \alpha \neq 1$, and the received signal $s_R(k)$ is

$$
s_R(k) = \alpha e^{-j\phi} s(k) + n(k)
$$

(3)

where $\alpha$ is fading factor and $\phi$ is the phase shift. It is assumed that the fading coefficient is constant in a bit interval, i.e., the channel fading is sufficiently slow that the phase shift $\phi$ can be reduced to zero by the receiver’s phase locked loop. Then, the received signal $s_R(k)$ is coherently demodulated using a correlator composed of a multiplier and adder. The signal at output of the multiplier is

$$
y_i(k) = [\alpha(k) + n(k)] \sqrt{2E_c / M} \cos \Omega_k
$$

(4)

The samples of this signal are added in the chip interval (corresponds to integration in continuous time systems). Suppose the first bit generated by the source is +1. Then the output of spreader is the non-inverted spreading sequence $c_i(k)$, i.e., $m(k)$=$c_i(k)$. The output of the adder in $I$ branch (an integrator in continuous time systems) is the random sample $z_i$ (one realization of a random process $Z_i$) is obtained, i.e.,

$$
z_i = \sum_{k=1}^{M} y_i(k) = \alpha \sqrt{E_c} \cdot c_{i1} + \sqrt{E_n} \cdot n_{i1}
$$

for $i = 2, 4, \ldots, \beta$, and in $Q$ branch as

![Fig. 1 Block schematic of communication system](image-url)
for $i = 2, 4, \ldots, \beta$. The multiplexer (MUX) is used to combine in-phase and quadrature sequences back into a $2\beta$-chip sequence $z_i$

$$z_i = \alpha \sqrt{E_c} \cdot c_i + \sqrt{E_n} \cdot n_i$$ \hspace{1cm} (5)

where $i = 1, 2, 3, 4, \ldots, 2\beta$ and $n_i$ are samples of the in-phase and quadrature baseband noise having zero mean and unit variance. The addition was performed in each chip interval and the obtained values $z_i$ represent a sample of the chip, i.e., its soft value in the related chip interval. The first term in the sample is the signal part and the second one is the noise part. In the correlator block, a locally generated reference chip sequences ($c_{i\alpha}, i = 1, 2, 3, 4, \ldots, 2\beta$), obtained from the sequences synchronization block, is multiplied with the incoming $z_i$ sequence and then the products are added chip-by-chip inside the bit interval. The resulting sum for the first assumed positive message bit sent is

$$w_i = \sum_{i=1}^{2\beta} z_i c_{i\alpha} = \alpha \sqrt{E_c} \sum_{i=1}^{2\beta} c_{i\alpha}^2 + \sqrt{E_n} \sum_{i=1}^{2\beta} n_i \cdot c_{i\alpha} = A + B$$ \hspace{1cm} (6)

This value is a random sample of a random variable $W_i$ defined for the first bit received. If the source generates binary bits from the alphabet $\pm 1$, the threshold value in the decision circuit needs to be set to $z_{\text{thr}} = 0$. Due to the central limit theorem (CLT), this random variable can be approximated by the Gaussian random variable. In addition, if powers of all chips are equal, then the mean of $W_i$ is

$$\eta_{w_i} = E\{w_i\} = E\{\sum_{i=1}^{2\beta} z_i c_{i\alpha}\} = \sqrt{E_c} \cdot E\{\alpha\} \sum_{i=1}^{2\beta} E(c_{i\alpha}^2)$$ \hspace{1cm} (7)

because, due to the statistical independence of the noise and the spreading sequence, the second term is zero. The variance of $w_i$ in general form is

$$\sigma_{w_i}^2 = E\{w_i^2\} - \eta_{w_i}^2 = E\{(A + B)^2\} - \eta_{w_i}^2$$ \hspace{1cm} (8)

where, for equal chip powers, we may have from (6)

$$E\{A^2\} = \left[\frac{\alpha \sqrt{E_c} \sum_{i=1}^{2\beta} c_{i\alpha}^2}{\sqrt{E_n}}\right]^2$$

$$= 2\beta \alpha^2 E_c \left[E(c_{i\alpha}^4) + (2\beta - 1)E(c_{i\alpha}^2)E(c_{i\beta}^2)\right]$$ \hspace{1cm} (9)

and

$$E\{B^2\} = E\left[\sqrt{E_n} \sum_{i=1}^{2\beta} n_i \cdot c_{i\alpha}\right]^2 = 2\beta E_n E(c_{i\alpha}^2)$$ \hspace{1cm} (10)

If the average powers of all chips are equal, i.e.,

$$E\{c_{i\alpha}^2\} = E\{c_{i\beta}^2\} = P_c$$

then, from (7), (8), (9) and (10), the variance can be expressed in this form

$$\sigma_{w_i}^2 = 2\beta E_c \left[E(\alpha^2)E(c_{i\alpha}^4) - E^2(\alpha)P_c\right]$$

$$+ 2\beta (2\beta - 1)E_n [E(\alpha^2) - E^2(\alpha) + 2\beta E_\alpha P_c]$$

which is further simplified for the case of binary chip spreading sequences ($P_c = 1$) to this form

$$\sigma_{w_i}^2 = (2\beta)^2 E_c \sigma_a^2 + 2\beta E_n$$ \hspace{1cm} (11)

The probability of bit error can be derived as

$$p_{\text{ber}} = \frac{1}{2} \text{erfc} \left[\frac{2\sigma_{w_i}^2}{\eta_{w_i}}\right]^{-1/2} = \frac{1}{2} \text{erfc} \left[\frac{2(2\beta)^2 E_n \sigma_{wa}^2 + 4\beta E_n}{(2\beta)^2 E_c \eta_{wa}}\right]^{-1/2}$$

$$= \frac{1}{2} \text{erfc} \left[\frac{2\sigma_{wa}^2}{\eta_{wa}} + \frac{2E_n}{2\beta \eta_{wa} E_c}\right]^{-1/2}$$

The energy of a bit is $E_b = 2\beta E_c$. The communication system is analyzed for discrete time domain signal representation, i.e., each chip and related noise sample are generated once for each chip interval and then repeated (interpolated) $M$ times in the interval to allow the discrete time modulation of the carrier. For $M$ repeated samples of noise in a chip interval the energy is $E_N = M \sigma^2$, where the variance is $\sigma^2 = B N_0$ and the bandwidth $B = 1/2 T_c = 1/2 M$. Thus, the energy is calculated to be $E_N = N_0/2$.

The density function of the fading coefficient $\alpha$ is Rayleigh expressed in this form

$$f_\alpha (\alpha) = \frac{2\alpha}{b} e^{-\alpha^2/b}$$

with the mean $(\pi b/4)^{1/2}$ and variance $b(4/\pi)^{1/2}$. It is assumed that $E(\alpha^2) = 1$. Having this assumption in mind the probability of error is
where the energy of a bit is related to the energy of a chip as \( E_b = 2\beta E_c \).

3 Interleaver communication system

The fading significantly increases bit error rate inside communication systems. It is assumed that a particular fading coefficient affects each bit transmitted. If interleaver/deinterleaver blocks are included into the transceiver structure, as shown in Fig. 1, the effects on fading can be spread in a bit interval.

In this paper it will be assumed that the block interleaver of \( 2\beta \times 2\beta \) size is employed. Thus the chips for each bit are written into the interleaver row wise and taken out column wise at the transmitter side. The opposite operation is performed at the receiver side to re-order the chips and return them to be in corresponding bit intervals. Thus, the samples at the output of multiplexer (MUX) are the chips affected with independent fading coefficients, which can be expressed in this form

\[
z_i = \alpha_i \sqrt{E_c} \cdot c_{i} + \sqrt{E_N} \cdot n_i
\]

This soft value is applied to the chip sequence correlator input. The output sample of the first correlator is a realization of a random variable expressed as

\[
w_i = \sum_{i=1}^{2\beta} z_i c_{i} = \sqrt{E_c} \sum_{i=1}^{2\beta} \alpha_i c_{i} + \sqrt{E_N} \sum_{i=1}^{2\beta} n_i \cdot c_{i} = A + B
\]

The mean and variance of this random variable, assuming binary chips of unit chip powers, are

\[
\eta_w = \mathbb{E}\{w_i\} = 2\beta \sqrt{E_c} \eta_{\alpha_i}
\]

and

\[
\sigma^2_w = \mathbb{E}\{\sum_{i=1}^{2\beta} E\{c_{i}^2\} - \sum_{i=1}^{2\beta} E\{c_{i}\}^2\} + 2\beta E_N
\]

\[
= \sum_{i=1}^{2\beta} \sigma^2_{c_{i}} + 2\beta E_N = 2\beta E_c \sigma^2_{\alpha_i} + 2\beta E_N
\]

The probability of bit error can be calculated as

\[
p_{be} = \frac{1}{2} \text{erfc} \left[ \frac{2(4 - \pi) + 4 \left( \frac{E_b}{N_0} \right)^{-1/2}}{\pi} \right]
\]

\[
= \frac{1}{2} \text{erfc} \left[ \frac{2\alpha^2_{\alpha_i}}{\eta_{\alpha_i}} \right] = \frac{1}{2} \text{erfc} \left[ \frac{4\beta E_c \sigma^2_{\alpha_i} + 4\beta E_N}{(2\beta \sqrt{E_c} \eta_{\alpha_i})} \right]^{-1/2}
\]

Therefore, if the source generates binary bits and the spreading sequence is in binary form, we may have the probability of error in this form

\[
p_{be} = \frac{1}{2} \text{erfc} \left[ \frac{2 (4 - \pi) + 4 \left( \frac{E_b}{N_0} \right)^{-1/2}}{\pi} \right]
\]

4 Simulation Results and discussions

Fig. 3 presents theoretical BER curve (Blue) and the curve obtained by simulation (Red) in the case when Rayleigh fading is present in the channel. The BER is significantly worse than in the case when the AWGN, which is represented by the third (Black) curve in Fig. 3. This is the reason a method of fading mitigation using interleavers is investigated in this paper. The BER achieved improvements are shown in Fig 4.
Fig. 4 Theoretical BER curves for receiver with interleaver in presence of AWGN and fading: theoretical (Blue) for fading, theoretical (Red) for fading with interleaver and theoretical for noise only (black).

The theoretical BER curve for the receiver with interleaver in presence of AWGN and fading is presented in Fig. 4 in Blue colour. The theoretical curve with interleaver and deinterleaver blocks, according to expression (19), is plotted in Red colour, while the curve for AWGN presence in the channel is presented in Black colour. For the BER value that is approximately $2.6 \times 10^{-2}$, the saving in SNR is approximately 6 DB, as shown in Fig. 4. This saving increases significantly when the BER decreases.

Fig. 4 also presents the results of simulation in the system with interleaver [10]. The theoretical curve, based on the closed form derivative (18) shows a BER improvement from $2.4 \times 10^{-2}$ to $10^{-4}$ (approximately two order of magnitude) for the required SNR = 10 dB. In summary it can be said that, by using interleaver influence of fading is decorrelated and the fading BER curve tends to the curve that is obtained when only AWGN noise is present in the channel (Black curve).

As the theoretical expression (18) shows, further improvement could be achieved if the spreading factor is increased. Therefore, the analyzed chip interleaving technique is efficient technique to mitigate fading. It contributes directly to the power saving and can be implemented in the transceivers of the physical layer in wireless sensor networks. By implementing this technique, for a desired BER in the system, a smaller value of SNR will be required if the system includes interleaver and deinterleaver blocks.

5 Conclusions

This paper presented the procedure of deriving closed form expressions for the probability of error of a spread spectrum system that uses spreading sequences defined for application in wireless sensor networks. In contrast to the usual practice used in communication theory, where signals are presented and processed in continuous time domain that is not suitable for practical implementation in digital technology, in this paper all signals are presented and processed in discrete time domain. Therefore the presented signal processing is suitable for direct implementation in digital technology, DSP or FPGA for example. The communication system is analyzed in the presence of noise and fading. The interleaver and deinterleaver processing is proven to be excellent technique to mitigate fading in the channel. The theoretically derived formulas are confirmed by simulation.
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