Effects of Gradient Mechanical Properties on Human Visual Accommodation studied using Nonlinear Finite Element Method (FEM)

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Abstract: Studies have confirmed that the primary cause of ‘old sightedness’ (presbyopia; the inability to focus up close for reading) is gradual lens stiffening. It has been suggested that refilling the contents of a presbyopic crystalline lens could restore the accommodative ability of the lens. Phaco-ersatz replaces the hardened content of the presbyopic lens with a flexible polymer to restore the ability of the lens to change shape. For this surgery to be successful, a thorough understanding of the effects of substituting the natural lens material with a gel is critical. In particular, the natural lens possesses a gradient in its mechanical property that cannot be reproduced by an isotropic gel. The impact on accommodation of a loss in mechanical gradient in phaco-ersatz has not been studied. We utilized the Finite Element Method (FEM) to build an axisymmetric model of the accommodative system and to investigate the contribution of gradient mechanical properties to the accommodative response of the lens and to predict the outcome of refilling the lens with a material of isotropic elasticity and refractive index. Using the lens aspect ratio as the criterion, an equivalent isotropic elasticity is found. Results show that the equivalent elasticity, to some extent, depends on the gradient elasticity profile implemented into the model. We showed that this method yields an equivalent elasticity in agreement with previous empirical studies. Numerical ray-tracing using a custom-made MATLAB™ code to assess the accommodative response of both models reveals that lens refilling is capable of restoring a major portion of accommodation.

Key-Words: Gradient Mechanical Properties; FEM; Accommodation; Lens; Presbyopia; Ray-tracing

1 Introduction

Visual accommodation is a biomechanical process in the eye that facilitates switching focus between near vision and distant vision. This is accomplished by changing the shape of the crystalline lens through radial forces applied by the ciliary muscle, consequently changing the optical power of the lens. For distant vision, the ciliary muscle relaxes, and the zonular fibres (see Fig. 1) become taut causing the lens to decrease its optical power by becoming flatter. During accommodation to near vision, ciliary muscle contraction releases the tension on the zonules and the lens takes on a more curved geometry due to its elastic behaviour.

With age, the eye progressively loses the ability to accommodate, leading to a condition known as presbyopia. Studies have shown that presbyopia is primarily caused by gradual lens hardening, making the lens almost undeformable under the accommodative stimulus [1-4]. This suggests that replacing the presbyopic lens contents with a polymer gel of Young’s modulus similar to or softer than that of a young lens, a procedure called lens refilling or phaco-ersatz could restore accommodative ability [5, 6].

![Fig. 1- Main parts of the accommodative system of the human eye: Disaccommodated (Left) Accommodated (Right)](image-url)
In order to precisely control the outcome of phaco-ersatz, it is critical to investigate the effects of different optical and mechanical properties of the natural and refilled lens on the resultant accommodative response. One particular aspect that deserves attention is that the natural lens is heterogeneous, with a gradient of optical and mechanical properties [4, 7]. In phaco-ersatz, the natural inhomogeneous lens is replaced with a homogeneous material. The impact of not reproducing the mechanical properties gradient on accommodation has not been studied. The current paper aims to assess the mechanical and optical effects of altering/eliminating the natural gradient mechanical gradient on the accommodative response using numerical methods.

2 Methods/Modelling

Two gradient elasticity distributions were implemented into the FE model. First, the Dynamic Mechanical Analyzer (DMA) results of Weeber et al [4] were used for the lens internal layers. Second, based on the model of Burd et al [8] that assumes an isotropic cortex and an isotropic nucleus for the lens, elasticity of the corresponding layers were linearly distributed in a way to yield average Young’s moduli similar to that of the cortex and nucleus of that model. For this purpose the minimum elasticity and maximum elasticity of the Burd model was used for the innermost and outermost layers. While the lens was assumed quasi-incompressible with Poisson’s ratio of 0.49, a Poisson’s ratio of 0.47 was assumed for capsule and zonules in accordance with the literature [8]. It should be noted that using a Poisson’s ratio of 0.5 in the FE model was not feasible as it leads to bulk modulus of infinity for the lens. Table 1 summarises the material properties used. The two elasticity profiles used here are known to be the most cited results for the Young’s modulus distribution of the lens.

### Table 1 – Material properties assigned – E1 to E11 denote the Young’s moduli of the inner most layer to the outermost layer.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>E1 to E5 (Pa)</td>
<td>119, 152, 209</td>
<td>183, 365, 547</td>
</tr>
<tr>
<td>(Pa)</td>
<td>429, 554</td>
<td>730, 912</td>
</tr>
<tr>
<td>E6 to E11 (Pa)</td>
<td>840, 840, 554</td>
<td>976, 1953, 2929</td>
</tr>
<tr>
<td>(Pa)</td>
<td>224, 155, 134</td>
<td>3905, 4882, 5858</td>
</tr>
<tr>
<td>Zonules</td>
<td>1.00 kPa</td>
<td>1.00 kPa</td>
</tr>
<tr>
<td>Capsule</td>
<td>1.27 kPa</td>
<td>1.27 kPa</td>
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2.1 Finite Element Model

The lens capsule geometry is modelled using 5\textsuperscript{th} order polynomials taken from Burd et al. The lens internal layers are then modelled by offsetting the outer geometry under two rules; first, layers are to be distributed in such a way that one layer has its tip at the equatorial half-diameter of 2.85 mm (to define the nucleus-cortex boundary). Second, the distance between the equators of two consecutive layers (offset ratio) should gradually increase by 2% offset (except for the most internal layer). This resulted in the lens stroma being modelled using 11 layers (5 intra-nucleus layers). The ciliary body and zonules were modelled using data extracted from optical coherence tomography (OCT) images [9, 10]. Zonular fibers seem to be connected to the insertion points that are not at the tip of the ciliary body but instead they are on a set of points that form a curve (the red curve in Fig.2A) (personal correspondence with Adrian Glasser). Thus, 10 zonules were connected to the curve from OCT images and distributed equally on lens equatorial region (Fig.2).

![Fig.2 – Zonular configuration: the red curve represents the zonular insertion points on the ciliary muscle (A) Discretised FE model including the internal layers (B)](image)

Following a h-convergence study, the lens was meshed using quadrilateral elements that were refined near the capsular region. Contact elements were also placed between the intra-lenticular layers. Fig.2B shows details of the final FE model.

2.2 Optical Ray-tracing Model

In order to be able to perform ray-tracing on uncommon surfaces such as the anterior and posterior surface of the lens, we developed a ray-tracing code in MATLAB\textsuperscript{™} which calculates the refractions at the aqueous-anterior lens and vitreous-posterior lens surfaces using Snell’s law of refraction. This code facilitates performing ray-
tracing at refractive surfaces with complex curvatures.

The refractive index of the lens is known to be non-uniform throughout the lens. However, studies have shown that assigning an equivalent value to the refractive index is sufficient for the purpose of ray-tracing and assessing accommodation. In the current study, a uniform equivalent refractive index of 1.42 [11] is used in the lens model together with equal aqueous and vitreous refractive indices of 1.333.

The back vertex optical power at each step is calculated from the distance between the lens apex and the back focal plane; the latter defined as the plane that produces the minimal RMS spot size for light ray intercepts. The accommodative amplitude is then found by subtracting the undeformed lens optical power and dis-accommodated lens optical power.

2.3 Loading/Boundary conditions

The accommodative stimulus is applied in terms of a non-deformable displacement (0 < δ_ciliary < 200 µm) vector on the ciliary muscle. Since the lens is assumed to be axisymmetric about its optical axis, the central axis was set to have no degree of freedom in the horizontal direction whereas it is free to move along the vertical axis. The internal layers of the lens are bonded through use of contact elements at the boundaries. The model is then solved non-linearly in ANSYS™ (Academic Research, Release 13.0, ANSYS Inc., PA, USA).

The aspect ratio (equatorial diameter / polar thickness) for each lens was found at maximum stretch in the gradient profiles. Then lens contents were replaced with a homogeneous material. This isotropic elasticity is then chosen as variable with the aspect ratio of the gradient being the iteration criterion. Finally, the homogeneous lens is again stretched using the force found from the gradient lens and accommodative response and von-Mises stress were calculated.

There were two approaches possible for finding the equivalent isotropic elasticity: first, calibrating the models based on ciliary displacement (displacement-based method) and second, based on the ciliary force (force-based method). Our preliminary studies show that the displacement-based method does not result in meaningful equivalent elasticities since the FE software applies unrestricted force as required to displace the ciliary up to the inputted displacement whereas in the real accommodation system this is not the case, instead, the ciliary force is restricted by the capability of the muscle. Hence, it was decided to use the force-based method.

3 Results

To ensure robustness of the results, a sensitivity study was performed to make sure that slight variations in the zonular configuration does not have substantial effects on the overall response and stress distribution within the lens. Results show that such variations can only bring about changes in local deformations near the lens equatorial cap but have no significant effects on the more central, optically-relevant lens curvatures.

The deformation vectors for the disaccommodated lens at the maximum deformation can be seen in Fig.3 for the elasticity profile based on Burd et al. The Burd profile appears to behave as a stiffer lens, undergoing less deformation under accommodative forces.

![Fig.3 – Deformation vectors of the fully-stretched lens (δ=200µm). Burd gradient (Top) Weeber gradient (Bottom) – The wireframe corresponds to baseline geometry.](image)

In order to compare lens deformations in the lenses with different elasticity profiles the total change in the lens equatorial diameter and polar thickness may be evaluated as the major dimensions of the lens (Fig.4). Results demonstrate that using the lens aspect ratio as the criterion for finding the equivalent Young’s modulus leads to almost the same change in the lens equatorial diameter in the gradient and associated equivalent lenses. Moreover, a very close change in thickness was
found for the heterogeneous lens and its equivalent homogeneous lens. Overall, the lens diameter change seems to be less sensitive to the elasticity profile and mechanical gradient used than the lens polar thickness.

Fig. 4 – Change in lens diameter and thickness during disaccommodation for the isotropic and gradient profiles – The lens diameter increases with stretch whereas the thickness decreases during disaccommodation. (D=diameter, T=thickness)

Fig. 5 shows the iterations for finding the equivalent elasticity of the lens for both gradients. The equivalent Young’s moduli for Burd profile and Weeber profile were found to be 680 Pa and 360 Pa, respectively. As expected, with increase in the lens elasticity the lens is less able to undergo shape changes, which means that stiffer lenses should have aspect ratio closer to the unstretched lens. Also, it can be inferred that the curves in Fig.5 have an asymptote which crosses the vertical axis at the aspect ratio of the unstretched lens.

An example (for the Burd et al model) of the ray-tracing outputs for stretched and unstretched lenses is shown in Fig.6. In agreement with the Helmholtzian theory of accommodation, the focal length increases with stretch which leads to a monotonic decrease in the optical power.

Fig. 5 – Iterative process for finding the equivalent isotropic elasticity: circles mark the location of the equivalent Young’s moduli based on each elasticity profile.

Fig. 7 shows the variation of accommodation found by ray-tracing. It can be seen that where the aspect ratios are close the accommodation amplitudes also match.

In order to compare the stiffness of the isotropic and gradient models, a reverse simulation was carried out; the accommodative stimuli are applied in terms of force rather than displacement while the ciliary body displacement is recorded. Results are shown in Fig.8. It can be seen that even with inclusion of the gradient mechanical properties, none of the models show any sign of nonlinear load-response. Furthermore, the natural lens (Weeber gradient profile) behaved as the softest lens among the ones attempted.

Fig. 6 – Numerical ray-tracing output for the Burd gradient model. Accommodated/unstretched (Top) Disaccommodated /stretched state (Bottom)

Fig. 7 – Accommodation amplitude for gradient and the equivalent profiles. Profiles with closer aspect ratio show less difference in the accommodation amplitude.

Fig. 8 – The ciliary displacements associated with different amounts of accommodative forces applied. The model with less displacement-force line slope has more overall stiffness.
We observed that the lens has negative aberrations (rays farther from the optical axis being focused farther from the lens than paraxial rays). This compares favourably with the literature [12]. Additionally, results show that during accommodation, longitudinal aberration shifts towards more negative values which is in agreement with previous studies [12, 13]. However, the magnitude of aberrations, especially for the undeformed lens, appears higher than the values reported by experimental studies [14, 15].

4 Conclusion

In the current study nonlinear finite element method and numerical ray-tracing are combined to establish an opto-mechanical model of the human accommodative system. This model was used to investigate the contribution of gradient mechanical properties on accommodation. Two sets of mechanical properties were tested based on elasticity data taken from the literature. For each of the gradient profiles the aspect ratio is found and employing an iterative process, the equivalent elasticity of the isotropic infill that yields the same aspect ratio in the deformed state is found. This elasticity is then re-inputted into the model to calculate accommodation.

Our results show that the aspect ratio is a reasonable representative of the lens shape based on the accommodative amplitudes calculated for the equivalent mechanical models. Our comparison of the accommodative responses reveal that using the aspect ratio as the criteria, a major portion of accommodation can be maintained using an infill material of isotropic Young’s modulus.

While the method employed in current study could be widely used to study various aspects of accommodation and presbyopia including accommodation amplitude and aberrations, it is worthwhile for future studies to explore the contribution of the natural and synthetic lens gradient refractive index to accommodation, in combination with the gradient mechanical property studied here.

Acknowledgements

This work was supported in part by NEI Grants: R01EY14225, R01EY021834; P30EY14801 (Center Grant); the Australian Federal Government through the Cooperative Research Centres Programme; the Florida Lions Eye Bank; and Research to Prevent Blindness.

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