Applicability of Constant Amplitude Fatigue Data to Life Predictions under Variable Amplitude Service Loading

JUSTIN LINDSEY AND ALI FATEMI
Mechanical, Industrial and Manufacturing Engineering Department
The University of Toledo
2801 West Bancroft Street, Toledo, Ohio 43606
UNITED STATES
afatemi@eng.utoledo.edu http://www.eng.utoledo.edu/mime/faculty_staff/faculty/afatemi

Abstract: - Variable amplitude loading of components or structures can result in failures which occur much sooner than would be predicted by commonly used life prediction models, utilizing fatigue data generated under constant amplitude loading. The goal of this study was to investigate the effect of periodic overloads on fatigue behaviour of steels. Life predictions were performed by implementing the linear damage rule along with the constant amplitude fatigue data and curves and then compared to experimental results. Presence of periodic overloads in a load history resulted in increased damage for small cycles not only at strain levels above the fatigue limit, but also in damage for cycles below the fatigue limit. It is suggested to generate fatigue curves using a periodic overload scheme representative of a worst-case scenario for application to fatigue design and life predictions for variable amplitude service loading conditions.

Key-Words: - Fatigue Life Prediction, Periodic Overloads, Variable Loading, Cumulative Fatigue Damage

1 Introduction

Many structures are subjected to varying types of cyclic loading. As a result, fatigue failure is one of the most common types of failure and must be taken into consideration in the design of many structures and components. Fatigue properties can be determined through basic fatigue testing, and models can be used to approximate the behavior of the material under the condition of use.

Additionally, the majority of components are not subjected to constant amplitude loading. Most loading histories are complex and can involve variable amplitude loading. Evaluation of the behavior of a component or structure under such load histories can be very costly and time consuming. In order to minimize the testing of components or structures subjected to variable amplitude loading, an accurate life prediction model is required. This requires a reliable damage quantification parameter, a cumulative damage rule, along with constant amplitude material fatigue data.

Since service load histories typically contain overloads, from a practical point of view, it is important to understand how a material responds to overloads, which can occur at any time during fatigue life. A shortening of fatigue life and a reduction in the fatigue limit due to the application of overloads has been observed, particularly if these overloads are periodic [1].

DuQuesnay et al. [2] performed periodic compressive overload tests at specific load levels and introduced the idea of an equivalent life to failure for the smaller fully-reversed cycles in each test using the Linear Damage Rule (LDR) [3]. The equivalent life to failure for the material was then plotted versus the stress amplitude of the smaller cycles in the loading history. The plotting of this data superimposed with the constant amplitude data for the same material identifies a trend where the data produced in the presence of periodic overloads deviates from constant amplitude stress-life curve. It was speculated that a decrease in the crack closure level following the compressive overloads may be responsible for the increased rate of damage accumulation. They also demonstrated that the interactive effect of the overload cycle on the smaller cycles can significantly increase the damage done by the smaller cycles.

Pompetzki et al. investigated the effects of compressive under-loads and tensile overloads on specimens of aluminum [4] and steel [5]. The concept of interactive damage was used in order to compare the results. The interactive damage is based on the assumption that all variable amplitude testing should result in a damage summation equal to unity. Therefore, the difference between the damage summation and unity is considered to be the damage...
that resulted from the interaction of the overload cycles on the smaller subsequent cycles.

Testing performed by Colin and Fatemi [6] on aluminum specimens showed that the application of a tensile overload would result in an increase in fatigue life over the application of a compressive overload. The tensile overloads were applied infrequently with 1,000 small cycles per block and the loading history resulted in a 20% increase in fatigue life over the compressive overload history. For long cracks, the acceleration period of damage is very short while the subsequent delay can cause crack arrest or retardation. For short cracks, however, the acceleration in crack growth is the most important consequence of tensile overload, while the delay is of secondary importance [5].

Topper and Lam [7] devised a method to increase the accuracy of life predictions for variable amplitude loading by attempting to estimate the effective strain range that a material would experience after an initial overload. They made the assumption that when analyzing variable amplitude service histories, the crack opening stress level from the largest cycle in the history persists throughout the history after its initial application. The succeeding smaller cycle stresses in the history are then compared against the crack opening stress value of the initial overload cycle to identify an estimate of the magnitude of the smaller cycle stress which is fully effective.

Many cumulative fatigue damage models have been suggested to help account for the nonlinearity of damage accumulation [8]. LDR is one of the most commonly used cumulative damage theories [3], represented by:

$$\sum \frac{n_i}{N_{f_i}} = D$$

(1)

where $D$ is the damage, $n_i$ is the number of cycles applied to the specimen at a certain amplitude in a variable amplitude load history, and $N_{f_i}$ is the expected life at this amplitude in constant amplitude loading. Failure is predicted to occur when the sum of the ratios is equal to unity. The LDR assumes that damage accumulates in a linear manner which has proven to be a non-conservative assumption due to some situations where damage accumulation has been shown to be nonlinear. Despite its shortcomings, many service load histories are such that sequence effects either cancel each other or are entirely unpredictable.

In this paper, first the experimental program carried out is described. This includes description of the materials and specimen geometry used, characterization of material cyclic deformation and fatigue behaviors, and the loading conditions for overload tests. Experimental results are then presented, along with life predictions for the overload tests. Finally, conclusions from the study based on the experimental observations and the analysis conducted are presented.

### 2 Experimental Program and Results

#### 2.1 Materials and Specimen Geometry

Test specimens of steel 8620 were heat treated using a procedure that would simulate a carburizing cycle to obtain hardness levels of 30 HRC and 36 HRC. Some additional data were also generated for 8622 steel carburized in atmosphere as well as in vacuum. Identical round specimens were used for monotonic tension and fatigue tests. The specimen configuration and dimensions are shown in Fig. 1. This configuration is similar to the specimen geometry recommended by ASTM Standard E606 [9]. After machining, the specimens were then polished to 0.2 μm surface finish prior to testing. A closed-loop servo-controlled hydraulic axial load frame in conjunction with a digital servo-controller was used to conduct the testing. An extensometer rated as ASTM class B1 [10] was used to control or measure strain.

![Fig. 1 Configuration and dimensions of specimen used (all dimensions in mm).](image-url)
2.2 Monotonic Tension and Constant Amplitude Fatigue Tests

Superimposed curves of experimental monotonic tension and cyclic stress-strain curves for the 8620 steel at 36 HRC are shown in Fig. 2. Similar curves were obtained for the 8620 steel at 30 HRC and for 8622 steel. The cyclic curve can be represented with the Ramberg-Osgood type relation given by:

\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_p}{2} + \frac{\Delta \sigma}{2E} e^{\frac{\Delta \sigma}{2K'}}^{1/n'}
\]

where values of the modulus of elasticity \( E \), cyclic strength coefficient \( K' \) and cyclic strain hardening exponent \( n' \) are listed in Table 1. As can be seen from Fig. 2, the material exhibits cyclic softening since the cyclic curve is below the tension curve.

![Figure 2](image)

**Fig. 2** Superimposed monotonic tension and cyclic stress-strain curves for 8620 (36 HRC) steel.

Table 1 Monotonic tension and cyclic fatigue properties of 8620 steel at two hardness levels.

<table>
<thead>
<tr>
<th>Monotonic Properties</th>
<th>30</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (HRC)</td>
<td>212</td>
<td>210</td>
</tr>
<tr>
<td>Modulus of elasticity, ( E ) (GPa)</td>
<td>694</td>
<td>796</td>
</tr>
<tr>
<td>Ultimate tensile strength, ( S_u ) (MPa)</td>
<td>991</td>
<td>1145</td>
</tr>
<tr>
<td>Percent elongation, %EL</td>
<td>32</td>
<td>36</td>
</tr>
<tr>
<td>Percent reduction in area, %RA</td>
<td>54</td>
<td>51</td>
</tr>
<tr>
<td>Strength coefficient, ( K ) (MPa)</td>
<td>1,624</td>
<td>2,132</td>
</tr>
<tr>
<td>Strain hardening exponent, ( n )</td>
<td>0.140</td>
<td>0.165</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cyclic Properties</th>
<th>( \varepsilon' ) (GPa)</th>
<th>( \sigma' ) (MPa)</th>
<th>( b )</th>
<th>( c )</th>
<th>( K' ) (MPa)</th>
<th>( S_y' ) (MPa)</th>
<th>( n' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclic modulus of elasticity, ( E' ) (GPa)</td>
<td>203</td>
<td>210</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Fatigue strength coefficient, ( \sigma' ) (MPa)</td>
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<tr>
<td>Fatigue strength exponent, ( b )</td>
<td>-0.101</td>
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<tr>
<td>Fatigue ductility coefficient, ( \varepsilon' )</td>
<td>0.469</td>
<td>0.595</td>
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<tr>
<td>Fatigue ductility exponent, ( c )</td>
<td>-0.547</td>
<td>-0.587</td>
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<tr>
<td>Cyclic strength coefficient, ( K' ) (MPa)</td>
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<tr>
<td>Cyclic strain hardening exponent, ( n' )</td>
<td>0.183</td>
<td>0.174</td>
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<tr>
<td>Cyclic yield strength, ( S_y' ) (MPa)</td>
<td>601</td>
<td>705</td>
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</tbody>
</table>

Constant amplitude fully-reversed fatigue tests were conducted in strain control for all the materials and conditions to obtain basic fatigue properties in the strain-life equation. This equation is given by:

\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_p}{2} + \frac{\Delta \sigma}{2E} e^{\frac{\Delta \sigma}{2K'}}^{1/n'}
\]

where definitions of the constants in this equation are given in Table 1.

Fig. 3 presents the elastic, plastic, and total strain-life curves for the 8620 steel at 36 HRC and the corresponding superimposed fatigue data and fits. Similar data and fits were obtained for the other materials and conditions. Values of the strain-life fatigue constants in equation (3) obtained from data fits in Fig. 3 for the two hardness levels of 8620 steel are listed in Table 1.

![Figure 3](image)

**Fig. 3** Total, elastic, and plastic strain amplitude versus reversals to failure data and fits for 8620 steel at 36 HRC.

2.3 Periodic Overload Fatigue Tests

Two input signals were used for the periodic overload fatigue tests. The first input signal consisted of a periodic fully-reversed overload of the type shown in Fig. 4(a). The load history in these tests consisted of repeated blocks made up of one fully-reversed overload cycle followed by a group of smaller constant amplitude cycles having the same maximum stress as the overload cycle.

The second input signal consisted of a periodic fully-reversed overload of the type shown in Fig. 4(b). The load history in these tests consisted of repeated blocks made up of one fully-reversed overload cycle followed by a group of smaller fully-reversed constant amplitude cycles. In both cases, the overload cycles were applied at frequent intervals to maintain a larger effective strain range resulting in the subsequent cycles being fully effective.
With the overload histories utilized in this study, as the large cycles become more frequent, the fraction of the total damage done by them increases. The fully-reversed strain amplitude for the overload cycle corresponded to about $10^4$ cycles to failure as recommended by Topper and Lam [7]. The number of small cycles per block, $n_{sc}$, were adjusted so that they cause about 80% to 90% of the damage per block.

Small cycle strain levels were selected near to or below the runout level of the constant amplitude tests (i.e. $N_f \sim 10^6$). The lowest small cycle strain amplitudes tested were between 20% and 40% of the respective runout strain amplitude in fully-reversed constant amplitude testing. The number of small cycles per overload cycle ranged between 16 and 3076.

Fig. 4 Loading histories for periodic overload fatigue tests (a) with small cycle mean stress or strain and, (b) without small cycle mean stress or strain.

2.4 Periodic Overload Fatigue Test Results
The periodic overload test results were plotted using the strain amplitude of the small cycles in the overload block and the calculated equivalent life to failure. The equivalent fatigue life for the smaller cycles was obtained using the linear damage rule:

$$\frac{B_f n_{ol}}{N_{f,ol}} + \frac{B_f n_{sc}}{N_{f,sc(eq)}} = 1$$

where $B_f$ is the number of loading history blocks that were repeated before specimen failure, $n_{ol}$ is the number of overload cycles in a loading history block, $N_{f,ol}$ is the number of cycles to failure if only overloads were applied in a test, $n_{sc}$ is the number of smaller cycles in a loading history block, and $N_{f,sc(eq)}$ is the computed equivalent fatigue life for the smaller cycles. The calculation of the equivalent life to failure attempts to take into account damage of the overload cycle by assuming that overload damage can be fully accounted for through the calculation of its damage ratio.

The equivalent fatigue life data for the 8620 steel at the two hardness levels are superimposed on the constant amplitude strain-life curves in Fig. 5(a). The periodic overload data show a distinct deviation from the strain-life curve for both hardness levels.

Fig. 5 Periodic overload data for 8620 steel at two hardness levels superimposed over (a) the strain-life curve and, (b) the SWT curve.
A plot of the Smith-Watson-Topper (SWT) parameter [11] for both the constant amplitude and overload data provides another method of comparison between the two sets of data, where the mean stress present in the small cycles is assumed to be taken into account. This parameter is given by:

\[
\sigma_{\text{max}} \varepsilon_a = \frac{1}{E} \left[ (\sigma_f')^2 (2N_f)^{2b} + \sigma_f' \varepsilon_f' E (2N_f)^{b+c} \right] \quad (5)
\]

where \( \sigma_{\text{max}} = \sigma_a + \sigma_m \). This plot is shown in Fig. 5(b).

### 3 Periodic Overload Fatigue Life Predictions

Life predictions are computed using the linear damage rule combined with strain-life or SWT curves. In damage accumulation calculations, cycles below the fatigue limit, defined at \(2 \times 10^6\) reversals (i.e. a horizontal line at \(2N_f > 2 \times 10^6\) reversals) for steels were considered as non-damaging \((D = 0)\).

The strain-life model given by equation (3) was used for life predictions. For each material, at strain amplitudes in the high cycle fatigue region, a large reduction in the life to failure in the presence of periodic overloads can be observed from Fig. 5(a). Below the fatigue limit (defined at \(1 \times 10^6\) cycles), this data demonstrates the amount of damage that occurs at strain amplitudes that are normally considered to have no effect on the fatigue life of specimens or components subjected to variable amplitude loading histories.

Predictions of the blocks to failure for each specimen were made using the strain-life equation and linear damage rule (LDR). Predictions for specimens subjected to the periodic overload (POL) with mean stress (w/MS) load history show that the predicted numbers of blocks to fatigue failure are significantly longer than experimental fatigue lives. Fig. 6(a) displays the results of the life predictions for 8620 and 8622 steels based on the strain-life equation (3) and LDR. In this figure, the predicted number of blocks to failure is plotted versus the observed number of blocks to failure. Additionally, there are reference lines added to the plot that denote life predictions which are factors of \(\pm 2\), \(\pm 5\), and \(\pm 10\) from that observed in the experiments.

It can be seen from Fig. 6(a) that the majority of the predictions are at least five times longer than the experimental results, with a number of predictions beyond an order of magnitude difference. It should be mentioned, however, that this approach with the strain-life equation does not take into account the mean stress present during the small cycles of the loading history.

Life predictions of POL tests were also performed using the SWT equation (5) along with the LDR. Fig. 5(b) shows the periodic overload data superimposed with the SWT equation, providing a representation of how well this equation was able to correlate the periodic overload data, which also contained small cycle mean stress. The ability of the SWT parameter to correlate the POL data appears to vary from one material or condition to another.

![Fig. 6 Predicted versus observed number of blocks to failure for using the LDR and (a) strain-life curve and, (b) SWT-life curve.](image)

Life predictions of specimens subjected to the POL w/MS loading history ranged from about one to 20 times longer than experimental results. Fig. 6(b) provides a representation of the life predictions.
using the SWT parameter and LDR for all materials in this study. It can be seen that the use of the SWT parameter increases the accuracy of life predictions for some specimens, but there still remains predictions for test conditions which were unaffected or nearly unaffected by the use of a mean stress parameter for life prediction. This outcome was observed in situations where the small cycle loading condition was below the fatigue limit for both strain-life and SWT curves.

Fatigue lives of periodic overload tests, with or without mean stresses, are compared with the constant amplitude fully-reversed fatigue lives at the same small cycle strain amplitude in Fig. 7. This plot shows these comparisons for the two strain levels used for steel 8620. For an applied strain of 0.175%, an arrow is used to denote that the constant amplitude curve predicts no failure at this strain level.

Fig. 7  Comparison of experimental fatigue lives of specimens subjected to two different strain levels of 0.25% and 0.175% for 8620 steel at 30 HRC.

Fatigue life comparisons in Fig. 7 indicate that the majority of the reduction in life results from the application of the periodic overloads. The tensile mean stress did cause a reduction in life when applied without overload cycles, but it is seen in Fig. 7 that when the mean stress is removed from the POL history, there is only a small difference in life as compared with the POL w/MS tests. This result is thought to occur due to the crack opening stress resulting from the overload cycle remaining below the minimum stress of subsequent small cycles in both the POL w/MS and POL wo/MS histories. This would result in the strain range remaining fully effective for both load histories. Applying the same concept to the mean stress test without POL cycles, the results imply that the crack opening stress may be above the minimum stress, resulting in only a portion of the strain range being effective in causing fatigue damage.

4 Summary and Conclusions

This study investigated the fatigue behavior of steel specimens subjected to fully-reversed periodic overloads and evaluated commonly used methods of fatigue life prediction in the realm of periodic overloads. A periodic overload history that is considered to be a worst case scenario of the amount of damage resulting from the following small cycles was employed, in which a periodic fully-reversed overload is applied followed by a number of small cycles in the presence of mean stress. Based on the fatigue tests performed, as well as the analysis conducted, the following conclusions can be made:

1. The presence of periodic overloads in a load history results in increased damage for small cycles not only at strain levels above the fatigue limit, but also in damage for cycles below the fatigue limit. Such cycles are commonly but incorrectly assumed (by virtue of the linear damage rule) to have no effect on fatigue life in variable amplitude load histories.

2. Use of the LDR and SWT mean stress parameter increased the accuracy of life predictions for some cases. However, predictions for most test conditions were unaffected or nearly unaffected by the use of the SWT parameter. This outcome was observed in situations where the small cycle loading condition was below the fatigue limit for both strain-life and SWT curves.

3. It is recommended to generate fatigue curves using a periodic overload scheme representative of a worst-case scenario for application to fatigue design and life predictions for variable amplitude service loading conditions.

References:


