Voltage Stability Assessment Using Bus P-Q Curve

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Abstract: - The idea of P-V and Q-V curve is used to determine the maximal reactive margin at load buses to avoid voltage instability. Sometimes the voltage stability study may be limited to identify the violation of the bus voltage constraints. In this paper the p-q curve for the critical bus voltage magnitude is created. Using this p-q curve the probability of the critical voltage violation is estimated for uniformly distributed active and reactive power at a given load bus. The p-q curve is created on the basis of bus impedance, which can be measured or calculated. To illustrate the usefulness of p-q idea the simple numerical example is presented. The paper describes also the importance of reactive power control basing on the failures and control problems in the Polish Transmission Networks in summer 2006.

Key-Words: - Power System Control, Reactive Power Control, Load Flow Control, Voltage Stability.

1 Introduction

Assessing and mitigating problems associated with voltage security remains a critical concern for many power system planners and operators. Since it is well understood that voltage security is driven by the balance of reactive power in a system, it is of particular interest to find out what areas in a system may suffer reactive power deficiencies under some conditions and to obtaining information regarding how system voltage stability can be improved most effectively. Operation near the voltage stability limits is impractical and a sufficient power or voltage margin is needed. Practically, the idea of P-V and Q-V curve is used to determine the minimal margin to avoid voltage collapse [7,14,19].

Distance to voltage instability may be measured in terms of physical quantities, such as reactive power reserve or voltage margin at a given bus. Vu Khoi, M.M. Begovic, D. Novosel and M.M. Saha proposed using local measurements of bus voltage and current for the estimation of voltage collapse proximity [7]. According to their idea the Thevenin impedance is equal to the bus load impedance at the point of voltage collapse. In this paper the idea of using Thevenin impedance to bus voltage study is extended by taking into account the bus load [17,18].

This paper is devoted to the analysis of voltage limits. The main question is how far we are from the voltage instability and how to consider the randomness of loads. The original p-q curve is applied here to solve such a task.

The idea of using p-q curve for voltage collapse analysis was presented in this paper. The p-q curve for the critical bus voltage magnitude is created. Using this p-q curve the critical voltage violation is estimated for uniformly distributed active and reactive power at a given load bus.

The p-q curve is created on the basis of bus impedance. The mathematical background of the proposed idea is presented. To illustrate the usefulness of p-q idea the simple numerical example is presented.

2 COUNTRY-WIDE ABSENCE OF ELECTRICAL SUPPLY – A BLACKOUT

The quality of the electrical energy supply can be evaluated basing on a number of parameters [8], [9]. However, the most important will be always the presence of electrical energy and the number and duration of interrupts. If there is no voltage in the socket nobody will care about harmonics, sags or surges. A long term, wide-spread interrupt - a blackout leads usually to catastrophic losses. It is difficult to imagine that in all the country there is no electrical supply. In reality such things have already happened a number of times [3], [5]. One of the reason leading to a blackout is reactive power that went out of the control. When consumption of electrical energy is high, the demand on inductive reactive power increases usually at the same proportion. In this moment, the transmission lines (that are well loaded) introduce an extra inductive reactive power. The local sources of capacitive reactive power become insufficient. It is necessary to deliver more of the reactive power from generators in power plants. It might happen that they are already fully loaded and the reactive power will have to be delivered from more distant places or from abroad. Transmission of reactive power will load more the lines, which in turn will introduce more reactive power. The voltage on customer side will decrease further. Local control of voltage by means of autotransformers will lead to increase of current (to get the same power) and this in turn will increase voltage drops in lines. In one moment this process can go like avalanche reducing voltage to zero. In mean time most of the generators in power plants will switch off due to unacceptably low voltage what of course will deteriorate the situation. In continental Europe most of the power plant are based on heat and steam turbines. If a generation unit in such power plant is stopped and cool down it requires time and electrical energy to start operation again. If the other power plants are also off - the blackout is permanent [1], [2], [4].

The difficulties showed up on June 26, 2006. The prediction for power consumption on this day was 18200 MW (in the morning peak) what was much higher compared with June in last year or previous years. This power was planned to be supplied from 75 generation units. Above these, there were a hot power reserve of 1350 MW (in this 237 MW second-reserve, 656 MW minute-reserve) and a cold reserve of about 2600 MW. In the north-east Poland there is not any grid-generation. The closest to this region is Ostroleka Power Plant, which in that time from three 200 MW units has two in operation and one set off for maintenance. In early morning of the 26th one unit in Power Plant Patnow had to be switched off and before noon four other units (two in Kozienice P. P. and two in Laziska P. P.) were switched off as well. All these unites were the main supplier to the north-east region of Poland. At 7 o'clock 570 MW of power was lost. At the same time the consumption prediction appeared to be wrong - the consumption was 600 MW higher and there was also much higher demand on reactive power. At 13 o'clock there was an unbalance of 1100 MW. In mean time one unit (in Dolna Odra P. P.) had been activated. However further activation from cold-reserve required more time (about 6 hours) because of technological reasons. Unusual heat wave spreading throughout the country caused deterioration of the operational conditions in power plants. Due to lack of sufficient amount of cooling water and exceeded water temperature levels the generating capacities of some power plants systematically decreased. That situation concerned mainly the power plants located in the central and northern part of Poland, the loadings of some transmission lines reached the acceptable limits what in turn cause the necessity of generation decrease in power plants located outside the mentioned region. The control of reactive power became critical [9,10,11].

3 BUS LOAD FLOW EQUATIONS USING THEVENIN'S CIRCUIT

Thevenin's theorem states that in the linear electric circuit the effect of the load change at a given bus can be represented by a simple circuit with emf $\underline{E}_T$ and the bus impedance $\underline{Z}_T$.

The basic circuit resulting from Thevenin's theorem is shown in Fig.1. Knowing the Thevenin's bus impedance $\underline{Z}_T = R_T + jX_T$, load bus voltage V, active P and reactive bus power Q one can calculate the magnitude of Thevenin's emf using the following formula

$$E_T = \sqrt{\left(V + \frac{PR_T + QX_T}{V}\right)^2 + \left(\frac{PX_T - QR_T}{V}\right)^2} \quad (1)$$

Using complex notification we have

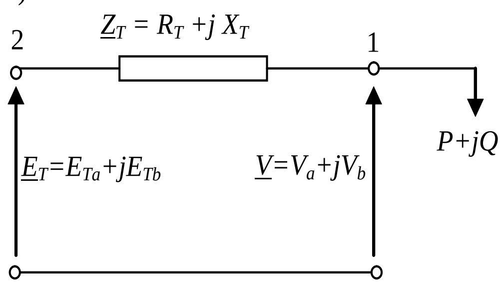
$$\underline{V} = V_a + jV_b \;\; and \;\; V = \sqrt{V_a^2 + V_b^2} \quad (2)$$

$$\underline{E}_T = E_a + jE_b \;\; and \;\; E_T = \sqrt{E_{Ta}^2 + E_{Tb}^2} \quad (3)$$

The complex admittance of branch connecting load bus with Thevenin's emf bus equals

$$G_T + jB_T = 1/(R_T + jX_T) \quad (4)$$

1.a)



1.b)

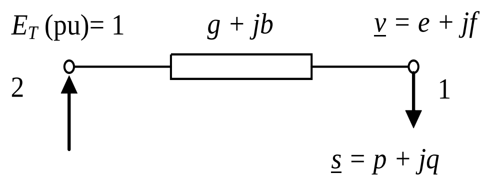


Fig.1. The scheme of 2-bus Thevenin's network, 1.a) simple Thevenin's circuit, 1.b) scheme for load flow study.

The load flow equations for load bus have the following form

$$P = V^2 G_{11} + (V_a E_a + V_b E_b) G_{12} + (-V_a E_b + V_b E_a) B_{12} \quad (5)$$

$$Q = -V^2 B_{11} - (V_a E_a + V_b E_b) B_{12} + (-V_a E_b + V_b E_a) G_{12} \quad (6)$$

where

$$G_{11} = G_T \ \ and \ \ G_{12} = -G_T \quad (7)$$

$$B_{11} = B_T \ \ and \ \ B_{12} = -B_T \quad (8)$$

Let the bus 2 with Thenenin's emf be the slack bus. Then we have $E_a = E_T$, $E_b = 0$ and

$$P = V^2 G_T - V_a G_T - V_b B_T \quad (9)$$

$$Q = -V^2 B_T + V_a B_T - V_b G_T \quad (10)$$

To simplify all considerations the load bus per unit system is introduced as follows

$$Z_b = Z_T = \sqrt{R_T^2 + X_T^2} \quad (11)$$

$$V_{base} = E_T \quad (12)$$

$$S_b = E_T^2 / Z_b \quad (13)$$

where symbol b means the base value.

Dividing both side of load flow equations by S_b we obtain

$$p = v^2 g - eg - fb \quad (14)$$

$$q = -v^2 b + eb - fg \quad (15)$$

where

$$p = P / S_b \ \ and \ \ q = Q / S_b \quad (16)$$

$$v = V / E_T \quad (17)$$

$$e = V_a / E_T \ \ and \ \ f = V_b / E_T \quad (18)$$

$$g = G_T Z_T \ \ and \ \ b = B_T Z_T \quad (19)$$

Note that the following relations exist in the new bus load per unit system

$$r + jx = R_T / Z_T + jX_T / Z_T \quad (20)$$

$$z^2 = r^2 + x^2 = \frac{R_T^2}{Z_T^2} + \frac{X_T^2}{Z_T^2} = 1 \quad (21)$$

$$\underline{y} = 1 / \underline{z} = 1 / (r + jx) = (r - jx) / z^2 = r - jx \quad (22)$$

$$\underline{y} = g + jb \quad (23)$$

and finally $g = r \ \ and \ \ b = -x$.

Hence the load flow equations in load bus per unit are as follows

$$p = v^2 r - er + fx \quad (24)$$

$$q = v^2 x - ex - fr \quad (25)$$

3.1 The P-Q Curve for the Critical Bus Voltage

From the point of view of voltage stability the voltage magnitude at a given bus must be in the range of upper and lower voltage limit

$$V_{lower} \le V \le V_{upper} \quad (26)$$

Especially the lower value is the critical value V_{cr} from the point of view of avoiding voltage instability in the power system. Hence the bus voltage must be greater than the critical value $V > V_{cr}$ .Using the new per unit system we can write

$$v > v_{cr} \quad (27)$$

where $v_{cr} = V_{cr} / E_T$.

According to the above assumptions load flow equations for load bus critical solutions *(e,f)* depend on the critical voltage magnitude v_{cr}. The load flow equations can be analyzed as a critical p-q curve composed of *(p,q)* values, which are related to the critical bus voltage magnitude v_{cr}. To find the formula of the critical p-q curve we must eliminate the rectangular components of *e* and *f* from the load flow equations (24) and (25). To find *e* we can make the following multiplications

$$rp = r^2 v^2 - r^2 e + rxf \quad (28)$$

$$xq = x^2 v^2 - x^2 e - rxf \quad (29)$$

and the following addition

$$rp + xq = z^2 v^2 - z^2 e \quad (30)$$

For $z = 1$ we have finally $e = v^2 - (rp + xq)$.

To find *f* we can made the following multiplications

$$rp = rxv^2 - rxe - x^2 f \quad (31)$$

$$rq = rxv^2 - rxe - r^2 f \quad (32)$$

and the following subtraction

$$xp - rq = z^2 f \quad (40)$$

For $z = 1$ we have finally $f = xp - rq$.

Substituting the obtained formula of *e* and *f* to the formula of v_{cr} we have finally:

$$v_{cr}^2 = v_{cr}^4 - 2v_{cr}^2 rp - 2v_{cr}^2 xq + p^2 + q^2 \quad (41)$$

Hence, we obtain the following formula of P-Q curve for a critical voltage magnitude v_{cr}

$$p^2 + q^2 - 2v_{cr}^2 rp - 2v_{cr}^2 xq + v_{cr}^4 - v_{cr}^2 = 0 \quad (42)$$

From the above formula we can obtain the quadratic equations for the specific *p*

$$q^2 - 2v_{cr}^2 xq + p^2 - 2v_{cr}^2 rp + v_{cr}^4 - v_{cr}^2 = 0 \quad (43)$$

The example of p-q curve is shown in Fig.2. The p-q curve can be transformed into P-Q curve after multiplication *p* and *q* by S_b .

Equation (43) is quadratic and has two real solutions according to the value of the equation discriminant Δ:

$$\Delta = 4v_{cr}^4 x^2 - 4p^2 + 8v_{cr}^2 rp - 4v_{cr}^4 + 4v_{cr}^2 \quad (44)$$

and finally we obtain two parts of p-q curve

ISBN: 978-960-474-356-8

$$q_{1,2} = 0.5\left(2v_{cr}^2 x -/+ \sqrt{\Delta}\right) \qquad (45)$$

The lower part of the p-q curve is associated with the consumed power, because a reactive consumed power at bus is treated in load flow equations as a negative value

$$q_{lower} = v_{cr}^2 x - \sqrt{-p^2 + 2v_{cr}^2 rp + v_{cr}^2 - v_{cr}^4 r^2} \qquad (46)$$

The upper part of the p-q curve relates to positive values, i.e. to reactive generation at a given bus

$$q_{upper} = v_{cr}^2 x + \sqrt{-p^2 + 2v_{cr}^2 rp + v_{cr}^2 - v_{cr}^4 r^2} \qquad (47)$$

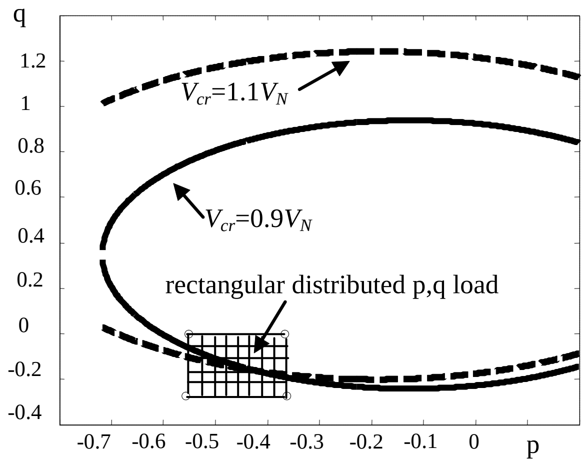


Fig.2. The example p-q curve at a load bus. Symbol V_N means the nominal voltage.

4 THE PROBABILITY OF THE VIOLATION OF THE CRITICAL VOLTAGE

Let's assume that the load at bus is uniformly distributed between their min and max

$$p_{\min} \le p \le p_{\max} \qquad (48)$$

$$q_{\min} \le q \le q_{\max} \qquad (49)$$

The probability of the violation of the critical voltage magnitude at load bus can be calculated using the outside area and the rectangular area, see Fig.3.

To find the probability of the violation of the critical voltage the lower part of p-q curve should be used, Fig.3. Using the geometrical definition of probability we have

$$p_{vcr} = S_{outside} / S \qquad (50)$$

where S means the area of rectangular

$$S = (p_{\max} - p_{\min})(q_{\max} - q_{\min}) \qquad (51)$$

The outside area equals

$$S_{outside} = S - S_{ABCD} \qquad (52)$$

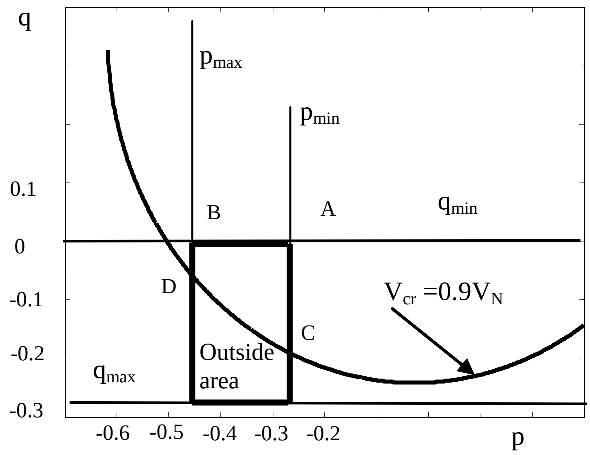


Fig.3. The p-q curve and the rectangular distributed load.

The area of *ABC*D figure can be computed using the definite integral formula in the following way

$$S_{ABCD} = S1 - S2 = \int_{p\max}^{p\min} q_{\min} dp - \int_{p\max}^{p\min} q_{lower} dp \qquad (53)$$

where

$$S1 = q_{\min}(p_{\min} - p_{\max}) \qquad (54)$$

$$\begin{aligned} S2 = {} & v_{cr}^2 x (p_{\min} - p_{\max}) + \\ & - \frac{(-p_{\min} + v_{cr}^2 r)}{2}\sqrt{W_{\min}} + \\ & + \frac{(-p_{\max} + v_{cr}^2 r)}{2}\sqrt{W_{\max}} + \\ & + \frac{v_{cr}^2}{2}\arcsin\frac{(-p_{\min} + v_{cr}^2 r)}{v_{cr}} + \\ & - \frac{v_{cr}^2}{2}\arcsin\frac{(-p_{\max} + v_{cr}^2 r)}{v_{cr}} \end{aligned} \qquad (55)$$

$$W_{\min} = -p_{\min}^2 + 2v_{cr}^2 r p_{\min} + v_{cr}^2 - v_{cr}^4 r^2 \qquad (56)$$

$$W_{\max} = -p_{\max}^2 + 2v_{cr}^2 r p_{\max} + v_{cr}^2 - v_{cr}^4 r^2 \qquad (57)$$

4.1 Numerical example

Thevenin's complex impedance seen from the 400 kV load bus has been obtained by the load flow study in 400/220 kV transmission grid

$$\underline{Z}_T = R_T + jX_T = (-125.12 + 31.40\ )\ \Omega$$

The bus voltage magnitude at the analysed load bus determined by load flow computation equals

$$V = 410 \text{ kV}$$

while the nominal voltage has the following value

$$V_N = 400 \text{ kV}$$

The critical voltage magnitude equals

$$V_{cr} = 0.9V_N = 360 \text{ kV}$$

ISBN: 978-960-474-356-8

The minimal and maximal active and reactive load at the analysed bus equal

$$P_{\min} = 400 \text{ MW} \quad \text{and} \quad P_{\max} = 600 \text{ MW}$$

$$Q_{\min} = 0 \text{ MVAR} \quad \text{and} \quad P_{\max} = 300 \text{ MVAR}$$

Knowing the Thevenin's bus impedance $\underline{Z}_T = R_T + jX_T$, load bus voltage V, active P and reactive bus power Q we calculate the magnitude of Thevenin's emf

$$E_T = \sqrt{\left(V + \frac{PR_T + QX_T}{V}\right)^2 + \left(\frac{PX_T - QR_T}{V}\right)^2} = 572.9 \text{ kV}$$

To simplify all considerations the load bus per unit system is introduced

$$Z_b = Z_T = \sqrt{R_T^2 + X_T^2} = 342.1\ \Omega$$

$$V_{base} = E_T = 572.9 \text{ kV}$$

$$S_b = E_T^2 / Z_b = 959.4 \text{ MVA}$$

The value of analysed variables in load by per unit systems are as follows

$$p_{\min} = P_{\min} / S_b = \text{-0.4169}$$

$$p_{\max} = P_{\max} / S_b = \text{-0.6254}$$

$$q_{\min} = Q_{\min} / S_b = 0; \quad q_{\max} = Q_{\max} / S_b = \text{-0.3127}$$

$$v_{cr} = V_{cr} / E_T = 0.6284$$

$$r = R_T / Z_T = \text{-0.3657}; \quad x = X_T / Z_T = 0.9307$$

Now we can calculate the probability of the violation of the critical voltage magnitude.

The rectangular area

$$S = (p_{\max} - p_{\min})(q_{\max} - q_{\min}) = 0.0652$$

The inside area

$$S_{ABC} = \int_{p_{\min}}^{p_{\max}} (q_{lower} - q_{\min}) dp = 0.0270$$

The outside area

$$S_{outside} = S - S_{ABCD} = 0.0382$$

The probability of the violation of the critical voltage magnitude

$$p_{vcr} = S_{outside} / S = 0.58$$

5 Conclusion

The proposed p-q curve method is simple and may be based on local measurements of bus impedance. It enables calculating the probability of voltage limit violation at a given load bus. The greater the probability the weaker the bus is from the point of view of voltage stability.

To find the formula of p-q curve a new load bus per unit system must be introduced. The transformation from p-q curve to P-Q curve can be easily made by multiplication p and q value by the base power of the analyzed load bus.

The probability of voltage limit violation is estimated as the quotient of relevant area outside and inside the specific p-q curve.

It is also very important that the form of the p-q curve in the load bus per unit has the same form for each power system bus.

The approach based on p-q or P-Q curve may be attractive in planning and operating because it is quick and requires only small modifications of a standard load flow program.

References:

[1] Bhattacharya and J. Zhong „Reactive power as an ancillary service", *IEEE Trans. on Power Systems*, vol. 16, pp. 294-300, May 2001.

[2] Bhattacharya K., Zhong J. „Reactive power management in deregulated electricity markets - a review", in conference proceedings: *IEEE Power Eng. Soc. Winter Meeting*, New York, 2002.

[3] C.J. Hatziadoniu, N. Nikolov and F. Pourboghrat "Power Conditioner Control and Protection for Distributed Generators and Storage", *IEEE Trans. on Power Systems*, vol. 18, pp. 83-90, Feb. 2003.

[4] G. Chicco and G. Gross "Allocation of the reactive power support requirements in multitransaction networks", *IEEE Transactions on Power Systems*, vol. 17, pp. 243-249, May 2002.

[5] K.N. Shubhanga and A. M. Kulkarni "Application of structure preserving energy margin sensitivity to determine the effectiveness of shunt and series FACTS devices", *IEEE Trans. on Power Systems*, vol. 17, pp. 730-738, Aug. 2002.

[6] P. I. Abril and J. A. G. Quintero "VAR compensation by sequential quadratic programming", *IEEE Trans. on Power Systems*, vol. 18, pp. 36-41, Feb. 2003.

[7] Vu Khoi, M. M. Begovic, D. Novosel and M. M. Saha, "Use of local measurements to estimate voltage -stability margin", *IEEE Trans. on Power Systems*, vol. 14, pp. 1029-1035, Aug. 1999.

[8] Y. Lu and A. Abur "Static security enhancement via optimal utilization of thyristor-controlled series capacitors", *IEEE Trans. on Power Systems*, vol. 17, pp. 324-329, May 2002.

[9] Zhong and K. Bhattacharya "Toward a competitive market for reactive power", *IEEE Trans. on Power Systems*, vol. 17, pp. 1206-1215, Nov. 2003.

[10] I. Dobson, T. VanCutsem, C. Vournas, C. L. DeMarco, M. Venkatasubramanian, T.

Overbye, C.A. Canizares, “Voltage Stability Assessment: Concepts, Practices and Tools”, *IEEE-PES Power Systems Stability Subcommittee Special Publication* SP101PSS, IEEE-PES General Meeting, Toronto 2003.

[11] I. Musirin, T. K. Abdul Rahman, “Estimating Maximum Load ability for Weak Bus Identification Using FVSI”, *IEEE Power Engineering Review*, Nov. 2002, pp. 50-52.

[12] Real Time System Operation 2006 – 2007; *Prepared By: Lawrence Berkeley National Laboratory*

[13] P. Nagendra, T. Datta, S. Halder, S. Paul, “Power System Voltage Stability Assessment Using Network Equivalents- A Review”, *Journal of Applied Sciences*, 2010.

[14] J. Zhao, Y. Yang, Z. Gao, “A Review on Online Voltage Stability Monitoring Indices and Methods based on Local Phasor Measurements”, 17th *Power Systems Computation Conference*, Stockholm, Sweden, August 22-26, 2011.

[15] Yuri V. Makarov, Zhao Yang Dong, David J. Hill, “On Convexity of Power Flow Feasibility Boundary”, *IEEE Transaction on Power Systems*, Vol. 23, No. 2, May 2008, pp. 811-813.

[16] Bernard C. Lesieutre, Ian A. Hiskens, “Convexity of the Set of Feasible Injections and Revenue Adequacy in FTR Markets”, *IEEE Transaction on Power Systems*, Vol. 20, No. 4, November 2005, pp. 1790-1798

[17] Ian A. Hiskens, Robert J. Davy, “Exploring the Power Flow Solution Space Boundary”, *IEEE Transaction on Power Systems*, Vol. 16, No. 3, August 2001, pp. 389-395.

[18] Ivan Smon, G. Verbic, F. Gubina, “Local Voltage Stability Index using Tellegen’s Theorem”, *IEEE Transactions on Power Systems*, Vol. 21, No. 3, August 2006.

[19] Analysis to Power Systems”, *Lecture Notes in Control and Information Sciences*, 2003, Volume 293/2003, Springer