On Some Nonlinear Partial Differential and Integro-Differential Diffusion Models

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Abstract: Existence, uniqueness, long-time behavior of solutions and algorithm of numerical resolution of initial-boundary value problem for one integro-differential system are studied. Attention is paid to construction and analysis of decomposition algorithms with respect to physical processes for one-dimensional nonlinear partial differential model based on Maxwell's system. Semi-discrete averaged models are constructed and investigated for this system. Finite difference schemes are studied. Investigated systems arise in modeling of process of the penetration of a magnetic field in a substance.

Key–Words: Nonlinear partial differential and integro-differential systems; existence and uniqueness of solutions; long-time behavior; decomposition algorithms; semi-discrete averaged models finite difference schemes

1 Introduction

In mathematical modeling of many natural phenomena and processes can be described by the initial-boundary value problems posed for non-stationary differential and integro-differential models (see, for example, [5], [6], [12], [35], [36], [39], [40], [43], [46], [48], [49] and references therein).

Investigation and numerical resolution of these problems are the actual sphere of mathematical physics and numerical analysis. One such model is obtained at mathematical modeling of processes of electro-magnetic field penetration in the substance. In the quasi-stationary approximation, the corresponding system of Maxwell's equations has the form [36]:

$$\frac{\partial H}{\partial t} = -rot(\nu_m rot H), \tag{1}$$

$$\frac{\partial \theta}{\partial t} = \nu_m \left(rot H\right)^2, \tag{2}$$

where $H = (H_1, H_2, H_3)$ is a vector of the magnetic field, θ is temperature, ν_m characterizes the electro-conductivity of the substance. Equations (1) describe the process of diffusion of the magnetic field and equation (2) - change of the temperature at the expense of Joule's heating. If ν_m depends on temperature θ, i.e., $\nu_m = \nu_m(\theta)$, then the system (1), (2) can be rewritten in the following form [11]:

$$\frac{\partial H}{\partial t} = -rot\left[a\left(\int\limits_0^t |rot H|^2 \, d\tau\right) rot H\right], \tag{3}$$

where function $a = a(S)$ is defined for $S \in [0, \infty)$.

Note that integro-differential parabolic models of (3) type are complex and still yields to the investigation only for special cases (see, for example, [3], [4], [7] - [9], [11], [15] - [20], [22], [23], [27] - [30], [32], [33], [37] - [39], [41] and references therein).

Study of the models of type (3) have begun in the work [11]. In this work, in particular, are proved the theorems of existence of solution of the first boundary value problem for scalar and one-dimensional space case while $a(S) = 1 + S$ and uniqueness for more general cases. One-dimensional scalar variant for the case $a(S) = (1+S)^p$, $0 < p \leq 1$ is studied in [9]. Investigations for multidimensional space cases at first are carried out in the works [7] and [8]. Multidimensional space cases are also discussed in the following works [4], [16], [37], [38].

Long-time behavior of solutions of initial-boundary value problems for (3) type models are studied in the works [3], [15] - [20], [22], [23], [27] - [30], [32], [33] and in a number of other works as well. In these works main attentions, are paid to one-dimensional analogs.

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One must note that for the cylindrical conductors to the study of modeling of physical process of penetrating of the electromagnetic field some amounts of works were also devoted. In this case above-mentioned type models, written in cylindrical coordinates, are studied in [24]. To the investigation of periodic problem for one-dimensional (3) type model in cylindrical coordinates the work [41] is also devoted.

Interest to above-mentioned differential and integro-differential models is more and more arising and initial-boundary value problems with different kinds of boundary and initial conditions are considered.

Particular attention should be paid to construction of numerical solutions and to their importance for differential and integro-differential models. This issue for integro-differential models see, for example, [5], [14], [15], [19], [21], [25] - [28], [30] - [32], [34], [41], [43], [44] and references therein.

Settling of semi-discrete and finite difference schemes, finite element analogues and Galerkin method algorithm for (3) type one-dimensional integro-differential models are given in [14], [15], [19], [21], [25] - [28], [30] - [32], [34], [41] and in the other works as well.

Let us note that studying of even one-dimensional systems with two-component vector of the magnetic field have their essential complexities and its investigation and numerical resolution are continuing very intensively (see, for example, [3], [15], [17], [18], [23], [26], [29] - [34]).

Taking into account thermal conductivity the same process of penetration of magnetic field into a substance is described by the following system:

$$\frac{\partial H}{\partial t} = -rot\ (\nu_m rot\, H), \qquad \frac{\partial \theta}{\partial t} = \nu_m (rot\, H)^2 + div(\kappa\, grad\, \theta), \tag{4}$$

where κ is coefficient of heat conductivity. As a rule this coefficient is function of argument θ as well.

The literature on the questions of existence, uniqueness, regularity of the solutions and numerical resolution of the initial-boundary value problems to the (4) type systems is very rich (see, for example, [1], [2], [6], [46], [48], [49] and references therein).

Beside of essential nonlinearity, complexities of the mentioned system (4) is caused by its multi-dimensionality. This circumstance is complicating to get numerical results for concrete real problems. Naturally arises the possibility of reduction to suitable one-dimensional models.

Beginning from the classical work [10] the appropriate algorithms are constructed and studied for many models (see, for example, [13], [42], [45] and references therein).

Complex nonlinearity dictates also to split along the physical process and investigate basic model by them. In particular, it is logical to split system (4) into following two models:

$$\frac{\partial \tilde{H}}{\partial t} = -rot\,(\nu_m(\tilde{\theta}) rot\, \tilde{H}), \qquad \frac{\partial \tilde{\theta}}{\partial t} = \nu_m(\tilde{\theta})(rot\, \tilde{H})^2 \tag{5}$$

and

$$\frac{\partial \tilde{\tilde{\theta}}}{\partial t} = div\,(\kappa(\tilde{\tilde{\theta}}) grad\, \tilde{\tilde{\theta}}). \tag{6}$$

In (5) Joule's rule, while in (6) process of thermal conductivity are considered.

Investigation of splitting along the physical processes in one-dimensional case is the natural beginning of studding this issue. In this direction the first step was made in the works [1], [2].

Purpose of the present work is to describe results received basically just for the one-dimensional (3) system with two-component magnetic field vector. Investigation of initial-boundary value problem as well as settle of algorithms of construction of approximation solutions are aim of this note too.

Our aim is also to construct and study additive analogues bases on models (5) and (6) for one-dimensional analog of system (1) with one-component magnetic field.

2 Existence, Uniqueness and Long-time Behavior of the Solution for the Integro-Differential Problem

Let us consider the cylinder $(0, 1) \times (0, \infty)$. If the magnetic field has the form $H = (0, U, V)$, $U = U(x, t)$, $V = V(x, t)$, then from (3) we obtain the following system of nonlinear integro-differential equations:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial U}{\partial x}\right], \qquad \frac{\partial V}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial V}{\partial x}\right], \tag{7}$$

where

$$S(x,t) = \int_0^t \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] d\tau. \tag{8}$$

Let us consider the following boundary and initial conditions:

$$U(0,t) = V(0,t) = 0,$$
$$\left.\frac{\partial U(x,t)}{\partial x}\right|_{x=1} = \left.\frac{\partial V(x,t)}{\partial x}\right|_{x=1} = 0, \quad (9)$$

$$U(x,0) = U_0(x), V(x,0) = V_0(x), \quad (10)$$

where U_0, V_0 are given functions.

We should note that boundary conditions (9) are used here taking into account the physical problem considered in [24].

The following statement of existence and uniqueness of the solution takes place.

Theorem 1 *If* $a(S) = (1+S)^p$, $-1/2 < p < 0$ *or* $0 < p \le 1$ *and* U_0, $V_0 \in H^2(0,1)$, $U_0(0) = V_0(0) = \left.\frac{dU_0(x)}{dx}\right|_{x=1} = \left.\frac{dV_0(x)}{dx}\right|_{x=1} = 0$, *then where exists unique solution* (U,V) *of the problem (7) - (10) such that:* $U,V \in L_2\left(0,\infty; H^2(0,1)\right)$, $U_{xt}, V_{xt} \in L_2\left(0,\infty; L_2(0,1)\right)$.

We use usual $L_2(0,1)$ and Sobolev spaces $H^k(0,1)$. The symbols C in this section denote various positive constants independent of t.

The existence part of the Theorem 1 is proved using Galerkin's modified method and compactness arguments as in [40], [47] for nonlinear parabolic equations.

To study long-time behavior of solution of the problem (7) - (10) is also very important.

Theorem 2 *If* $a(S) = (1+S)^p$, $-1/2 < p < 0$ *or* $0 < p \le 1$ *and* $U_0, V_0 \in H^3(0,1)$, $U_0(0) = V_0(0) = \left.\frac{dU_0(x)}{dx}\right|_{x=1} = \left.\frac{dV_0(x)}{dx}\right|_{x=1} = 0$, *then for the solution of problem (7) - (10) the following estimates hold as* $t \to \infty$*:*

$$\left|\frac{\partial U(x,t)}{\partial x}\right| + \left|\frac{\partial V(x,t)}{\partial x}\right| \le C \exp\left(-\frac{t}{2}\right),$$

$$\left|\frac{\partial U(x,t)}{\partial t}\right| + \left|\frac{\partial V(x,t)}{\partial t}\right| \le C \exp\left(-\frac{t}{2}\right),$$

uniformly in x *on* $[0,1]$.

Results of Theorem 2 show that asymptotic behavior of the solution has an exponential character. Let us note that same results are true for problem with first type homogeneous conditions on whole boundary (see, for example, [15], [32] and references therein).

3 Finite Difference Scheme for Integro-differential Model

Now, consider the problem (7) - (10) in the cylinder $[0,1] \times [0,T]$, where T is given positive constant, for the case $p = 1$.

On $[0,1] \times [0,T]$ let us introduce a grid with mesh points denoted by $(x_i, t_j) = (ih, j\tau)$, where $i = 0,1,...,M$; $j = 0,1,...,N$, with $h = 1/M$, $\tau = T/N$. The initial line is denoted by $j = 0$. The discrete approximation at (x_i, t_j) is designed by u_i^j, v_i^j and the exact solution to problems (7) - (10) by U_i^j, V_i^j. We will use the following known notations:

$$r_{t,i}^j = \frac{r_i^{j+1} - r_i^j}{\tau},\ r_{x,i}^j = \frac{r_{i+1}^j - r_i^j}{h},\ r_{\bar{x},i}^j = \frac{r_i^j - r_{i-1}^j}{h}.$$

For problem (7) - (10) let us consider the finite difference scheme:

$$u_{t,i}^j - \left\{\left(1 + \tau \sum_{k=1}^{j+1} \left[(u_{\bar{x},i}^k)^2 + (v_{\bar{x},i}^k)^2\right]\right) u_{\bar{x},i}^{j+1}\right\}_x = 0,$$
$$v_{t,i}^j - \left\{\left(1 + \tau \sum_{k=1}^{j+1} \left[(u_{\bar{x},i}^k)^2 + (v_{\bar{x},i}^k)^2\right]\right) v_{\bar{x},i}^{j+1}\right\}_x = 0,$$
$$i = 1,2,...,M-1;\ j = 0,1,...,N-1, \quad (11)$$
$$u_0^j = v_0^j = 0,\ u_{\bar{x},M}^j = v_{\bar{x},M}^j = 0,\ \ j = 0,1,...,N,$$
$$u_i^0 = U_{0,i},\ ,v_i^0 = V_{0,i},\ \ i = 0,1,...,M.$$

Theorem 3 *If problem (7) - (10) has sufficiently smooth solution* $U = U(x,t)$, $V = V(x,t)$, *then the solution* $u^j = (u_1^j, u_2^j, \dots, u_M^j)$, $v^j = (v_1^j, v_2^j, \dots, v_M^j)$, $j = 1,2,\dots,N$ *of the difference scheme (11) tends to the solution of continuous problem* $U^j = (U_1^j, U_2^j, \dots, U_M^j)$, $V^j = (V_1^j, V_2^j, \dots, V_M^j)$, $j = 1,2,\dots,N$ *correspondingly as* $\tau \to 0$, $h \to 0$ *and the following estimates are true:*

$$\|u^j - U^j\|_h \le C(\tau + h), \quad \|v^j - V^j\|_h \le C(\tau + h).$$

Note that in Theorem 3 C is independent of h and τ and $\|\cdot\|_h$ denotes discrete analog of the $L_2(0,1)$ space norm.

Note that for solving the difference schemes (11) Newton's iterative process is used.

4 Decomposition Algorithms with Respect to Physical Processes for the Differential Problem

In the domain $\Omega \times (0,T)$, where $\Omega = (0,1)$ let us consider the following problem:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(V^{\alpha}\frac{\partial U}{\partial x}\right),$$

$$\frac{\partial V}{\partial t}=V^{\alpha}\left(\frac{\partial U}{\partial x}\right)^2+\frac{\partial^2 V}{\partial x^2}, \tag{12}$$

$$U(x,t)=\frac{\partial V(x,t)}{\partial x}=0,\ (x,t)\in\partial\Omega\times(0,T),$$

$$U(x,0)=U_0(x),\ V(x,0)=V_0(x)\geq v_0>0,$$

where $-1/2\leq\alpha\leq 1/2, \alpha\neq 0, U_0$ and V_0 are known functions defined on $[0,1]$ and v_0 is constant.

If we denote $V^{\frac{1}{2}}=W$, $2\alpha=\gamma$, then problem (12) can be rewritten in the following equivalent form [1]:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(W^{\gamma}\frac{\partial U}{\partial x}\right), \tag{13}$$

$$\frac{\partial W}{\partial t}=\frac{1}{2}W^{\gamma-1}\left(\frac{\partial U}{\partial x}\right)^2+\frac{\partial^2 W}{\partial x^2}+\frac{1}{W}\left(\frac{\partial W}{\partial x}\right)^2,$$

$$U(x,t)=\frac{\partial W(x,t)}{\partial x}=0,\ (x,t)\in\partial\Omega\times(0,T),$$

$$U(x,0)=U_0(x),\ W(x,0)=W_0(x)=V_0^{1/2}(x).$$

Let us introduce the notations:

$$y_t=\frac{y^{j+1}-y^j}{\tau},\ y_{1t}=\frac{y_1^{j+1}-y^j}{\tau},\ y_{2t}=\frac{y_2^{j+1}-y^j}{\tau},$$
$$y=\eta_1 y_1+\eta_2 y_2,\ \eta_1+\eta_2=1,\ \eta_1>0,\ \eta_2>0.$$

Correspond to the problem (13) following additive averaged semi-discrete scheme:

$$u_{1t}=\frac{d}{dx}\left(w_1^{\gamma}\frac{du_1}{dx}\right),$$
$$\eta_1 w_{1t}=\frac{1}{2}w_1^{\gamma-1}\left(\frac{du_1}{dx}\right)^2,$$

$$u_{2t}=\frac{d}{dx}\left(w_2^{\gamma}\frac{du_2}{dx}\right), \tag{14}$$
$$\eta_2 w_{2t}=\frac{d^2 w_2}{dx^2}+\frac{1}{w_2}\left(\frac{dw_2}{dx}\right)^2,$$

$$u_1^0=u_2^0=U_0,\quad w_1^0=w_2^0=W_0,$$

with suitable boundary conditions.

Theorem 4 *If problem (13) has a sufficiently smooth solution and $-1\leq\gamma\leq 1$, then the solution of the scheme (14) convergence to the solution of problem (13) as $\tau\to 0$, and the following estimate is true*

$$\left\|U(t_j)-u^j\right\|+\left\|W(t_j)-w^j\right\|=O(\tau^{\frac{1}{2}}).$$

Here $\|\cdot\|$ is an usual norm of the space $L_2(0,1)$.

Let us also correspond to the system (13) the following semi-discrete additive model:

$$u_t=\frac{d}{dx}\left[\left(\eta_1 w_1^{\gamma}+\eta_2 w_2^{\gamma}\right)\frac{du}{dx}\right],$$

$$\eta_1 w_{1t}=\frac{1}{2}w_1^{\gamma-1}\left(\frac{du}{dx}\right)^2, \tag{15}$$

$$\eta_2 w_{2t}=\frac{d^2 w_2}{dx^2}+\frac{1}{w_2}\left(\frac{dw_2}{dx}\right)^2,$$

with suitable initial and boundary conditions as well.

Let us note that the analogous result as Theorem 4 is valid for scheme (15).

Note also that the above mention result and result analogical to Theorem 4 with Dirichlet boundary conditions for function W in problem (13) are obtained in the works [1], [2].

At last we note that additive models analogical to (5) and (6) for the system (4) may be also constructed.

The fully discrete averaged finite difference schemes based on investigated (14) and (15) semi-discrete schemes are also constructed. The fully discrete finite difference schemes for the problems (12) and (13) analogical to schemes stated in [1], [2], [46] are also applied. Numerical experiments are done using all of these algorithms for problems (12) and (13).

5 Conclusion

Models of nonlinear differential and integro-differential equations associated with the penetration of a magnetic field in a substance are considered. Existence, uniqueness and long-time behavior of solution of initial-boundary value problem for integro-differential case are studied. The finite difference scheme are investigated for this model as well. The semi-discrete and fully discrete finite difference schemes for the one-dimensional differential problem are studied. Decomposition algorithms with respect to physical processes are constructed and investigated. Various numerical experiments for the studied schemes are carried out. The results of these numerical experiments agree with theoretical researches.

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