Active Vibration Control of Satellite Panels using Piezoelectric Actuators and Sensors

M.M. ELMADANY, K.A. ALSAIF, M.A. FODA, A.A. ALBEDAH
Department of Mechanical Engineering
King Saud University
Riyadh, 11421
SAUDI ARABIA
mmadany@ksu.edu.sa

Abstract: - One of the most important problems in satellite design is of vibration control. High pointing accuracy is an important part of the overall design problem for satellite control system. However, the orbiting attitude slewing or manoeuvring operation will introduce certain levels of vibration to flexible appendages, which will deteriorate its pointing performance. This paper is concerned with the active vibration control of a satellite structure with flexible appendages representing a solar panel equipped with piezoelectric actuators and sensors. A classical control law, constant-gain negative velocity feedback, and optimal control law based on LQG theory are presented for actively damping the elastic oscillations using piezoelectric materials as actuators/sensors bonded on the surface of the flexible appendages. Numerical simulations are performed. The results show that both control strategies significantly reduce the vibration response of the satellite panels. The LQG optimal control is more effective than the velocity feedback due to the fact that the former changes both damping and natural frequencies of the system while the later changes only the system damping.

Key-Words: - Satellite panels**,** Active vibration control, Piezoelectric actuators and sensors, optimal control law

1 Introduction

Spacecrafts with flexible appendages, in orbiting, are important for the purpose of communication, remote sensing and variety of space-related research activities. Such spacecrafts have a central rigid body with attached appendages such as antennas and solar panels. Rapid rotational manoeuvres of spacecraft are essential to meet mission requirements. In rotational manoeuvres of such spacecrafts, elastic deformations in the flexible appendages are often presented. Therefore, designing a control system to provide fine pointing accuracy, while effectively suppressing the induced vibration, creates a challenging problem for spacecraft designers. In addition, micro-satellites with flexible appendages like deployable solar panels are very prone to vibration when excited by the reaction wheels or momentum wheels. Any infinitesimal amount of unbalance in the reaction wheels' rotors will impose a harmonic excitation which may interact with the solar panels structure. Therefore, minimizing the solar panels structural vibration interaction can control the jitter.

In order to meet the high precision requirement of flexible space structures, the application of active control for vibration suppression has attracted significant attentions and many achievements have been accomplished in the past two decades. Many control algorithms have been used to suppress the vibration, such as direct velocity feedback (DVFB) control [1], acceleration feedback control [2], and positive position feedback (PPF) control [3]. A range of controllers including linear quadratic Gaussian (LQG) and loop transfer recovery, and H_∞ based design methods have been designed and tested by researchers [4-9], requiring approximating the dynamics by a finite dimensional state space model.

Piezoelectric materials, such as lead zirconate titanate (PZT), have coupled mechanical and electrical properties, and exhibit mechanical deformation when subjected to an applied electric field, which is called the converse piezoelectric effect. They also generate a voltage or charge when subjected to a force or deformation, which is termed as the direct piezoelectric effect. The use of PZT as actuators/sensors is considered as one promising method for actively suppressing the residual micro-vibrations. Therefore, piezoelectric transducers have been used extensively as distributed sensors and actuators for vibration control of flexible structures [10]. The modeling technique applicable to the beam structure bonded with piezoceramic (PZT) sensors and actuators have been discussed by Fuller et al. [11]. Clark et al. [12] developed a dynamics model for the vibration response of a simply supported elastic rectangular plate using a piezoelectric patch of variable rectangular geometry. Chen and Shen [13] adopted independent modal space control (IMSC) for vibration control of piezoelectric active structures.

The design of an acceleration sensor based active vibration control for a cantilever beam with bonded piezoelectric patches was studied in [2]. The system's dynamic model considering the non-collocated placement of the acceleration sensor and the piezoelectric patch actuator was derived. An acceleration sensor based proportional feedback control algorithm and a sliding mode variable structure control algorithm with phase shifting technology were proposed, for suppressing the first two bending modes vibration of the beam. Active vibration control of a low frequency space frame platform was studied by incorporating piezoelectric actuators and sensors in the dynamic model, [14]. It was shown using simulation that a control strategy using optimal control was effective for vibration suppression under a wide variety of loading conditions.

This research work is concerned with the active vibration control of a satellite structure with flexible appendages representing a solar panel equipped with piezoelectric actuators and sensors. A classical control law, constant-gain negative velocity feedback, and optimal control law based on LQG theory are presented for actively damping the elastic oscillations using piezoelectric materials as actuators/sensors bonded on the surface of the flexible appendages.

2 Problem Formulation

Figure 1 shows the model of a flexible satellite. The model consists of a central point mass, which represents the satellite body, and two flexible appendages. The physical model used in the analysis is a free-free beam of finite length L with concentrated mass in the middle ($x=L/2$), together with a set of piezoelectric actuators bonded on the top face and sensors bonded on the bottom, as shown in Fig. 1. x_{i1}, x_{i2} are the locations of edges of the ith actuator/sensor; each actuator/sensor has thickness h_a and length l_a. The input to the system is the voltage applied to the actuators and the output is the strain-induced voltage generated by the sensors.

The main assumptions used in the analysis are: (i) The actuators are perfectly bonded to the beam; (ii) The stiffness of the bonding material and the PZT patches are neglected; (iii) The voltage is uniformly distributed along the piezoelectric actuators; and (iv) The poling direction of the piezoelectric material is in the direction of the transverse direction of the beam. As voltage is applied to the actuator, pure longitudinal strains are induced, resulting in pure moments at the two edges of the actuator.

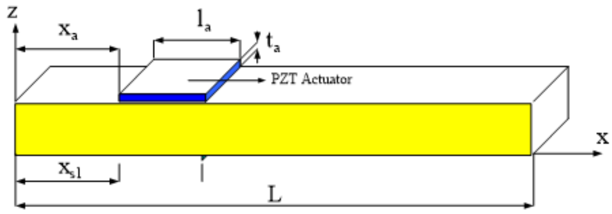

Fig. 1. A schematic diagram of a harmonically excited panel with piezoelectric patches.

The governing equation for a panel with bonded m piezoelectric patches has been obtained in [15] and is given below;

$$E_b I_b \frac{\partial^4 w}{\partial x^4} - \rho_b I_b \frac{\partial^4 w}{\partial x^2 \partial t^2} + \rho_b A_b \frac{\partial^2 w}{\partial t^2} = \operatorname{Re} F_0 e^{j\omega t} \delta(x - x_f)$$
$$- M_s \frac{\partial^2 w}{\partial t^2} \delta(x - L/2) + \sum_{i=1}^{i=m} \frac{\partial^2 M_i^a}{\partial x^2} \tag{1}$$

where ρ_b is the mass per unit volume of beam material, A_b is the cross-sectional area, I_b is area moment of inertia of cross-section and $w(x, t)$ is the transverse deflection at spatial point x along the beam at time t. δ (.) is the Dirac delta singularity function, M_s represents the mass of the satellite body.

$$M_i^a = \int_{h_b/2}^{h_b/2+h_a} \sigma_x b y dy = \int_{h_b/2}^{h_b/2+h_a} \frac{d_{31a} E_a V_i^a}{h_a} b y dy, \tag{2}$$

where h_a, and h_b are the actuator and beam thicknesses, respectively; d_{31a} and E_a are the piezoelectric strain constant and Young's modulus of the actuator; V_i^a is the voltage applied to actuator i. The evolution of the integral (2) results the following expression

$$M_i^a = b d_{31a} E_a r_a V_i^a, \tag{3}$$

where r_a denotes the distance measured from the neutral surface of the beam to the mid-plane of the actuator. The voltage distribution of actuator i can

ISBN: 978-1-61804-177-7

be expressed as V_i^a (x,t)= $V_i^a(t)[H_a(x\text{-}x_{i1}) - H_a(x\text{-}x_{i2})]$, where H_a is the Heaviside functions for generalized location and x_{i1} and x_{i2} are the end coordinates of the actuator *i*.

Assuming a uniform voltage on the electrode surface of actuator *k*, the moment can be expressed as

$$M_i^a = K_i^a V_i^a(t)\,, \tag{4}$$

where $K_i^a = bd_{31a}E_a r_a$.

The sensors are considered with the same length and axial position as the actuators, but vertically opposite, Fig. 1. On a sensor *i*, the open circuit voltage $V_i^s(t)$ due to the bending effect can be estimated by the normal strains in the axial direction ε_x of the beam. For each sensor, for a specific vibration mode, it is given by

$$\begin{aligned} V_i^s(t) &= \frac{h_s}{x_{i2}-x_{i1}}\int_{x_{i1}}^{x_{i2}} h_{31s}\varepsilon_x dx \\ &= \frac{h_s}{x_{i2}-x_{i1}}\int_{x_{i1}}^{x_{i2}} g_{31s}E_s r_s \frac{\partial^2 w}{\partial x^2}dx \\ &= K_i^s[\frac{\partial w}{\partial x}\,|_{x_{i2}} - \frac{\partial w}{\partial x}\,|_{x_{i1}}\,]\,, \end{aligned} \tag{5}$$

where h_s is the sensor thickness, and h_{31s} is the piezoelectric constant, $K_i^s = [h_s/(x_{i2} - x_{i1})]g_{31s}E_s r_s$; E_s is the Young's modulus of the sensor, r_s is the distance measured from the neutral axis of the beam to the midplane of the sensor layer, and g_{31s} is the piezoelectric stress constant.

The method of separation variables is applied to discretize the partial differential equation (1) into a set of ordinary differential equations for the numerical simulation. Then, w(x,t) is assumed to be expressed with eigenfunctions of a uniform free-free beam such that

$$w(x,t) = \sum_{j=1}^{n}\phi_j(x)q_j(t) \tag{6}$$

Where the space-dependent functions $\phi_j(x)$ (j=1,2,..,n) are the assumed mode shapes or admissible functions which must satisfy the geometric boundary conditions of the problem, and the time-dependent functions $q_j(t)$ (j=1,2,…,n) are a set of generalized coordinates.

The mode shapes of a free-free beam are used in the analysis, and they are given by

$$\begin{aligned}\phi_j(x) &= \cosh[(\beta_j L)x/L] + \cos[(\beta_j L)x/L] - \\ &\sigma_j[\sinh[(\beta_j L)x/L] + \sin[(\beta_j L)x/L]]\end{aligned} \tag{7}$$

where j=1,2,3,… and

$$\sigma_j = \frac{\cosh(\beta_j L) - \cos(\beta_j L)}{\sinh(\beta_j L) - \sin(\beta_j L)}$$

$$\beta_j L = [4.7300407;\, 7.8532046;\, 10.9956107;\, 14.1371655;\, 17.278797]$$

Equation (6.1) can be transformed into

$$M\ddot{q} + Kq = B_a V^a + B_f f \tag{8}$$

While the sensors voltage equation is given by

$$V^s = B_s q \tag{9}$$

Where

$$M_{jk} = M_{kj} = \rho_b A_b \int_0^L \phi_j \phi_k dx + M_s \phi_j(L/2)\phi_k(L/2),$$

$$K_{jk} = K_{kj} = E_b I_b \int_0^L \frac{d^2\phi_j}{dx^2}\frac{d^2\phi_k}{dx^2}dx\,,$$

$$B_{aji} = K_i^a\left(\frac{d\phi_j}{dx}\Big|_{x=x_{i2}} - \frac{d\phi_j}{dx}\Big|_{x=x_{i1}}\right),$$

$$B_{fj} = \phi_j(L/2)\,,$$

$$B_{sij} = K_s^i\left(\frac{d\phi_j}{dx}\Big|_{x=x_{i2}} - \frac{d\phi_j}{dx}\Big|_{x=x_{i1}}\right)$$

3 Controller Design

In order to design a controller to suppress the vibration of the satellite, the dynamic model of the satellite is expressed in state space form as

$$\dot{x} = Ax + Bv^a + Df \tag{10a}$$

$$y = Cx \tag{10b}$$

Where *x* is the state space variable, *A* is the system matrix, *B* is the control matrix, *D* is the disturbance matrix, y is the output vector, and C is the sensors distribution matrix. These matrices are given by

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad A = \begin{bmatrix} 0_{nxn} & I_{nxn} \\ -M^{-1}K & 0_{nxn} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{nxm} \\ M^{-1}B_a \end{bmatrix},$$

$$D = \begin{bmatrix} 0_{nx1} \\ M^{-1}B_f \end{bmatrix},$$

$$C = [B_s \quad 0_{mxn}]$$

Classical and optimal control laws are used in the analysis. The classical control law considered is constant-gain negative velocity feedback. The optimal control law considered is LQR/LTR scheme. In the classical control law, the gains are arbitrarily chosen, whereas in the optimal control law, an optimal control gain is obtained, which minimizes a performance index.

3.1 Constant-Gain Negative Velocity Feedback Control

In this method of control, the sensor voltage is differentiated so that strain rate (related to the velocity) information is obtained and the control voltage is given by

$$V^a = -G\dot{V}^s = -GB_s\dot{q}, \tag{11}$$

where *G* is an (*m*×*m*) feedback gain matrix. The resulting S/As control law for the system equation is expressed as

$$M\ddot{q} + B_aGB_s\dot{q} + Kq = B_f f \tag{12}$$

Using the sensor equations and the proposed control, Equation (6.10a) can be expressed in the corresponding closed-loop state space form as

$$\dot{x} = A_c x + Df, \tag{13}$$

where $A_c = \begin{bmatrix} 0_{nxn} & I_{nxn} \\ -M^{-1}K & -M^{-1}B_aGB_s \end{bmatrix}$

The velocity feedback can enhance the system damping and therefore effectively control the oscillation amplitude.

3.2 Linear Quadratic Gaussian Optimal Control

In modern control theory, gains system controller is selected depending upon the requirements of control system designer represented by controller cost and speed dynamic response.

Considering the process and measurement noise □$_1$(t) and □$_2$(t), the state space equation of the system can be written as

$$\dot{x} = Ax + Bv^a + \varepsilon_1 \tag{14a}$$

$$y = Cx + \varepsilon_2 \tag{14b}$$

where □$_1$ and □$_2$ are zero-mean Gaussian white noise, they can be expressed as:

$$E[\varepsilon_1(t)] = 0, \qquad E[\varepsilon_2(t)] = 0 \tag{15a}$$

$$E[\varepsilon_1(t)\varepsilon_1^T(\tau)] = Q_e\delta(t-\tau) \tag{15b}$$

$$E[\varepsilon_2(t)\varepsilon_2^T(\tau)] = R_e\delta(t-\tau) \tag{15c}$$

A spatial LQG controller attempts to minimize the vibration of the flexible satellite by minimizing a performance index (cost function) that relates to the spatial behavior of the composite system. The LQG controller consists of a state feedback control law designed by solving a linear quadratic regulator problem plus a Kalman filter (an estimator).

The control v^a is given by

$$v^a = -G_c\hat{x}, \tag{16}$$

where $\hat{x}$ is an estimated state which is introduced to estimate the full state vector *x* from the sensor output y and G_c is the control gain.

The optimal control gain G_c is determined by the minimization of the quadratic performance index given below

$$J = \int_0^\infty (x^TQ\,x + v^{a^T}Rv^a)dt \tag{17}$$

ISBN: 978-1-61804-177-7

where R is a positive definite matrix and Q is a positive semidefinite matrix. In this case, larger (relatively) elements in Q mean that more vibration suppression ability is demanded from the controller. The purpose of the second term in Eqn (11) is to account for the effort being expended by the control system, so that small reductions in the output response are not obtained at the expense of physically unreasonable actuator input levels.

With an appropriate selection of the weighting matrices, the optimal control gain is given by

$$G_c = -R^{-1}B^T P, \tag{18}$$

where P is the symmetric positive definite solution of the Riccati equation

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{19}$$

A state estimator (an observer) for equation (10) is assumed to have the form

$$\dot{\hat{x}} = A\hat{x} + Bv^a + \hat{G}(y - C\hat{x}), \tag{20}$$

where $\hat{G}$ is the Kalman filter gain matrix and can be determined by minimizing the expected value, $E[(x-\hat{x})^T(x-\hat{x})]$. For steady state case, the optimal observer gain matrix is given by

$$\hat{G} = SC^T R_e^{-1}, \tag{21}$$

where the matrix S satisfying the Riccati equation

$$AS + SA^T - SC^T R_e^{-1} CS + Q_e = 0 \tag{22}$$

4 Simulation Results

In the simulation, the first five vibration modes are considered to describe the satellite dynamics, and the system has the following properties.

Beam properties

L= 2 m, E_b =71 GPa, h_b = 0.01 m, b= 0.2 m, $\rho_b = 2770\ kg/m^3$

Piezoelectric actuator properties

E_p= 63 GPa, h_a = 0.001 m, l_a= 0.16 m,

d_{31a} = -1.66x10^{-10} m/V

Piezoelectric sensor properties

E_s= 63 GPa, h_s= 0.001 m, l_s= 0.16 m, g_{31s}= 0.01 Vm/N

Two actuator/sensor pairs are set up at x_{11}=0.4 m and x_{21}=1.4 m.

An applied sinusoidal force, f_o, of 5 N acting on the satellite mass at a frequency of 290.5 rad/s, which is near the third resonance frequency of the system, is assumed. The state weighting matrix Q and the control weighting matrix R were selected as $Q=Q_aI$, $R=R_aI$, where I is the unit matrix and the values of Q_a and R_a were determined by trial-and-error method to most effectively control the flexible appendages. In the case considered, $Q_a/R_a= 10^8$. The intensities of the assumed white noise Q_e and R_e are selected to be

$$Q_e = 0.001\begin{bmatrix} 0_{nxn} & 0_{nxn} \\ 0_{nxn} & I_{nxn} \end{bmatrix},\ R_e = 10^{-7} I_{2x2}$$

In the present work, MATLAB software has been used for solving the associated Riccati equations and obtaining the control gains in LQG control method.

Fig. 2 shows the response of the satellite structure for both controlled and uncontrolled systems. The vertical axis represents the normalized deflection w/w_o where $w_o= f_o\ L^3/E_bI_b$. The effectiveness of the active control strategy, based on LQG control method, in controlling the response of the flexible appendages subjected to harmonic load is demonstrated in Fig.2, which shows a significant reduction in the amplitude response.

The control performances with constant-gain negative velocity feedback with gain G = 2 together with LQG are shown in Fig. 3. It can be seen from the figure that the velocity feedback can enhance the system damping and therefore effectively control the amplitude response. It can be noted that LQG optimal control offers more effective control response compared with constant-gain velocity feedback.

5 Summary and Conclusions

In this work, the active vibration control of a satellite with flexible appendages, modelled as a free-free beam, and two sets of piezoelectric actuator and sensor layers bonded to the top and bottom faces of the beam has been studied. The active vibration control performance has been studied using a classical control law, constant-gain negative velocity feedback, and optimal control law based on LQG theory.

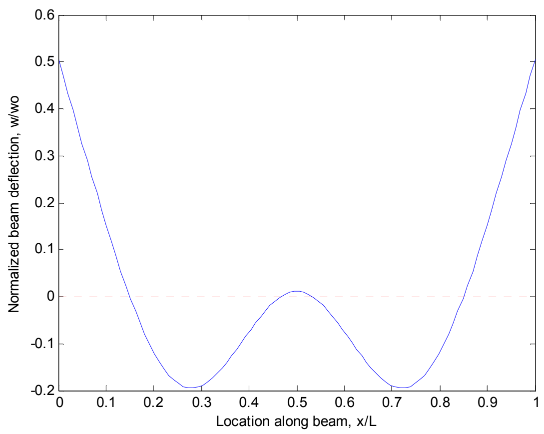

Fig. 2 Steady state displacement response for controlled (-------) and uncontrolled (——) beam

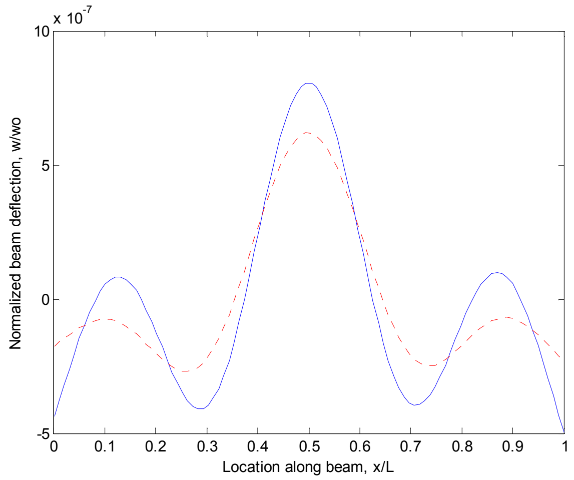

Fig. 3 Steady state displacement response for controlled beam using LQG (--------) and constant-gain negative velocity feedback (——)

From the results it can be noted that both control strategies significantly reduce the vibration response of the beam. The LQG optimal control is more effective than the velocity feedback due to the fact that the former changes both damping and natural frequencies of the system while the later changes only the system damping.

References:

[1] E. Shimon, Y. Richer, Y. Hurmuzlu, Theoretical and experimental study of efficient control of vibrations in a clamped square plate, *Journal of Sound and Vibration*, Vol. 282, 2005, pp. 453–473.

[2] Z. Qiu, J. Han, Z. Zhang, Y. Wang, Z. Wu, Active vibration control of a flexible beam using a non-collocated acceleration sensor and piezoelectric patch actuator, *Journal of Sound and Vibration*, Vol. 326, 2009, pp. 438–455.

[3] J.L. Fanson, T.K. Caughey, Positive position feedback control for large structure, *AIAA J*, Vol. 4, 1990, pp. 717–724.

[4] G.S. Aglietti, *Active control of microvibrations for equipment loaded spacecraft panels*, PhD thesis, School of Engineering Sciences, University of Southampton, UK, 1999.

[5] J. Stoustrup, G. S. Aglietti, E. Rogers, R.S. Langley, and S. B. Gabriel, S. B, H_∞ controllers for the rejection of microvibrations", *Proceedings European Control Conference (ECC99)*, 1999.

[6] V.S. Rao, S. Sana, An overview of control design methods for smart structural system, SPIE 4326, 2001, pp. 1–13.

[7] C. Charbonnel, H∞ and LMI attitude control design: towards performances and robustness enhancement, *Acta Astronautica*, Vol. 54, 2004, pp. 307 – 314.

[8] G.B. Maganti and S.N. Singh, Simplified adaptive control of an orbiting flexible spacecraft, *Acta Astronautica*, Vol. 61, 2007, pp. 575 – 589.

[9] B. Wu, X. Cao, and Z. Li, Multi-objective output-feedback control for microsatellite attitude control: An LMI approach, *Acta Astronautica*, Vol. 64, 2009, pp. 1021–1031.

[10] T. Bailey, J.E. Hubbard, Distributed piezoelectric-polymer active vibration control of a cantilever beam, *Journal of Guidance Control and Dynamics*, Vol. 8, 1985, pp. 605–611.

[11] C.R. Fuller, S.J. Elliott, P.A. Nelson, *Active Control of Vibration*, Academic Press, San Diego, CA92101, 1996.

[12] R.L. Clark, W.R. Saunders, G. Gibbs, *Adaptive Structures Dynamics and Control*, Wiley, Inc., 1998.

[13] CQ. Chen, and YP. Shen, Optimal control of active structures with piezoelectric modal sensors and actuators, *Smart Materials and Structures*, Vol. 6, 1997, pp. 403-409.

[14] D. Kamesh, R. Pandiyan, A. Ghosal, Modeling, design and analysis of low frequency platform for attenuating micro-vibration in spacecraft, *Journal of Sound and Vibration*, Vol. 329, 2010, pp. 3431–3450.

[15] K. Alsaif, M. Foda, M. ElMadany, and A. Albedah, *Impact of micro vibrations on remote sensing satellites and mitigation techniques*, Project No. 08-SPA237-2, KACST, 2011.