Vibration of Gold Nano-Beam with Variable Thermal Conductivity Subjected to Sinusoidal Heating

H. M. Youssef, K. A. Elsibai, A. A. El-Bary

Abstract - The non-Fourier effect in heat conduction and the coupling effect between temperature and strain rate are the two significant effects in the nano-scale beam. In the present work, the solution of vibration of gold nano-beam resonator induced by sinusoidal heating is developed in the context of generalized thermoelasticity with variable thermal conductivity. Laplace transform methods have been used. The inverse Laplace transform has been calculated numerically and the numerical results have been presented graphically with some comparisons to stand on the effects of the variability of the thermal conductivity and the angular thermal frequency parameter.

Key-Words -Thermoelasticity; Euler–Bernoulli equation; gold nano-beam; sinusoidal heating; variable thermal conductivity

1. INTRODUCTION
Recently, many discussions have been made to find out the elastic properties of nanostructured materials by atomistic simulations. Diao et al. [1] studied the effect of free surfaces on the structure and elastic properties of gold nanowires by atomistic simulations. Although the atomistic simulation is a good way to calculate the elastic constants of nanostructured materials, it is only applicable to homogeneous nanostructured materials (e.g., nano-plates, nano-beams, nano-wires, etc.) with limited number of atoms. Moreover, it is difficult to obtain the elastic properties of the heterogeneous nanostructured materials using atomistic simulations.

For these and other reasons, it is prudent to seek a more practical approach. One such approach would be to extend the classical theory of elasticity down to the nano-scale by including in it the hitherto neglected surface/interface effect. For this it is necessary first to cast the latter within the framework of continuum elasticity. Kidawa [2] has studied the problem of transverse vibrations of a beam induced by a mobile heat source. The analytical solution to the problem was obtained using the Green’s functions method.

Boley [3] analyzed the vibrations of a simply supported rectangular beam subjected to a thermal shock input distributed along its span. Manolis and Beskos [4] discussed the thermally induced vibration of structures consisting of beams, subjected to rapid surface heating. Al-Huniti et al. [5] investigated the thermally induced displacements and stresses of a rod using the Laplace transformation technique. Ai Kah Soh et al. studied the vibration of micro/nano-scale beam resonators induced by ultra-short-pulsed laser by considering the thermoelastic coupling term in [6] and [7]. The propagation characteristics of the longitudinal wave in nano-plates with small scale effects are studied by Wang et al. [8].

Elsibai and Youssef used the state-space approach to vibration of gold nano-beam induced by ramp type heating without energy dissipation in femtosecond scale [9], and Youssef and Elsibai discussed the vibration of gold nano-beam induced by different types of thermal loading in [10].

The physical property of a solid body related to application of heat energy is defined as a thermal property. Thermal properties explain the response of a material to the application of heat and one of the most important thermal properties is the thermal conductivity. Thermal conductivity K is ability of a material to transport heat energy through it from high temperature region to low temperature region and it is a microstructure sensitive property its value range for metals 20-400, for ceramics 2-50 and for polymers order of 0.3. Heat is transported in two ways – electronic contribution, vibrational (phonon) contribution. In metals, electronic contribution is very high. Thus metals have higher thermal conductivities. It is same as electrical conduction. Both conductivities are related through Wiedemann-Franz law: \[ K = \sigma L T \] where L is Lorentz constant and \( \sigma \) is the electrical conductivity. As different contributions to conduction vary with temperature, the above relation is valid to a limited extension for many metals. With increase in temperature, both number of carrier electrons and contribution of lattice vibrations increase. Thus thermal conductivity of a metal is expected to increase. However, because of greater lattice vibrations, electron mobility decreases. The combined effect of these factors leads to very different behaviour for different metals. For example: thermal conductivity of gold nano-beams...
iron initially decreases then increases slightly; thermal conductivity decreases with increase in temperature for aluminium; while it increases for platinum and gold [11].

The question arises: what are the effects of these variations on the lateral vibration, the temperature, the displacement, the strain, the stress and the strain-stress energy distributions in nano-beam resonator?

Youssef used the state-space approach to solve a problem of generalized thermoelasticity for an infinite material with a spherical cavity and variable thermal conductivity subjected to ramp-type heating [11]. Youssef and El-Bary in [12]-[14] and Erzat and Youssef in [15] discussed the effect of variability of the thermal conductivity on all the studied fields. Youssef and Elsibai discussed the vibration of gold nano-beam induced by different types of thermal loading [16].

In the present work, the solution of vibration of gold nano-beam resonator induced by sinusoidal heating is developed in the context of generalized thermoelasticity with variable thermal conductivity. Laplace transform methods are used to determine the lateral vibration, the temperature, the displacement, the stress and the strain-stress energy. The effect of the variability of thermal conductivity has been studied and represented graphically with some comparisons.

2. FORMULATION OF THE PROBLEM

Creating a beam with a rectangular cross section is the easiest when compared to other cross sections. Consider small flexural deflections of a thin elastic beam of length $\ell (0 \leq x \leq \ell)$, width $b \left( \frac{b}{2} \leq y \leq \frac{b}{2} \right)$ and thickness $h \left( \frac{h}{2} \leq z \leq \frac{h}{2} \right)$ as in fig. 1, for which the x, y and z axes are defined along the longitudinal, width and thickness directions of the beam, respectively. In equilibrium, the beam is unstrained, unstressed, no damping mechanism present, and at temperature $T_0$ everywhere [16].

In the present study, the usual Euler–Bernoulli assumption [16] is adopted, i.e., any plane cross-section, initially perpendicular to the axis of the beam remains plane and perpendicular to the neutral surface during bending. Thus, the displacement components $(u, v, w(x, t))$ are given by

$$u(x, y, z, t) = -z \frac{\partial w(x, t)}{\partial x}, \quad v(x, y, z, t) = 0 \quad (1)$$

Hence, the differential equation of thermally induced lateral vibration of the beam may be expressed in the form [22]:

$$\frac{\partial^4 w}{\partial x^4} + \frac{\rho A}{E I} \frac{\partial^2 w}{\partial t^2} + \alpha_T \frac{\partial^2 M_T}{\partial x^2} = 0 \quad (2)$$

where $E$ is Young’s modulus, $I = \frac{bh^3}{12}$ the inertial moment about x-axis, $\rho$ the density of the beam, $\alpha_T$ the coefficient of linear thermal expansion, $w(x, t)$ the lateral deflection, $x$ the distance along the length of the beam, $A = bh$ is the cross section area and $t$ the time and $M_T$ is the thermal moment, which is defined as:

$$M_T(x, z, t) = \frac{12}{h^3} \int_{-h/2}^{h/2} \theta(x, z, t) z \, dz \quad (3)$$

where $\theta(x, z, t) = (T - T_0)$ is the dynamical temperature increment of the resonator, in which $T(x, z, t)$ is the temperature distribution and $T_0$ the environmental temperature.

According to Lord-Shulman model (L-S), the non-Fourier heat conduction equation has the following form [16]:

$$\frac{\partial}{\partial x} \left( K(\theta) \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial z} \left( K(\theta) \frac{\partial \theta}{\partial z} \right) = \left( 1 + \frac{\tau}{\tau_0} \right) \left( \rho C_E(\theta) \dot{\theta} + \beta T_0 \dot{\theta} \right) \dot{\theta} \quad (4)$$

where $\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ is the volumetric strain, $C_E(\theta)$ is the specific heat with variable temperature at constant volume, $\tau_0$ the thermal relaxation time, $K(\theta)$ the thermal conductivity with variable temperature, $\rho$ is the density, $\beta = \frac{E \alpha_T}{1 - 2\nu}$ in which $\nu$ is Poisson’s ratio and dot above it means partial derivative with respect to time.

We have the relation [11].
\( \rho C_v(\theta) = \frac{K(\theta)}{\kappa} \),
\( (5) \)
where \( \kappa \) is the thermal diffusivity.

Hence, equation (4) will take the form
\[
\frac{\partial}{\partial x} \left( K(\theta) \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial z} \left( K(\theta) \frac{\partial \theta}{\partial z} \right) = \left( 1 + \tau_o \frac{\partial}{\partial t} \right) \left( \frac{K(\theta)}{\kappa} \theta + \beta T_0 e \right).
\( (6) \)

Considering the following mapping [11]-[15]:
\[
\theta = \frac{1}{K_o} \int_0^\theta K(\xi) \, d\xi,
\( (7) \)
where \( K_o \) is the thermal conductivity at the normal case.

Differentiating equation (7) with respect to the coordinates \( x \) and \( z \) respectively, we get
\[
K(\theta) \frac{\partial \theta}{\partial x} = K_o \frac{\partial \theta}{\partial x} \quad \text{and} \quad K(\theta) \frac{\partial \theta}{\partial z} = K_o \frac{\partial \theta}{\partial z}.
\( (8) \)

Differentiating the above equations with respect to the coordinates \( x \) and \( z \) respectively, we obtain
\[
K_o \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial}{\partial x} \left[ K(\theta) \frac{\partial \theta}{\partial x} \right]
\]
and
\[
K_o \frac{\partial^2 \theta}{\partial z^2} = \frac{\partial}{\partial z} \left[ K(\theta) \frac{\partial \theta}{\partial z} \right].
\( (9) \)

Differentiating equation (7) with respect to time, we get
\[
K_o \dot{\theta} = K(\theta) \dot{\theta}.
\( (10) \)

Applying the equations (8)-(10) in equation (6), we get
\[
\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2} = \left( \frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2} \right) \left( \frac{1}{\kappa} \theta + \beta T_0 e \right),
\( (11) \)
where there is no heat flow across the upper and lower surfaces of the beam, so that \( \frac{\partial \theta}{\partial z} = 0 \) at \( z = \pm h/2 \).

For a very thin beam and assuming the temperature varies in terms of a \( \sin(pz) \) function along the thickness direction, where \( p = \pi/h \), gives [16]:
\[
\theta(x, z, t) = \theta_1(x, t) \sin(pz).
\( (12) \)

Hence, equation (2) gives
\[
\frac{\partial^2 w}{\partial x^2} + \frac{\rho A}{EI} \frac{\partial^2 w}{\partial t^2} + \frac{12\alpha_T}{h^3} \frac{\partial^2 \theta_1}{\partial x^2} \int_{-h/2}^{h/2} z \sin(pz) \, dz = 0.
\( (13) \)

To get equation (14), we used the linearity condition of thermoelasticity such that \( \frac{\theta}{T_0} \ll 1 \), which gives
\[
\frac{\partial^2 \theta_1}{\partial x^2} = p^2 \frac{\partial^2 \theta_1}{\partial x^2}.
\( (14) \)

Equation (11) takes the following form
\[
\frac{\partial^2 \theta_1}{\partial x^2} \sin(pz) - p^2 \frac{\partial^2 \theta_1}{\partial x^2} = \left( \frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2} \right) \left( \frac{1}{\kappa} \frac{\theta_1}{K_o} \sin(pz) - \frac{\beta T_0}{K_o} \frac{\partial^2 w}{\partial x^2} \right).
\( (15) \)

After doing the integrations, equation (13) gives
\[
\frac{\partial^4 w}{\partial x^2} + \frac{\rho A}{EI} \frac{\partial^2 w}{\partial t^2} + \frac{24\alpha_T}{h^3} \frac{\partial^2 \theta_1}{\partial x^2} = 0.
\( (16) \)

In equation (15), we multiply the both sides by \( z \) and integrate with respect to \( z \) from \( -h/2 \) to \( h/2 \), then we obtain
\[
\left( \frac{\partial^2 \theta_1}{\partial x^2} - p^2 \frac{\partial^2 \theta_1}{\partial x^2} \right) = \left( \frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2} \right) \left( \epsilon \frac{\partial^2 \theta_1}{\partial x^2} - \frac{\beta T_0}{K_o} \frac{\partial^2 w}{24 K_o} \right),
\( (17) \)
where \( \epsilon = \frac{1}{\kappa} \).

Now, for simplicity we will use the following non-dimensional variables [14]:
\[
(x', \, w', \, h') = \epsilon c_o (x, \, w, \, h), \quad (t', \, \tau_o') = \epsilon c_o^2 (t, \, \tau_o).
\( (18) \)

Hence, we have
\[ \frac{\partial^4 w}{\partial x^4} + A_1 \frac{\partial^2 w}{\partial t^2} + A_2 \frac{\partial^2 \theta}{\partial x^2} = 0 , \]  \tag{19} 

and

\[ \frac{\partial^2 \theta}{\partial x^2} + A_3 \theta = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \theta - A_4 \frac{\partial^2 w}{\partial x^2} \right) , \]  \tag{20} 

where

\[ A_1 = \frac{12}{\pi^2}, \quad A_2 = \frac{24\alpha T}{\pi^2 h}, \quad A_3 = p^2, \quad A_4 = \frac{\pi^2 \beta h}{24K} . \]

We have dropped the prime for convenience.

3. FORMULATION IN THE LAPLACE TRANSFORM DOMAIN

Applying the Laplace transform for equations (19) and (20) which is defined by the formula

\[ F(s) = \int_0^\infty f(t)e^{-st} \, dt . \]

Hence, for the zero initial conditions of all the states functions, we obtain the following system

\[ \frac{d^4 \bar{w}}{dx^4} + A_1 s^2 \bar{w} + A_2 \frac{d^2 \bar{\theta}}{dx^2} = 0 , \]  \tag{21} 

and

\[ \frac{d^2 \bar{\theta}}{dx^2} - \left( A_3 + (s + \tau_0 s^2) \right) \bar{\theta} + A_4 \left( s + \tau_0 s^2 \right) \frac{d^2 \bar{w}}{dx^2} = 0 . \]  \tag{22} 

Now, we will consider the first end of the nano-beams \( x=0 \) is clamped and loaded thermally, which gives [16]:

\[ w(0,t) = \eta(0,t) = 0 , \]  \tag{23} 

and

\[ \theta(0,t) = f(t) . \]  \tag{24} 

Substituting from equation (24) into the mapping in (7), we obtain

\[ \bar{G}_i(0,t) = -\frac{1}{K} \int_0^{\xi(t)} K(\xi) \, d\xi . \]  \tag{25} 

After using Laplace transform, the above conditions take the forms

\[ \bar{w}(0,s) = \bar{\eta}(0,s) = 0 , \]  \tag{26} 

and

\[ \bar{G}_i(0,s) = G(s) . \]  \tag{27} 

where

\[ G(s) = \frac{1}{K} \int_0^\infty \left[ \int_0^{\xi(t)} K(\xi) \, d\xi \right] e^{-st} \, dt . \]  \tag{28} 

The other end of the beam has been considered to be clamped and at zero increment of temperature as follows:

\[ w(\ell,t) = \theta(\ell,t) = \eta(\ell,t) = 0 . \]  \tag{29} 

After using Laplace transform, we have

\[ \bar{w}(\ell,s) = \bar{\eta}(\ell,s) = \bar{\eta}(\ell,s) = 0 . \]  \tag{30} 

After some simplifications by using MAPLE software, we get the final solutions in the Laplace transform domain as follow:

\[ \bar{w}(x,s) = -\frac{\Delta \sinh \left( k_i \left( \ell - x \right) \right)}{\left( k_i^2 - k_{i-1}^2 \right) \left( k_i^2 - k_{i+1}^2 \right) \sinh(k_i\ell)} + \frac{\Delta \sinh \left( k_{i-1} \left( \ell - x \right) \right)}{\left( k_i^2 - k_{i-1}^2 \right) \left( k_i^2 - k_{i-2}^2 \right) \sinh(k_{i-1}\ell)} \]  \tag{31} 

and

\[ \bar{G}(z,x,s) = -\frac{\alpha \sinh(k_i\ell)}{\left( k_i^2 - k_{i-1}^2 \right) \left( k_i^2 - k_{i+1}^2 \right) \sinh(k_i\ell)} \left( k_i^2 - k_i^2 \right) \left( k_i^2 - k_{i+2}^2 \right) \sinh(k_{i+1}\ell) \] \tag{32} 

\[ -\frac{\alpha \sinh(k_i\ell)}{\left( k_i^2 - k_{i-1}^2 \right) \left( k_i^2 - k_{i+1}^2 \right) \sinh(k_{i-1}\ell)} \left( k_i^2 - k_i^2 \right) \left( k_i^2 - k_{i-2}^2 \right) \sinh(k_{i-1}\ell) . \]
\[ u(z, x, s) = -\frac{z\Delta k_1 \cosh(k_1(\ell - x))}{(k_1^2 - k_2^2)(k_2^2 - k_3^2)\sinh(k_1\ell)} \]
\[ -\frac{z\Delta k_2 \cosh(k_2(\ell - x))}{(k_2^2 - k_1^2)(k_3^2 - k_1^2)\sinh(k_2\ell)} \]
\[ -\frac{z\Delta k_3 \cosh(k_3(\ell - x))}{(k_3^2 - k_1^2)(k_3^2 - k_2^2)\sinh(k_3\ell)} \]
\[ (33) \]

\[ \bar{v}(z, x, s) = \frac{z\Delta k_1^2 \sinh(k_1(\ell - x))}{(k_1^2 - k_2^2)(k_1^2 - k_3^2)\sinh(k_1\ell)} \]
\[ +\frac{z\Delta k_2^2 \sinh(k_2(\ell - x))}{(k_2^2 - k_1^2)(k_2^2 - k_3^2)\sinh(k_2\ell)} \]
\[ +\frac{z\Delta k_3^2 \sinh(k_3(\ell - x))}{(k_3^2 - k_1^2)(k_3^2 - k_2^2)\sinh(k_3\ell)} \]
\[ (34) \]

where \( k_1, k_2, \) and \( k_3 \) are the roots of the following characteristic equation
\[ k^6 - 4k + mk^2 - n = 0, \]
\[ (35) \]

where \( l = \alpha_1 + \alpha_5, m = \alpha_1\alpha_5 - \alpha_2\alpha_4 + \alpha_3, n = \alpha_1\alpha_3, \]
\[ \alpha_1 = (A_1 + s + \tau_o s^2), \alpha_2 = A_2(s + \tau_o s^2), \alpha_3 = A_3s^2, \]
\[ \alpha_4 = A_2(A_1 + s + \tau_o s^2), \alpha_5 = A_3A_4(s + \tau_o s^2) \] and
\[ \Delta = \frac{G(s)}{\alpha_1\alpha_2}((\alpha_1 - k_1^2)(\alpha_1 - k_2^2)(\alpha_1 - k_3^2)). \]

To determine the function \( K(\theta) \), we will consider the thermal conductivity depends on the temperature with linear function in the form
\[ K(\theta) = K_0(1 + K_\theta \theta), \]
\[ (36) \]
and for the gold, we have \( K_\theta \geq 0 \) [17].

By using the mapping in (7), we get
\[ \theta_1 = \theta + \frac{K_{\theta}}{2} \theta^2. \]
\[ (37) \]

Hence, we have
\[ \theta_1(t, 0, t) = f(t) + \frac{K_{\theta}}{2} \left( f(t) \right)^2. \]
\[ (38) \]

For sinusoidal heating, we will consider
\[ f(t) = \theta_0 \sin(\omega t), \]
\[ (39) \]

where \( \theta_0 \) is amplitude and \( \omega \) is the angular thermal frequency.

\[ \theta_1(0, t) = \theta_0 \sin(\omega t) + \frac{K_{\theta}}{2} \left( \theta_0 \sin(\omega t) \right)^2. \]
\[ (40) \]

Then, we get
\[ G(s) = \frac{\theta_0\omega}{s^2 + \omega^2} + \frac{K_{\theta}\theta_0^2\omega^2}{s(s^2 + 4\omega^2)} . \]
\[ (41) \]

After obtaining \( \theta \), the temperature increment \( \theta \) can be obtained by solving equation (37) to get
\[ \theta = -1 + \frac{1}{K_{\theta}} \frac{\sqrt{1 - K_{\theta} \theta}}{K_\theta} \text{ where } (1 + 2K_{\theta} \theta) > 0 . \]
\[ (42) \]

4. THE STRAIN-STRESS ENERGY

The stress on the x-axis, according to Hooke’s law is:
\[ \sigma_{xx}(x, z, t) = E \left( e - \alpha_1 \theta \right). \]
\[ (43) \]

By using the non-dimensional variables in (18), we obtain the stress in the form
\[ \sigma_{xx}(x, z, t) = e - \alpha_1 \sigma_{xx} \theta. \]
\[ (44) \]

After using Laplace transform, the above equation takes the form:
\[ \bar{\sigma}_{xx}(x, z, s) = \bar{e} - \alpha_1 \bar{\sigma}_{xx} \theta. \]
\[ (45) \]

The strain energy which is generated on the beam is given by
\[ W(x, z, t) = \sum_{i,j=1}^{3} \frac{1}{2} \sigma_{ij} e_{ij} = \frac{1}{2} \sigma_{xx} e_{xx} = -\frac{1}{2} z \sigma_{xx} \eta . \]
\[ (46) \]

or, we can write it in the form
\[ W(x, z, t) = -\frac{1}{2} z \left[ L^{-1}(\bar{\sigma}_{xx}) \right] \left[ L^{-1}(\bar{\eta}) \right] , \]
\[ (46) \]

where \( L^{-1}[\bullet] \) is the inversion of Laplace transform.
5. NUMERICAL INVERSION OF THE LAPLACE TRANSFORM

In order to determine the solutions in the time domain, the Riemann-sum approximation method is used to obtain the numerical results. In this method, any function in Laplace domain can be inverted to the time domain as

\[ f(t) = \frac{\xi}{t} \left[ \frac{1}{2} \mathcal{L}^{-1}(\xi) + \text{Re} \sum_{n=1}^{N} (-i)^{n} \mathcal{L}^{-1} \left( \xi + i \frac{\pi n}{t} \right) \right] \]

where \( \text{Re} \) is the real part and \( i \) is imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of \( \xi \) satisfies the relation \( \xi t \approx 4.7 \) Tzou [17].

6. NUMERICAL RESULTS AND DISCUSSION

Now, we will consider a numerical example for which computational results are given. For this purpose, gold (Au) is taken as the thermoelastic material for which we take the following values of the different physical constants [16]:

\[ K_{o} = 318 \text{ W/(mK)}, \alpha_{T} = 14.2 \times 10^{-6} \text{ K}^{-1}, \]
\[ T_{o} = 293 \text{ K}, \rho = 1930 \text{ kg/m}^3, C_{p} = 130 \text{ J/(kgK)}, \]
\[ E = 180 \text{ GPa}, \nu = 0.44. \]

The aspect ratios of the beam are fixed as \( \ell/h = 10 \) and \( b/h = 1/2 \), when \( h \) is varied \( \ell \) and \( b \) change accordingly with \( h \).

For the nano-scale beam, we will take the range of the beam length \( \ell \left(1 - 100\right) \times 10^{-9} \text{ m} \), the original time \( t \) will be considered in the picoseconds \( (1 - 100) \times 10^{-12} \text{ sec} \) and the relaxation time \( \tau_{o} \) in the range \( (1 - 100) \times 10^{-14} \text{ sec} \).

The figures were prepared by using the non-dimensional variables which are defined in (18) for beam length \( \ell = 1.0 \), \( \theta_{o} = 1.0 \), \( z = h/6 \), \( t = 0.1 \) and \( K_{i} = 0.0, 0.2 \) and 0.5.

Figures 2-6 present the temperature, the lateral vibration, the stress, the displacement, and the strain-stress energy distribution when \( \omega = 10 \) with range of the coordinate \( x \left(0.0 \leq x \leq 0.5\right) \) with different values of the angular thermal frequency to stand on the behaviour of the wave propagation of all the studied fields. The figures show that, the variability of the thermal conductivity plays a vital role on the speed of the wave propagation, and increasing of the parameter \( K_{i} \) leads to increasing on all the state functions distributions. The damping of the strain-stress energy increase when the parameter \( K_{i} \) increases. Also, when \( K_{i} = 0.0 \) all the results are coincide with the classical results [16].

The figures 7-11, present the temperature, the lateral vibration, the stress, the displacement and the strain-stress energy distribution when \( K_{i} = 0.2 \) with wide range of the coordinate \( x \left(0.0 \leq x \leq 0.5\right) \) with different values of the angular thermal frequency \( \omega = (10, 20, 30) \) to stand on the behaviour of the wave propagation of all the studied fields. We deduced that, the peak points of the temperature, the lateral vibration and the displacement the stress and the strain-stress energy increase when the angular thermal frequency increases. The damping of the strain-stress energy increase when the value of the angular thermal frequency decreases.

7. CONCLUSION

1. The thermal conductivity has significant effects on the speed of the wave propagation of all the studied fields.
2. Thermal conductivity depends on the temperature with linear function in the form \( K(\theta) = K_{o}(1 + K_{i}\theta) \) where for gold \( K_{i} \geq 0 \).
3. The effect of the variability of the thermal conductivity decreasing when the length of the beam increasing.
4. The maximum value of the strain-stress energy increasing when the changing in the thermal conductivity with respect to the temperature increasing.
5. The changing in the thermal conductivity with respect to the temperature effects on the damping of the strain-stress energy of the beam.
6. The angular thermal frequency parameter of the sinusoidal heating effects on all the studied fields and the damping of the stress-strain energy increasing when this parameter decreasing.
7.

8. Acknowledgments

The authors are grateful for the supports for this work provided by the “Long-Term Comprehensive National Plan for Science, Technology and Innovation” through STU - Umm Al-Qura University by grant number 10-MAT1243-10.

9. REFERENCES


Fig. 4: The displacement distribution with variable thermal conductivity

Fig. 5: The stress distribution with variable thermal conductivity

Fig. 6: The stress-strain energy distribution with variable thermal conductivity

Fig. 7: The temperature distribution with variable angular thermal frequency

Fig. 8: The lateral vibration distribution with variable angular thermal frequency

Fig. 9: The displacement distribution with variable angular thermal frequency

Fig. 10: The stress distribution with variable angular thermal frequency
Fig. 11: The stress-strain energy distribution with variable angular thermal frequency