Viscous Free Surface Numerical Simulations of Oscillating SWATH Ship Sections

LUCA BONFIGLIO\textsuperscript{1,2}, STEFANO BRIZZOLARA\textsuperscript{1,2}, CHRYSSOSTOMOS CHRYSSOSTOMIDIS\textsuperscript{2}  
bonfi@mit.edu, stebriz@mit.edu, chrys@mit.edu  
\textsuperscript{1} University of Genoa  
Dept. of Naval and Electrical Engineering  
Via Montallegro 1, 16145 Genoa  
ITALY  
\textsuperscript{2} MIT Massachusset Institute of Technology  
Design Laboratory, MIT Sea Grant College Program  
292 Main Street 02139 Cambridge (MA)  
USA

This study presents a solution method based on a fully viscous non-linear flow solver used to evaluate the two dimensional hydrodynamic coefficients of damping and added mass appearing in the generalized linear equations of motion of a ship in a seaway. Experiments made on a circular and a SWATH section were used to validate the direct numerical solver of Navier Stokes equations, based on finite volume technique which is implemented using the open-source libraries of OpenFOAM. Numerical results agree very well with experimental measurements for these two quite different section shapes, validating the satisfactory accuracy of the proposed method.

Key–Words: Unsteady viscous solvers, free surface, OpenFOAM, SWATH

1 Introduction

Hydrodynamics design of a ship hull is normally based on the determination of forces induced on the hull which advances in a non realistic calm sea. Most ships require to be operative even in extremely rough seas; for those kind of vessels seakeeping studies allow the comparison among different hull geometries on the basis of the their motions in waves, with the aim to extend the operational capability in the highest sea states. The assumption that the greatest part of energy is dissipated through waves generation, makes the potential flow theory a valid hypothesis for the development of seakeeping predictions [1]. For particular ship hulls, such as SWATH or Semi-SWATH typologies, the energy loss due to viscous effects is absolutely non-negligible in comparison with the wave generation one. Assuming valid the superimposition principle, the solution of the problem can be achieved splitting it into different parts, and recovering the viscous effects only where the eddies generation affects in a non negligible way the results of the calculation, in order to develop a method which can be suitable in an early stage of design, and maintaining the computational efforts as low as possible. The present study is developed through a time domain model, based on frequency data obtained by a linear approach. The aim is to define the radiation forces exploiting the behavior of a body forced to oscillate in calm water. The motion of a ship as a rigid body and the external forces acting on the hull can be described with the following system of six equations in six unknowns:

\[
\ddot{\vec{\eta}}_j(t) = \sum_{k=1}^{6} M_{jk}\dot{\eta}_k \quad j = 1, 2, \ldots, 6 \quad (1)
\]

With k as the motions index and j as forces index.

Non-linear effects are important in severe sea states however an incident regular wave of amplitude $\zeta_a$, far from breaking, can be considered in a linear theory approach. In such a way the wave induced motions can be well described considering them linearly proportional to $\zeta_a$.

The hydrodynamic problem is normally dealt splitting it into two sub-problems:

1. The forces and the moments on the body when the ship is restrained from oscillating and subjected to incident regular waves. The hydrodynamic loads are called wave excitation loads; they can be splitted in the Froude-Kriloff and the diffraction forces and moments.

2. The forces and moments on the body when the structure is forced to oscillate with the wave excitation frequency in any rigid-body motion mode. There are no incident waves. The hydrodynamic loads are identified as added mass, damping and restoring terms.
Due to linearity the forces obtained in 1 and in 2 can be added to give the total hydrodynamic forces [2]. Using the hypothesis of small amplitude motions, the wave exciting forces ($\mathbf{F}_{Ej}$) can be expressed in terms of radiation and restoring forces as follow:

$$\mathbf{F}_{Ej} = \sum_{k=1}^{6} (M_{jk}\dot{\eta}_k) - \mathbf{F}_{RADj}(t) - \mathbf{F}_{RESj}(t) \quad (2)$$

where:

$$\mathbf{F}_{RADj} = - \sum_{k=1}^{6} (A_{jk}\dot{\eta}_k + B_{jk}\ddot{\eta}_k) \quad (3)$$

With term $-A_{jk}\dot{\eta}_k$ ($A_{jk}$ added mass matrix) are indicated hydrodynamic forces in phase with accelerations and with $-B_{jk}\ddot{\eta}_k$ ($B_{jk}$ damping matrix) the one in phases with velocity.

$$\mathbf{F}_{RESj} = - \sum_{k=1}^{6} C_{jk}\eta_k \quad (4)$$

With $-C_{jk}\eta_k$ ($C_{jk}$ restoring matrix) are indicated the hydrostatic forces.

In this way the equation (1) becomes:

$$\mathbf{F}_{Ej}(t) = \sum_{k=1}^{6} [(M_{jk} + A_{jk})\dot{\eta}_k + B_{jk}\ddot{\eta}_k + C_{jk}\eta_k] \quad (5)$$

Considering only two dimensions, it is possible to study motions of transverse sections which are characterized by three degree of freedom: sway, heave and roll.

Assuming sinusoidal motion and response, it is possible to change from time domain to frequency domain and using Euler’s formula for complex notation, the radiation forces can be described by the following relation:

$$F_{RADj} = \sum_{k=1}^{6} \xi_k [-\omega^2 a_{jk} + i \omega b_{jk}] \quad (6)$$

In the case of heave motion:

$$F_{RAD3} = \xi_3 [-\omega^2 a_{33} + i \omega b_{33}] \quad (7)$$

2 Numerical methods

Imposing an harmonic law of motion to a transverse section, the two dimensional restoring forces can be found by the integration over the section of the hydrodynamic pressure and the tangential strain fields which the incompressible viscous fluid exerts on the hull. Pressure and velocity fields can be described writing the conservation principles for the mass and the momentum:

$$\frac{dm}{dt} = 0 \quad \frac{d(mv)}{dt} = \sum \mathbf{f} \quad (8)$$

Expressing the mass as an extensive property, its rate of change can be expressed as the sum of the rate of change of the mass in the original control volume and the net flux of it through the boundaries (convective term); applying Gauss divergence theorem to the convective term it is possible to express the surface integral as a volume integral; considering an infinitesimal small control volume, the conservation law can be expressed as follow:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \quad (9)$$

Momentum associated with a given mass can be changed through the action of forces on the control mass. Expressing the momentum in terms of an extensive property and considering that the forces which can act on the fluid can be exerted on the surface of the control mass (surface forces) or throughout the volume of the control mass (body forces), the momentum conservation can be expressed by the following relation:

$$\frac{\partial}{\partial t} \int_{\Omega} \rho v d\Omega + \int_{S} \rho \mathbf{v} \cdot \mathbf{n} dS = \int_{S} \mathbf{T} \cdot \mathbf{n} ds + \int_{\Omega} \rho d\Omega \quad (10)$$

In the case of subsonic flow, the density may be assumed constant; in such a way the mass and the momentum conservation can be written as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (11)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} (\frac{\partial u_i}{\partial x_j}) - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + g_i \quad (12)$$

The integral form of conservation equations is at the basis of the Finite Volume technique. The computational domain is subdivided into a finite number of control volumes (cells) and the conservation equations can be applied to each cell as well as to the solution domain as a whole.

Considering a generic scalar quantity $\phi$, it is possible to define the integral form of the equation for the conservation of $\phi$ as:

$$\frac{\partial}{\partial t} \int_{S} \rho \phi d\Omega + \int_{S} \rho \phi \mathbf{v} \cdot \mathbf{n} dS = \sum f_{\phi} \quad (13)$$

Where $f_{\phi}$ represents the transport of the quantity $\phi$ by diffusive mechanism:

$$f_{\phi}^{\text{diff}} = \int_{S} \Gamma \nabla \phi \cdot \mathbf{n} dS \quad (14)$$
Where $\Gamma$ is the diffusivity for $\phi$.
Including also the effect of sources or sinks ($q_\phi$):
\[
\frac{\partial}{\partial \tau} \int_S \rho \phi d\Omega + \int_S \rho \phi \mathbf{v} \cdot n dS = \int_S \Gamma \nabla \phi \cdot n dS + \int_{\Omega} q_\phi d\Omega \tag{15}
\]
This equation applies to each cell and the sum of all the equations for all values gives the global conservation equation since surface integrals over inner faces cancel out.

To obtain an algebraic equation for a particular cell, the surface and volume integrals need to be approximated using quadrature formulas. The net flux through the cells boundary is the sum of integrals over the control volume faces:
\[
\int_S f dS = \sum_k \int_{S_k} f dS \tag{16}
\]
Where $f$ is generally representative of convective ($\rho \mathbf{v} \cdot \mathbf{n}$) or the diffusive ($\Gamma \nabla \phi \cdot \mathbf{n}$) flux vector in the direction normal to control volume face.

To calculate the surface integral in (16) it is necessary to know the integrand $f$ everywhere on the surface $S_k$; this information is not available, as the only known values are the nodal ones, so approximations must be introduced: first of all the integral is approximated in terms of the variable values at one or more locations on the cell face and then the cell-face values are approximated interpolating between the nodal values of the cells which share the face.

If the midpoint rule is applied:
\[
F_e = \int_{S_e} f dS = \overline{f_e} S_e \approx f_e S_e \tag{17}
\]
Using a linear interpolation (Central Differential Scheme), an expression for the value of the quantity $f$ at the cell face $e$ can be obtained using the values of two nearest nodes ($f_E$ and $f_P$).
\[
f_e = f_E \frac{x_e - x_P}{x_E - x_P} + f_P (1 - \frac{x_e - x_P}{x_E - x_P}) \tag{18}
\]
In the same way the volume integrals can be expressed through the product of the mean value of the integrand in the volume and the volume of the cell itself.

If the mean value it is approximated as the value of the integrand in the only point in which it is known:
\[
Q_P = \int_{\Omega} q d\Omega = \overline{q} \Delta \Omega \approx q_P \Delta \Omega \tag{19}
\]
where $q_P$ stands for the value of $q$ at the cell center; since all the variables are known at the centroid, no interpolation is required.

By summing all the flux approximations and source terms, it is possible to produce an algebraic equation: the number of equations and unknowns are both equal to the number of cells and the matrix of the system is well conditioned [3].

3 Results
The computational effort required to figure out the solution of the hydrodynamic problem grows with the increase of the number of cells used in the discretization. To discretize the computational domain a body fitted structured mesh will be used for the regions where the viscous effects are supposed to be more important: this includes the wavy free surface and the viscous boundary layer near the body. For regions other than the ones previously defined, a triangular unstructured mesh is adopted. Mesh morphing is used to describe the oscillatory motion of the body.

Vugt’s experiments made on a circular section forced to oscillate with a sinusoidal law of motion were used to validate the study [4]. The radiation force signals due to the sinusoidal oscillation are evaluated using OpenFOAM libraries; analyzing signals through the Fourier transform it is possible to pass from time domain to frequency domain, in which an amplitude and a phase angle between the force and the motion for each frequency are able to describe the forces. Multiplying the amplitude for the cosine and the sinus of the phase angle it is possible to obtain respectively added mass and damping coefficient, considering (6) and given a motion with an amplitude $\xi_k$ and a frequency of oscillation $\omega_k$:
\[
a_{jk} = -\frac{F_{jk}(\omega_k) \cos(\xi_k)}{\omega_k^2 \xi_k}, \quad b_{jk} = \frac{F_{jk}(\omega_k) \sin(\xi_k)}{\omega_k \xi_k} \tag{20}
\]

The comparison between the hydrodynamic coefficients obtained with the fully viscous method and the experiments shows a good agreement which confirm the good accuracy of the method.

![Circle Heave Motion - Added Mass Coefficient](image)

Fig. 1: Added Mass of a circular section - $a_{33}$

The interest on SWATH ships comes from their markedly reduced motion characteristics in rough seas. A large impulse for the development of SWATH ships happened in late eighties with the design of the T-AGOS 19 for the US Navy, since then different kind of cross-sections have been proposed. The hydrodynamic coefficients prediction is validated compar-
The section considered is the "golf club" shape, in which the strut is positioned outboard of the underwater body’s center-line such that the outside edge of the strut is tangent to the outside edge of the lower hull (Fig.3).

This shape has been of interest to ship operators since the offset strut might facilitate over the side operations, especially required for naval and oceanographic purpose.

The oscillation frequency was varied in the range from $3 \text{rad/s}$ to $15 \text{rad/s}$. The model was tested at two different drafts and different oscillation amplitudes: the design draft (0.226 m) was tested with an oscillation amplitude of 0.0127 m and the deep draft (0.277 m) was tested with an amplitude of 0.0127 m and 0.0381 m. The added mass and damping coefficients are presented in the non-dimensionalized form:

$$\alpha = \frac{a_{33}}{\rho A} \quad \delta = \frac{b_{33}}{\rho A \omega}$$

4 Conclusion

The comparison between the results obtained solving the Navier Stokes equations and the experimental ones shows that the viscous solver can replicate with high fidelity the resonance both in terms of peak value and frequency. The overestimation of added mass peak can be explained by mesh related problems (cell skewness change due to mesh morphing), which may lead to inaccurate solutions of the vorticity field. As the oscillation frequency increases, the forces measurement can be influenced by motions and vibrations due to high mechanical stresses on the structure used for the experiments. From the numerics, as the frequency in-
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Fig. 7: \( b_{33}: \xi = 0.0127m @ T = 0.277m \)

Fig. 8: \( a_{33}: \xi = 0.0381m @ T = 0.277m \)

Fig. 9: \( b_{33}: \xi = 0.0381m @ T = 0.277m \)

\[
\omega^2 = \frac{2\pi g}{\lambda} \tag{22}
\]

Fig. 10: G added mass: draught influence

Fig. 11: G damping coefficient: draught influence

Fig. 12: G added mass: amplitude influence

crease, the wavelength becomes shorter:

and mesh resolution should be properly adapted to capture the right wave profile with the volume of fluid method. Both these experimental and numerical issues can explain the discrepancies found in the high frequency range.

The influence of draught was investigated and the results obtained show that the proximity of the section bulb to the free surface produces an higher radiated wave in the mid range of frequency; when the sharp edge comes closer to the free surface the eddies generation increases and with it also viscous damping. Changes in the phase angle at low frequencies lead the influence of draught to be more evident in the
Fig. 13: G damping coefficient: amplitude influence

Part of forces proportional to the acceleration; at high frequencies, as draught decrease, the fluid accelerated becomes smaller leading to smaller value for added mass coefficient.

The hydrodynamic coefficients are not influenced by changes of the oscillation amplitude. This confirms the linear relationship between forces and amplitude in this relatively low amplitudes range. Further investigations will be done at higher amplitudes.

The sharp edge of the SWATH section gives a contribution to the damping by generating a vorticity field which dissipates energy, although the energy lost in the radiated waves has the greater importance in this case. In fact, the circular section which has no corners but a larger breadth close to the free surface, is able to generate higher radiated waves, and thus it has a higher damping coefficient than the SWATH’s one. Since the sides of the SWATH section are perpendicular to the heave motion direction, the acceleration to the fluid is mostly generated by the top and the bottom walls of the bulb. Their large extension and normal orientation to the motion direction (in comparison with the bottom of the circular section) explains the higher values of added mass of the SWATH.

The final goal of the research project is to develop a method which can allow seakeeping performance as an optimization criteria during early ship design stage. For this reason the estimation of hydrodynamic coefficients should be more accurate and free from empirically derived viscous corrections (with respect to methods based on potential flow) but faster with respect to a fully 3D viscous flow simulations. The excellent obtained in the validation of this method for the 2D sections presented in this papers represent a step forward towards the final goal.

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