Modeling and Analysis of the Electric Field and Potential Distribution in a Wire-Cylinder Air Gap

KONSTANTINOS N. KIOUSIS, ANTONIOS X. MORONIS
Technological Educational Institute (TEI) of Athens
Energy Technology Department
Ag. Spyridonos 12210 Aegaleo
GREECE
konstantinosq@gmail.com, amoronis@teiath.gr

Abstract: The electric field distribution in a wire-cylinder electrode arrangement in air, under high voltage dc application, has been studied by implementing the Finite Element Analysis (FEA). The applied mesh parameters have been optimized in terms of accuracy and processing power. Numerical analysis has been carried out on the field intensity, along the wire-cylinder gap axis, considering different geometrical characteristics of the electrodes, which play an important role in the spatial distribution of the electric field. The maximum field intensity was mainly associated to the wire-cylinder radii ratio, while the distance between the electrodes strongly affected the distribution along the axis of the gap. On the other hand, the potential distribution was almost independent of the gap length.

Key-Words: Finite Element Analysis (FEA), Electric Field, High voltage electrodes, Wire-cylinder

1 Introduction
The electric field strength is the key parameter that defines the behavior of insulating materials, thus the study of the electric field strength distribution is of great importance for the design and dimensioning of high voltage equipment [1-3]. The experimental measurement of the strength in air gaps is difficult and not quite accurate, due to the interference of the measuring devices which may affect the distribution of the electric field. On the other hand, computer methods can provide instant results and are capable of solving accurately complex conditions. Some of the most commonly used applications implement the Finite Element Analysis (FEA).

Despite the large number of computational studies of the electric field distribution in uniform and non-uniform electric fields at different electrode configurations (e.g. parallel planes, tip-plane, wire-plane, wire-wire etc.) found in literature [4-8], there is no study available for the wire-cylinder arrangement. On the other hand, experimental investigations, by means of corona discharge current [9-11], have already been conducted for wire-cylinder electrode pairs.

The goal of this paper is the fine modeling and analysis of the electric field strength, as well as the potential distribution in a wire-cylinder arrangement, considering the geometrical characteristics of the electrodes (e.g. the electrode gap and wire, cylinder radii).

2 Numerical Modeling
Dedicated open source simulation software FEMM (ver.4.2), implementing the Finite Element Analysis, has been used for the present study. In our case, the steady state electrostatics problem is governed by the well-known Gauss’s and Poisson’s equations:

\[ E = -\nabla V \]  \hfill (1)
\[ \nabla^2 V = -\frac{\rho}{\varepsilon_0} \]  \hfill (2)

where \( E [Vm] \) is the electric field, \( V [V] \) is the applied voltage, \( \rho \left[C/m^3\right] \) is the space charge density in the region, and \( \varepsilon_0 \) represents the dielectric permittivity of free space.

The electric field should satisfy the charge conservation law:

\[ \nabla \cdot j = 0 \]  \hfill (3)

where \( j \left[A/m^2\right] \) is the current density, defined as:

\[ j = \rho \cdot u = \rho \cdot \mu \cdot E \]  \hfill (4)

where \( u \left[m/s\right] \) represents the drift velocity across the electrode gap and \( \mu \left[m^2/V\cdot s\right] \) is the ion mobility.

Equations (1-4) can be combined to obtain:

\[ \nabla \left( \nabla^2 V \right) = 0 \]  \hfill (5)
3 Modeling Parameters

FEMM solves the nonlinear differential equation (5), for potential \( V \), over the user defined domain with user defined sources and boundary conditions. It discretizes the problem’s domain using triangular elements, which form a mesh consisting of a large number of nodes. The solution over each element is approximated by a linear interpolation of the values of potential at the three vertices of the triangle [12]. In such a computational analysis, the solver precision, the boundary conditions, the mesh distribution and the bounding box size which defines the domain of the problem, are of great importance for the accuracy of the results [13-15].

In our case, a two dimensional planar electrostatic problem has been defined, with a solver precision value of \( 10^{-8} \). Dirichlet conditions [16] have explicitly been defined in order to fix the voltage of the wire-cylinder surface in the problem’s domain. The stressed electrode was the wire, with prescribed voltage \( V_{\text{wire}} = 1000V \), while the cylindrical electrode was grounded (\( V_{\text{cylinder}} = 0V \)).

2.1 Electrode Geometry

The electrode pair and its critical dimensions (wire radius \( r \), cylinder radius \( R \) and electrode gap \( d \)), are shown in Fig. 1. Dimensions quite common in experimental investigations have been selected for the present study (\( r/R \div 1-600 \) and \( d \div 1-10 \) cm).

2.2 Bounding Box Size

Due to the symmetry of the wire-cylinder geometry along the axis of the gap, half-plane modeling has been applied and the problem’s domain was defined by the bounding box shown in Fig. 2. The box size, which sets the limits of the surrounding dielectric medium (in our case air), has been defined by the fixed distances \( A = k\cdot D \) between its sides and the electrodes. The distance \( D=2r+d+2R \) is the total length of the electrode’s assembly (air gap included) and \( k \) is a scaling constant.

The determination of the bounding box dimensions is critical, since a small box may cause deformation of the electric field and lead to errors, while on the other hand, a large box may unnecessarily demand a great amount of processing power and time.

Numerical analysis has been carried out on the electric field intensity for different \( k \) values in order to determine the ideal bounding box size. The geometrical dimensions of the electrodes were \( r=25\mu m, R=15mm \) and \( d=3cm \), while the mesh formation parameters were set on the program default values, as shown in table 1.

<table>
<thead>
<tr>
<th>Minimum angle (Degrees)</th>
<th>Maximum arc segment (Degrees)</th>
<th>Local element size (( \mu m ))</th>
<th>Mesh size (( \mu m ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>5</td>
<td>Auto</td>
<td>Auto</td>
</tr>
</tbody>
</table>

![Fig. 1 The wire-cylinder electrode pair](image1.png)

![Fig. 2 The wire-cylinder electrode pair and the bounding box where the mesh is applied](image2.png)

![Fig. 3 Maximum field intensity and applied mesh nodes with \( k \) (multiplier of distance \( D \), \( A = k \cdot D \))](image3.png)
The simulation results showed that the maximum electric field intensity values converge with the box size. Similar results have been obtained for other geometrical characteristics of the electrodes as well. Therefore, a suitable choice for the bounding box size, taking into account the number of nodes as well (see Fig. 3), would be for \( k=3 \) \((A=3D)\) and can be considered as adequate for further analysis.

### 2.3 Mesh Formation Parameters

There is a set of key parameters to the FEMM model, so as to ensure proper mesh formation. The mesh discretization at areas of interest (e.g. very close to the wire-cylinder electrode surfaces) depends mainly on two parameters, the *maximum arc segment* and the *minimum angle degrees*, which determine the size of the triangular elements near the outer surface of the electrodes, where the electric field and voltage gradients get their maximum values. Thus, very fine analysis by a dense mesh is appropriate for the accuracy of the results. On the other hand, the mesh distribution along the inter-electrode gap is a function of the *local element size along line* parameter. The density of the mesh elements in other areas, such as the space away from the electrode surfaces up to the user defined boundaries, is a function of another key parameter, the *mesh size*.

The mesh generation, using the default FEMM values for the mesh key parameters, is presented in Fig. 4. The wire-cylinder electrode pair in this case had dimensions \( r=25 \mu m, R=15mm, d=3cm \) and the bounding box size had a fixed distance \( A=3D \) away from the electrodes.

![Fig. 4 Generated mesh for the default values of the mesh formation parameters](image)

It can be seen that the mesh is not quite dense in the areas of interest and the results may lack of accuracy. The default mesh generation consists of 732 nodes and 1303 elements. Mesh optimization is needed in this case by adjusting FEMM mesh formation parameters.

### 2.4 Optimized Mesh Justification

All simulation results have been compared with other reference results coming from two well-known geometries, where analytical formulas for the maximum electric field intensity exist and resemble the wire-cylinder arrangement (wire-wire geometry, recent researches in information science and applications). Analytical study of the influence of the mesh parameters has been carried out by running a large number of simulations using different *maximum arc segment* and *minimum angles of the triangular mesh*, as well as *local elements size along line* and *mesh sizes*, in order to accomplish convergence of the results. In this way an optimal mesh has been developed, in terms of accuracy, processing power and time consumption, with the key parameter values given in table 2.

<table>
<thead>
<tr>
<th>FEMM key parameter</th>
<th>Selected values (optimized)</th>
<th>Default values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Angle (degrees)</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>Maximum Arc Segment (degrees)</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>Local Element Size Along Line (µm)</td>
<td>10</td>
<td>auto</td>
</tr>
<tr>
<td>Mesh Size (µm)</td>
<td>auto</td>
<td>auto</td>
</tr>
<tr>
<td>Nodes</td>
<td>25858</td>
<td>732</td>
</tr>
<tr>
<td>Elements</td>
<td>47870</td>
<td>1303</td>
</tr>
</tbody>
</table>

An example of the optimized mesh formation is shown below, in Fig. 5. The higher mesh density in the areas of interest can be easily seen (i.e. near the high voltage and the grounded electrode, as well as along the wire-cylinder gap axis).

![Fig. 5 Optimized mesh formation and detail of the area of interest where the fine mesh generation is shown](image)
where \( r/R=1 \) and wire-plane geometry, where \( r/R\rightarrow0 \) [7]. In this way, the accuracy of the optimized mesh simulation results could be easily tested.

In the case of two identical wires of radius \( r \), fixed parallel to each other at distance \( d \), the maximum electric field intensity is given by the following formula:

\[
E_{\text{max Wire-Wire}} = \frac{V}{d} \cdot \ln \left( \frac{\sqrt{\left(\frac{d}{2r}\right)^2 + \left(\frac{d}{2r}\right)^2}}{1 + \left(\frac{d}{2r}\right)^2 + \left(\frac{d}{2r}\right)^2} \right)
\]  

(6)

On the other hand, the analytical formula for the maximum field intensity, in the case of a wire-plane geometry, is:

\[
E_{\text{max Wire-Plane}} = \frac{V}{r \cdot (d-r)} \cdot \frac{\sqrt{d^2+r^2}}{t_{\ln} \left( \frac{d^2+r^2}{r^2} \right)}
\]  

(7)

\( FEMM \) simulations that have been conducted with the optimized applied mesh, for geometrical characteristics of the electrodes within the limits of our study, have provided results which are in good agreement with theoretical expectations in all cases. A comparison between simulation and theoretical results for the maximum electric field intensity in the case of wire-wire and wire-plane electrode arrangements is given in tables 3 and 4 respectively.

**Table 3** Theoretical and simulation results for \( E_{\text{max}} \) in the case of wire-wire arrangement (equation 6)

<table>
<thead>
<tr>
<th>( r(\mu m) )</th>
<th>( d(cm) )</th>
<th>Theoretical ( E_{\text{max}} ) (V/m)</th>
<th>FEMM ( E_{\text{max}} ) (V/m)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3</td>
<td>2.83 ( 10^6 )</td>
<td>2.8 ( 10^6 )</td>
<td>0.78</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>807 ( 10^4 )</td>
<td>799 ( 10^4 )</td>
<td>0.93</td>
</tr>
<tr>
<td>250</td>
<td>7</td>
<td>357 ( 10^3 )</td>
<td>353 ( 10^3 )</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Table 4** Theoretical and simulation results for \( E_{\text{max}} \) in the case of wire-plane arrangement (equation 7)

<table>
<thead>
<tr>
<th>( r(\mu m) )</th>
<th>( d(cm) )</th>
<th>Theoretical ( E_{\text{max}} ) (V/m)</th>
<th>FEMM ( E_{\text{max}} ) (V/m)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3</td>
<td>5.08 ( 10^6 )</td>
<td>5.14 ( 10^6 )</td>
<td>1.16</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>1.45 ( 10^6 )</td>
<td>1.44 ( 10^6 )</td>
<td>0.94</td>
</tr>
<tr>
<td>250</td>
<td>7</td>
<td>638 ( 10^4 )</td>
<td>634 ( 10^4 )</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The \( FEMM \) simulation results, along with the relative error between theoretical and \( FEMM \) values which is generally low (see tables 3 and 4), provide strong evidence for the accuracy of the applied method.

### 4 Wire-Cylinder Simulation Results

Analysis of the wire-cylinder arrangement has been carried out for different electrode radii and gap values \((r/R \approx 1-600 \text{ and } d \div 1-10 \text{ cm})\).

As expected, the maximum electric field intensity has been located at the surface of the high voltage electrode (wire), while the minimum electric field magnitude along the gap has been identified closer to the grounded electrode (cylinder). The electric field, as well as the potential distribution, is shown in Fig. 6.

**Fig. 6** (a) Electric field intensity and (b) Potential distribution

\( r=25\mu m, R=15mm, d=3cm \text{ and } A = 3 \cdot D \)

For identical electrode gaps \( d \), it becomes clear that \( FEMM \) simulation results for the maximum electric field intensity in the case of the wire-cylinder arrangement, should lie between the lower theoretical limit defined by the wire-wire electrode geometry \((r/R=1)\) and the upper limit defined by the wire-plane setup \((r/R=0)\), according to equations 6 and 7 respectively.

**Fig. 7** \( E_{\text{max}} \) correlation between the wire-cylinder configuration \((r/R\rightarrow0)\) and the wire-wire \((r/R=1)\), wire-plane arrangements \((r/R=0)\)

\( r=50\mu m \text{ and } d=5cm \)
The field intensity and the potential distribution along the axis of the gap, for \( r/R = 600 \) and different gaps, are shown in figures 8 and 9 respectively. The field intensity seems to fall abruptly after 10% of each gap (especially in the case of small gaps) and remains below the mean field intensity in all cases. Large gaps produce more homogeneous fields, thus \( E_{\text{min}} \) moves closer to the center of the distance between the electrodes as the gap increases (see figure 8).

On the other hand, as shown in figure 9, the potential distribution along the axis of the gap changes slightly for different distances between the wire-cylinder electrodes.

Investigation has been carried out for other \( r/R \) and \( d \) values and similar results have been obtained, as well. As far as the maximum electric field intensity is concerned, figure 10 is an indicative representation of the dependence of \( E_{\text{max}} \) on the wire-cylinder radii ratio \( (r/R) \) in a constant air gap. As expected, the maximum electric field intensity values have been derived in the case of the higher field inhomogeneity \( (r/R \rightarrow \text{max}) \).

In many practical cases of non-uniform electric fields, where analytical formulas for the maximum field intensity do not exist, a field enhancement factor \( f \) is introduced for the maximum stress estimation:

\[
f = \frac{E_{\text{max}}}{E_{\text{m}}}
\]  
(8)

In the wire-cylinder case, the variation of the field enhancement factor \( f \) with the \( r/R \) ratio in a constant electrode gap \( d \) is presented below in figure 11.

The maximum value of the field enhancement factor \( (f=104) \), has been also found in the case of the maximum field inhomogeneity. The field enhancement factor for \( r/R = 600 \) was high enough considering other geometries [7].
4 Conclusion

A study for the electric field, as well as the potential distribution, along the axis of a typical wire-cylinder air gap has been carried out. Dedicated open source simulation software FEMM (ver. 4.2), implementing the Finite Element Analysis (FEA), has been used for the present study. Prior to the final simulations, the key parameters of the applied mesh formation have been investigated in order to ensure accuracy and, at the same time, keep low the demand for computing power and time.

For accuracy evaluation purposes, the optimized mesh has been tested with other well-known geometries (e.g. wire-wire and wire-plane arrangement), for which analytical expressions for the maximum electric field intensity may be found in bibliography. These tests have shown that the application of the selected mesh has given simulation results which were in very good agreement with theoretical expectations.

Detailed analysis has been carried out in the case of the wire-cylinder arrangement, considering different geometrical characteristics of the electrodes. The electric field strength, as well as the potential distribution, has been examined from the standpoint of the wire-cylinder electrode gap \( d \) and the radii ratio \( r/R \).

Simulation results have shown that the electric field strength is strongly affected by the electrode geometry and arrangement. The maximum field intensity was mainly associated to the wire-cylinder radii ratio, while the gap length strongly affected the distribution along the gap. On the other hand, the potential distribution along the axis of the gap remains almost the same regardless of the gap distance.

References: