Abstract: - In recent years, several strategies have been proposed for dealing with complex geometry, curved boundaries to study particle-fluid interaction in the frame of lattice Boltzmann method. Curved boundary treatments have been suggested as a means of improving the accuracy of the stair-shaped approximation conventionally used in LB simulations. In this research, the variation of lift and drag forces are studied for different shape of particles in three distinctive methods. Result comparison shows that some of these methods are more adaptable for force evaluating on distinct surfaces.

Key-Words: - Lattice Boltzmann, BGK model, distribution function, curved boundary, two-way interaction, hard sphere model

1 Introduction

From its birth the lattice Boltzmann method (LBM) has been gradually proved itself as alternative numerical methods for simulating various types of fluid flows. It has been used extensively in different field of computational fluid dynamics (CFD), such as microfluidics, bio-fluidics, particulate flows and multiphase flows [1]. Intrinsic characteristics of LBM method such as parallel computation [2] and easiness in handling complicated geometries [3] make it more attractive than conventional continuum-based CFD methods. Likewise all numerical method boundary conditions give meaningfulness, accuracy, and numerical stability to LBM simulations. Bounce-back (BB) boundary condition was proposed to simulate no-slip condition on solid surface. Practically, two schemes of BB are used in simulations, namely “on site” type and “mid-plane” type. The former is simpler but has first order of accuracy [4] but the later provides second-order accuracy in both space and time [5].

Curve boundary (CB) techniques have been investigated widely in order to the accuracy of stair-shaped which is employed in traditional LB simulation. Three CB techniques are proposed to deal with curve boundary. Both first [4] and second [5] methods are based on interpolation scheme but apply different mesh structure to execute distribution function throughout computational domain, non-uniform and uniform respectively. The third one utilized immersed boundary treatment [6]. Interpolation-based types are the most commonly employed method for simulating curved boundary problems in LBM because of their inherent reliability and paramount numerical accuracy [5]. Consequently, present study tries to simulate flow over different shapes of blockage in the channel by using three curve boundary models.

2 Lattice Boltzmann Method

Lattice Boltzmann method has been raised from kinetic theory by using the principle of distribution function \( f(x,t) \). Distribution function indicates the probability of existence of particles at position \( x \) at time \( t \) with velocity \( c_i \) over time difference \( \Delta t \) in direction of \( i \). Then lattice Boltzmann equation (LBE) incorporating with Bhatnagar–Gross–Krook (BGK) approximation is presented as

\[
 f_{\alpha}(x + e_{\alpha} \Delta t, t + \Delta t) - f_{\alpha} = -\frac{1}{\tau} \left( f_{\alpha} - f_{\alpha}^{eq} \right) \quad (1)
\]

In Eq. (1) local distribution function is depicted by \( f_{\alpha}^{eq} \), which is called Maxwell-Boltzmann equilibrium function. \( \tau \) is called single relaxation factor. Practically, LBE is used in two separated steps
Collision step:
\[ f_a'(x,t) = f_a - \frac{1}{t}(f_a - f_a^{(eq)}). \]  
(2)

Propagation step:
\[ f_a(x + e_a \Delta t, t + \Delta t) = f_a'(x,t). \]  
(3)

Macroscopic properties fluid (velocity and density) can be obtained from distribution function by the following relations:
\[ \rho = \sum f_a. \]  
(4)
\[ u = \frac{1}{\rho} \sum f_a e_a. \]  
(5)

3 Boundary Condition Treatment

In present study solid curved surfaces are treated by applying three interpolation-based CB schemes to find unknown distribution functions, namely, Lallemand’s model [5], Yin’s model [7] and Mei’s model [8]. In these techniques, interpolation is performed on nodes near solid boundary rather than whole computational domain. An arbitrary curve boundary is depicted in figure (1). \( x_b \) is shown as intersection between solid node \( (x_s) \) and fluid node \( (x_f) \). The fraction of link sited in fluid part can be determined as \( \Delta \). Distribution function \( f_a(x_b,t) \) is unknown \( (f_a(x_b,t) = f_a(x_f,t + \Delta t)) \). These models try to interpolate this value by using information of neighboring nodes.

\[ \Delta = \frac{x_b - x_f}{x_s - x_f}, \quad \text{where} \quad 0 \leq \Delta \leq 1. \]  
(4)

A. Lallemand’s model

Lallemand’s ideal [5] used combination of simple bounce back and quadratic interpolation. This model is extension of Bouzidi’s model [9]. Based on where the solid boundary located i.e. \( \Delta < \frac{1}{2} \) or \( \Delta \geq \frac{1}{2} \) Lallemand’s model can be formulated as: Eq.(5) and Eq.(6) is for \( \Delta < \frac{1}{2} \) and \( \Delta \geq \frac{1}{2} \)

\[ f_a(x_f,t + \Delta t) = \Delta(1 + 2\Delta)f_a'(x_s,t + \Delta t) + (1 - 4\Delta^2)f_a'(x_f,t + \Delta t) - \Delta(1 - 2\Delta)f_a'(x_{ff},t + \Delta t) + 3w_a'(e_{a'},u_b). \]  
(5)

\[ f_a(x_f,t + \Delta t) = \frac{1}{(1 + 2\Delta)}f_a(x_s,t + \Delta t) + \frac{(2\Delta - 1)}{(2\Delta + 1)}f_a(x_{ff},t + \Delta t) + \frac{3w_a'(e_{a'},u_b)}{\Delta(2\Delta + 1)}. \]  
(6)

In above formulas \( e_{a'} = -e_a \), \( w_a \) is weighting factor and \( u_b \) is the velocity of moving boundary.

B. Mei’s model

Mei’s scheme [8] is an extended form of Filippova and Ha"{n}el’s (FH) work in order to improved numerical stability and computational accuracy. FH’s treatment is accurate for slow-flow condition i.e. \( \frac{1}{\alpha t} \ll 1 \). Thus Mei proposed different nodes to calculate \( f_a(x_b,t) \).

\[ f_a(x_f,t + \Delta t) = (1 - \chi)f_a'(x_f,t) + \chi f_a^{**}(x_s,t). \]  
(7)

where \( f_a^{**} \) is fictitious equilibrium distribution function and is defined as below:

\[ f_a^{**}(x_s,t) = w_{a'}\rho(x_f)[1 + 3(e_{a'} \cdot u_{sp}) + 4.5(e_{a'} \cdot u_f)^2 - 1.5u_f^2]. \]  
(8)

Base on the value of \( \Delta \) weighting factor \( \chi \) and fictitious velocity \( u_{sp} \) have forms

\[ u_{sp} = u_{ff} \cdot \chi = \frac{\omega(2\Delta - 1)}{(1 - 2\omega)} \cdot u_{sp}, \quad \Delta < \frac{1}{2} \]  
(9)

\[ u_{sp} = \left(1 - \frac{3}{2\Delta}\right)u_f + \frac{3}{2\Delta}u_b, \]  
(10)

\[ \chi = \frac{2\omega(2\Delta - 1)}{(2\omega - 1)} \cdot \Delta \geq \frac{1}{2} \]
C. Yin’s model

In despite of two pervious schemes, Yin’s method does not interpolate the boundary distribution function. It calculates the velocity \( u_m \) at the midpoint \( x_m \) through inter-/extrapolation procedure from \( u_b \) at the boundary node \( x_b \). Then modifies bounce back scheme which is proposed by Ladd [10] to treat moving boundaries.

\[
f_a(x_f, t + \Delta t) = \overline{f_{a'}} - 6 \rho_w a' (u_m \cdot e_a').
\] 

where \( \overline{f_{a'}} \) is post-collision distribution function. Linear interpolation is used to obtain midpoint velocity.

\[
u_m = \frac{\frac{1}{2} u_b + (\frac{1}{2} - \Delta) u_f}{1 - \Delta}, \quad \Delta < \frac{1}{2}
\]

\[
u_m = \frac{\frac{1}{2} u_b + (\Delta - \frac{1}{2}) u_{ff}}{2 - \Delta}, \quad \Delta \geq \frac{1}{2}
\]

The hydrodynamic force exerted on the solid particle at any boundary node can be obtained from the following expression:

\[
F(x, t) = e_a' \left[ f_{a'}(x_f, t) + f_a(x_f, t + \Delta t) \right].
\]

4 Problem Statement and Validations

Four different particle geometries (circular, square, triangular and elliptic) are chosen to simulate a channel flow with stationary obstacles. Hydraulic diameter (\( d_H \)) around 20 lattice units is set for all shapes. The particles are positioned in the centre of the channel, and fluid is passed over them (Fig 2). Parabolic velocity profile (\( Re = \frac{U_{MAX} H}{\nu} = 90 \)) and constant pressure boundary condition is defined at inlet and outlet of channel respectively. Computational domain of channel is \( L \times H = 500 \times 100 \) lattice units.

To validate our code, moving circular cylinder in infinite channel along centreline is simulated using curve boundaries method and velocity profile (Fig. 3) is compared with Aidun et al [11]. Fig. 3 shows that there is a good agreement between present methods and Aidun et al.

5 Results and Discussions

Fig. 4 (a-d) presents the velocity contours obtained by different curve boundary treatments. As shown in Fig. 4 (a-c) velocity contours are similar to each other and there is no significant difference between quadratic interpolation scheme (Lallemand’s model [5]) and other two linear interpolation methods. Velocity contours for triangular particle is presented in figure 4d.
Mei’s model [8] shows instability result for this geometry. This instability accrued because of value of deltas ($\Delta$) are more than 1 ($\Delta > 1$) for majority of links. Yin’s model [7] with linear interpolation scheme for velocity presents less accurate results than quadratic interpolation scheme.

Variation of velocity in x direction along centreline is presented in Fig. 5 for different obstacles using quadratic interpolation model. Velocity profile different shapes except triangular block have similar velocity profile.

Fig. 6(a-d) indicates the variation of drag force ($F_x$) over different time-steps. As exhibited in Fig 6(a-d) two methods which based on interpolation of distribution function i.e. Lallemand’s model [5] and Mei’s model [8], have similar results and there is significant difference between these methods and Yin’s model [7] which is based on interpolation of velocity.

Figure 3. x-component of velocity profile along centerline of the channel

Figure 4. Velocity contours a) circle b) square c)ellipse d) triangular

6 Conclusion

In this study, simulation of different shape of particles has been investigated by using three curved boundary models. The results show the quadratic interpolation scheme is more robust and stable in dealing with different blockage.
References: