Swarm Optimization of an Active Vibration Controller for Flexible Manipulator

HANIM MOHD YATIM and INTAN Z. MAT DARUS
Department of System, Dynamics & Control
Universiti Teknologi Malaysia
81310 Skudai, Johor Bahru
MALAYSIA
hanim.yatim@gmail.com; intan@fkm.utm.my

Abstract: This paper presents the development of an optimal PID controller for vibration control of flexible manipulator structures using particle swarm optimization (PSO) for tuning the PID parameters. The flexible manipulator system is first modeled using finite difference (FD) approach and identification using global search of PSO. The control structure comprises conventional PID controller and an intelligent PID incorporated PSO controller for position and vibration control. Behavior of system response including hub angle and end-point displacement are recorded and assessed. The validation of the algorithm is presented in time and frequency domains. It was noted that proposed controller is effective to move the flexible link to the desired position with suppression of vibration at end-point of a flexible manipulator structure.

Key-Words: finite difference algorithm, identification, PID control; particle swarm optimization; single-link flexible manipulator

1 Introduction

In these recent years, reduction of unwanted vibration of flexible manipulator has received substantial attention. The increased utilization of flexible manipulator has been motivated by the requirement of industrial applications since it offer several advantages over rigid manipulator [1,2,3]. However, flexible manipulator systems are known to demonstrate an intrinsic property of vibration when subjected to disturbance forces [4,5]. This highly non-linear dynamics of the system make control of flexible manipulators become complicated which involves complex processes. In this paper, modeling of single-link flexible manipulator is considered using the PSO technique. The vibration reduction of a flexible manipulator is the main area of interest in this work. It is important to control this non-linear dynamics of the system as the flexible manipulator need to have reasonable accuracy in response of input commands.

A number of approaches have been proposed and developed for control of flexible manipulator. However, PID control is still one of the most widely used in industries even with the introduction of many control theories and approaches. This includes by Mahamood and Pedro [6] which implemented PD-PID controller for control of two link flexible manipulator. A hybrid controller incorporated collocated PD controller and a feedforward controller based on input shaping has also been develop for vibration suppression and end-point trajectory tracking of flexible robot manipulator [7].

In PID control, finding optimum PID parameters is very important in order to give a satisfactory system response. Several investigations have applied intelligent techniques for optimization of PID parameters such in [8] which applied Genetic Algorithm to tune PID controller for flexible manipulator in vertical motion. However, this paper focused only on horizontal motion where global search method of PSO is utilized for control of single-link flexible manipulator.

PSO was first introduced by Kennedy and Eberhart in 1995 is a stochastic optimization technique inspired by social behaviour of bird flocking or fish schooling [9,10]. PSO implementing an information sharing approach within the search space in order to find an optimum solution. The main strength of PSO is it has only a few parameters to adjust which possesses the properties of easy implementation and fast convergence. PSO has been found to be robust and widely applied in continuous and discrete optimization problems and promisingly suitable for both scientific and engineering applications [11,12,13].

This paper presents an investigation into the use of PSO to optimize the PID controller gains. Initially, a dynamic model of the flexible manipulator structure is developed using the finite difference approach. The model is then identified using PSO. The main objective is to develop an optimal PID controller for vibration control of flexible manipulator. Simulation results are presented in time and frequency domains.
manipulator structure based on FD method is developed. Then, PSO modeling is employed to obtain the transfer function of the system responses. Subsequently a control structure with two PID controllers is proposed for position tracking and end-point vibration reduction. The proposed controller is implemented in Matlab simulation environment and its performance is assessed.

2 Simulation of Flexible Manipulator Structure

In this work, single link flexible manipulator is considered. A schematic representation of the flexible manipulator has been depicted in Fig. 1.

The single-link flexible manipulator can be modelled as a pinned-free flexible beam. The pinned end of the flexible beam of length $L$ is attached to the hub with inertia $I_h$, where an input control torque $τ(t)$ is applied at the hub by a motor and payload mass $M_p$ is attached at the free end. $E$, $I$ and $ρ$ represent the Young Modulus, second moment of inertia and mass density per unit length of the flexible manipulator respectively.

![Fig 1. Schematic diagram of the flexible manipulator system](image)

Let $X_0OY_0$ and $XOY$ be the stationary and moving coordinates respectively. For an angular displacement $θ(t)$ and an elastic deflection $v(x,t)$, the total (net) displacement $w(x,t)$ of a point along the manipulator at a distance $x$ from the hub can be described as a function of both rigid body motion and elastic deflection measured along the line $OX$.

$$w(x,t) = xθ(t) + v(x,t)$$ (1)

The governing equation of the manipulator with the associated boundary and initials conditions can thus be obtained as [3,14,15]

$$EI \frac{d^4 w(x,t)}{dx^4} + ρ \frac{d^2 w(x,t)}{dt^2} = τ(t)$$ (2)

$$w(0,t) = 0$$

$$I_h \frac{d^3 w(0,t)}{dx^3} - EI \frac{d^2 w(0,t)}{dx^2} = τ(t)$$

$$M_p \frac{d^2 w(L,t)}{dt^2} - EI \frac{d^2 w(L,t)}{dx^2} = 0$$ (3)

$$EI \frac{d^2 w(L,t)}{dx^2} = 0, \quad w(x,0) = 0, \quad \frac{∂w(x,0)}{∂x} = 0$$

The fourth-order PDE in (2) represents the dynamic equation describing the motion of the flexible manipulator. Note that the model in equation (2) does not incorporate damping. This can next be solved using a FD method [15].

A simulation algorithm characterizing the dynamic behavior of the structure is thus developed using finite difference (FD) method to obtain the numerical solution of the PDE in (2). The FD method computes solutions of the model only at discrete points. This involves dividing the manipulator into $n$ sections each of length $Δx$ and deflection of each section at sample times $Δt$. Thus, a solution of the PDE is obtained using the central difference approximations for the partial derivative terms of the response $w(x,t)$ of the manipulator at points $x = iΔx$ and $t = jΔt$.

A solution of the PDE in (2) can be obtained as,

$$w_{i,j+1} = p[w_{i+1,j} + w_{i-1,j}] + q[w_{i+1,j} + w_{i-1,j}]$$

$$+ r w_{i,j} - w_{i,j+1} + \frac{Δt^2}{ρ} τ(i,j)$$ (4)

where,

$$r = 2 - \frac{6EIΔt^2}{ρΔx^4}; \quad q = \frac{4EIΔt^2}{ρΔx^2}; \quad p = \frac{EIΔt^2}{ρΔx^4}$$

Equation (4) gives the displacement of section $i$ of the manipulator at time step $j+1$ where difference equation corresponding to each nodal point can be acquired. From this equation, to obtain the displacement at $w_{n-i,j+1}$ and $w_{n,i+1}$ the displacement of the fictitious point $w_{n+2,j}$ and $w_{n+1,j}$ are required. These can be obtained by utilizing the known boundary conditions of the flexible manipulator. This highly non-linear dynamic of the system is thus presents a good benchmark problem to test and explore modern identification and control methodologies.
3 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a population based, stochastic optimization technique introduced by Kennedy and Eberhart [9,10]. The operating procedure of a PSO can be described through the stages shown in Fig.2.

![Fig. 2. Flowchart of PSO algorithm](image)

PSO is initialized with a group of random particles, move in the search space of an optimization problem. Each particle is associated with a position and a velocity. The particles evaluate their positions based on a global fitness value and have a memory to remember their best position in the search space. At each iteration, particles are updated with two ‘best’ values. The first one is called pbest (\(P_{p}^{i}\)), which is the best position a particle has visited so far. Another ‘best’ value is the global best or gbest (\(P_{gd}\)), obtained so far by any particle in the population. After finding the pbest and gbest, particle is then accelerated toward those two best values by updating the particle position and velocity for the next iteration using the following set of equations [9,16]:

\[
\begin{align*}
\dot{v}_{id}(t) &= k_{vd}(t-1) + C_{1} \cdot \text{rand.}(P_{id} - x_{id}(t-1)) \\
& \quad + C_{2} \cdot \text{rand.}(P_{gd} - x_{id}(t-1)) \\
\dot{x}_{id}(t) &= x_{id}(t-1) + v_{id}(t)
\end{align*}
\]

\(\dot{v}_{id}(t)\) and \(x_{id}(t)\) are the current velocity and position vector of the \(i\)-th particle in the \(d\)-dimensional search space respectively. \(C_{1}\) and \(C_{2}\) are learning factor usually \(C_{1} = C_{2} = 2\) and \text{rand.} is the random number between 0 and 1.

\(k\) is the inertia serves as memory of the previous direction, preventing the particle from drastically changing direction. High value of \(k\) promote global exploration and exploitation, while low value of \(k\) leads to a local search. The common approach is to provide balance between global and local search by linearly decrease \(k\) during the search process. Decreases the inertia over time can be expressed as

\[
k(t) = k_{start} - \frac{(k_{start} - k_{end})}{T_{max}} \cdot t
\]

\(T_{max}\) is the maximum number of time step the swarm is allowed to search.

4 Parametric Identification

Parametric system identification is to construct a mathematical model that is equivalent to the identified system based on measured data. The purpose of system identification is to establish a mathematical model and estimate model parameters [17].

A number of conventional approaches for identification can be found including least squares (LS) and recursive least squares (RLS). However, these conventional methods have a potential risk to be stuck on the local minima, which often result in poorly identified model [18]. PSO is one of a few intelligent techniques commonly used for identification. PSO identification technique is developed in order to counter the problem of convergence to local minima.

In this work, parametric identification is conducted using ARX model where the model of the system can be described as

\[
y(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t) + \frac{\xi(t)}{A(z^{-1})}
\]

\(y(t)\) and \(u(t)\) is the system output and input vector respectively, where,

\[
A(z^{-1}) = 1 + a_{1}z^{-1} + \ldots + a_{n}z^{-n}
\]

\[
B(z^{-1}) = b_{0} + b_{1}z^{-1} + \ldots + b_{n}z^{-(n-1)}
\]

\(\xi(t)\) = 0 (white noise)

\(n\) = model order of the system

The global search technique of PSO is used to identify the parameters of a flexible manipulator structure based on the method of minimization of the mean square error (MSE) using the objective function

\[
\text{MSE} = \frac{1}{S} \sum_{i=1}^{S} [\xi(t)]^{2}
\]
where, $S$ represents the number of samples and $e(t)$ is the predicted error between the desired output and estimated model output.

### 5 Control Schemes

In this work, the proposed control structure for vibration control of the flexible manipulator is introduced. Initially, a collocated PID control is developed for rigid body motion. Subsequently an intelligent PID tuning by PSO controller is implemented for flexible body motion. These two loops are combined together to give control inputs to the flexible manipulator system. A block diagram of the control scheme is shown in Fig. 3.

For rigid body motion control, $\theta_d(t)$, $\theta(t)$ and $e(t)$ represents reference hub angle, hub angle and error of the system respectively. PID control system is adopted to position the flexible link to the desired angle of demand. The hub angle signals are fed back and used for control purpose. The control signal $U_m(s)$ can be obtained as

$$U_m(s) = \left[ K_p + \frac{K_i}{s} + K_d s \right] e(s) \quad (10)$$

where, $s$ is the Laplace variable. Error function of the system $e(s)$ defined as

$$e(s) = \theta_d(s) - \theta(s) \quad (11)$$

The closedloop transfer function is therefore obtained as

$$\frac{\theta(s)}{\theta_d(s)} = \frac{\left( K_p + \frac{K_i}{s} + K_d s \right) H(s)}{1 + \left( K_p + \frac{K_i}{s} + K_d s \right) H(s)} \quad (12)$$

where $H(s)$ is the open loop transfer function from the input torque to the hub angle obtained through the parametric identification process. In this study, PID parameter is determined from the autotuning function in Matlab software since PID controller is good for tracking control.

For flexible body motion control, $K_{ps}$, $K_i$ and $K_{ds}$ are proportional, integral and derivative gains respectively. $y(t)$ and $e_v(t)$ represents end-point displacement and error of the system respectively and $y_d(t)$ represents reference end-point displacement which is set to zero as the control objective to have zero vibration. An intelligent PID control is utilized to suppress an unwanted vibration at the end-point of flexible link. The end-point displacement is fed back and thus controls input $U_p(s)$ can be defined as

$$U_p(s) = \left[ K_{ps} + \frac{K_i}{s} + K_{ds} s \right] e_v(s) \quad (13)$$

Since the reference end-point displacement was set to zero signal, the error function $e_v(t)$ is defined as

$$e_v(s) = |0 - y(s)| \quad (14)$$

The closed loop transfer function is therefore obtained as

$$\frac{y(s)}{y_d(s)} = \frac{\left( K_{ps} + \frac{K_i}{s} + K_{ds} s \right) G(s)}{1 + \left( K_{ps} + \frac{K_i}{s} + K_{ds} s \right) G(s)} \quad (15)$$

where $G(s)$ is the open loop transfer function from the input torque to the end-point displacement obtained through the parametric identification process. To design a particular control loop, the $K_{ps}$, $K_i$ and $K_{ds}$ controller parameters were tuned using a global optimization method of PSO so that control input $U_p(s)$ provides acceptable performance of flexible manipulator system.
6 Implementation and results

A thin aluminium alloy of flexible manipulator with dimensions 0.96 x 0.01923 x 0.0032004 m³, mass density per area, \( \rho \), 2710 kg/m², Young’s Modulus, \( E \), \( 7.11 \times 10^{10} \) N/m² and second moment of inertia, \( I \), \( 5.1924 \times 10^{-11} \) m² is considered [1]. The response of the flexible manipulator is monitored for duration of 3.0 seconds with sampling time 0.2 ms and the system behaviour is observed and recorded at the hub and end point of the manipulator.

The first three modes of vibration of the manipulator calculated based on equation (16)[19] are obtained at 12.60 Hz, 40.83 Hz and 85.19 Hz where first mode being dominant mode.

\[
f_i = \frac{\lambda_i^2}{2\pi l^2} \left( \frac{EI}{m} \right)^{1/2} \quad i = 1, 2, 3 \quad (16)
\]

For simplicity purposes, the effects of hub inertia and mass payload are neglected. Throughout this simulation, a bang-bang torque input with an amplitude \( \pm 0.3 \) Nm and duration of 0.5 seconds is applied at the hub of the manipulator as shown in Fig. 4.

6.1 PSO modeling

Investigations were then carried out for modelling with PSO algorithm using the input-output data obtained from dynamic simulation of flexible manipulator system. The model achieved the best result with an order of 2. The PSO was designed with 50 individuals in each iteration with maximum number of iterations was set to 100. Figure 5 shows the algorithm convergence for hub angle and endpoint displacement for 100th generation. The best MSE results of PSO algorithm achieved are \( 1.5725 \times 10^{-4} \) and \( 4.6084 \times 10^{-5} \) for hub angle and endpoint displacement response respectively with their transfer function obtained as

Hub angle transfer function:

\[
\frac{0.02205s + 0.00097}{s^2 - 4.978s + 0.001696}
\]

End-point displacement transfer function:

\[
\frac{-3.714s - 0.04183}{s^2 + 3.522s - 0.009484}
\]
Figs. 6 and 7 show the simulated output of hub angle and end-point displacement in time and frequency domains. It is noted that the simulated output using PSO match the actual output very well for the time domains mapping of both response. This is further demonstrated by the frequency domain plots which show that the model has successfully characterized the system dynamics especially the first three modes of vibration. The first vibration mode captured by PSO is recorded at 11.66 Hz, which is very close to the theoretical vibration mode with percentage of error 7.46. The corresponding correlation tests for identification using PSO were found to be within 95% confidence interval indicating adequate model fit.

6.2 Control Results

Proposed control schemes are implemented and tested within Matlab environment of the flexible manipulator. The flexible link of the manipulator is required to follow a step input of 19º. Hub angle and end-point displacement responses of the system is measured and analyzed to study the performance of the proposed controllers. The performances are assessed in terms of input tracking and vibration suppression achieved with the controllers.

PID parameters for rigid motion control obtained using autotuning function in Matlab software are \( K_p = 1.96 \times 10^5 \), \( K_i = 5000.41 \) and \( K_d = 100.39 \).

Figure 8 shows the simulated output of hub angle response. It reveals that the system response successfully track the desired hub angle. The manipulator reached the desired hub angle with zero steady state error, zero overshoot and settling time, \( T_s \) of 0.2641 s.

In this study, 30 particles are considered with 50 iterations for PSO algorithm to tune the PID controller. \( k_{start} \) and \( k_{end} \) are set as 0.9 and 0.25 respectively. For purposes of comparison, the flexible body motion also control by PID controller where the parameters tuned by autotuning function in Matlab software. Table 1 shows the optimal PID parameters achieve where \( K_{p2}, K_{i2} \) and \( K_{d2} \) obtained using autotuning function in Matlab software and \( K_{p1}, K_{i2} \) and \( K_{d2} \) obtained using global search of PSO optimization method.

<table>
<thead>
<tr>
<th>PID gains</th>
<th>Parameters</th>
<th>PID gains</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{p2} )</td>
<td>-10.00</td>
<td>( K_{p1} )</td>
<td>5.25 x 10^4</td>
</tr>
<tr>
<td>( K_{i2} )</td>
<td>-0.13</td>
<td>( K_{i2} )</td>
<td>3.95</td>
</tr>
<tr>
<td>( K_{d2} )</td>
<td>0.00</td>
<td>( K_{d2} )</td>
<td>29.90</td>
</tr>
</tbody>
</table>

(a) Actual and predicted PSO output in time domain

(b) Spectral density of output

Figure 6. Hub angle modeling using PSO

(a) Actual and predicted PSO output in time domain

(b) Spectral density of output

Figure 7. End-point displacement modeling using PSO
Figure 9 shows the simulated output of vibration suppression of PID controller in time and frequency domains. It is noted from time domain mapping, PID controller actively suppress the unwanted vibration at end-point. This is further demonstrated by the frequency domain plots which show the significant amount of spectral attenuation at the resonant mode.

The spectral attenuation achieved by PID-PSO controller were 48.3 dB, 67 dB and 77.2 dB at the first, second and third modes respectively. Evidently, conventional PID controller was achieved the spectral attenuation were 8.8 dB, 23.4 dB and 22.6 dB for the first three resonant modes respectively. This reveals that the developed PSO for tuning PID controller has performed far better in the suppression of vibration of the flexible manipulator structure as compared with the conventional PID controller. Therefore optimizations of PID parameters are crucial in order to improve overall performance of the system.

7 Conclusion
This paper has presented design of an optimum PID controller using PSO for control of a flexible manipulator system. Initially, flexible manipulator system has been modelled using PSO modelling techniques in order to obtain the transfer function of the system response. A number of validation tests were carried out through input/output mapping, mean square error and correlations tests. Subsequently, hybrid PID controller was employed for control of flexible manipulator. The optimum gains that acquired through global search of PSO technique has been tested on the control structure. System responses including input tracking and vibration suppression at the end-point has been evaluated. Simulation results reveals that proposed controller has successfully position the flexible link to the desired position with reduction of vibration at end-point.

Acknowledgement
The authors wish to thank the Ministry of Higher Education (MoHE) and Universiti Teknologi Malaysia (UTM) for providing the research grant and facilities. This research is supported using UTM Research University grant, Vote No. 00H11

References: