A non-linear earthquake Intensity-Magnitude relationship. Preliminary results using macroseismic observations of the Balkan Area.

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Abstract: A classical analysis of the concepts of earthquake intensity and magnitude estimations shows that a straight line fitting is a first order approximation of the intensity-magnitude relationship. It is the scope of the present work to exam a nonlinear function, as the inverse of a sigmoid one, instead of a straight line in order to fit the intensity – magnitude observations. The approach has been applied to a data set from the Balkan area with results, which are compared with those obtained from linear regression methods. We note that the proposed function could be applied well not only for interpolation purposes but also for extrapolation to quantify the effects of historical earthquakes for which no instrumental data are available.

Key-Words: Earthquake Intensity Magnitude macroseismic Balkan

1 Introduction

Earthquake is one of the natural disasters which can cause tremendous damage to live and properties. To have a better understanding, various aspects of earthquakes have been studied over the years [1]. Macroseismic observations were traditionally used as a measure of the strength of the earthquakes. Accompanied by the intensity, attenuation analysis, the relationship between intensity and earthquake magnitude has been a major area of concern which requires further investigation [2]. The Balkan area is characterized by a high seismic activity [3] and numerous strong disaster earthquakes occurred in this area since the ancient times, which caused great damage to populated areas. As a consequence, in order to contribute to seismic hazard estimation in the Balkan area the use of macroseismic data is focused to attenuation problems [4]. Due to the continuous high culture in this area there is detailed information on the distribution of damage, which can be used as the basis for the compilation of macroseismic observations for studies related to the attenuation of seismic intensities, along with the calibration of historical earthquake events [4-7].

The present paper concerns the Balkan area, which includes Greece, Albania, Yugoslavia, Bulgaria, and western Turkey characterized mainly by normal faulting in the inner part, thrust faulting along the collision of Eurasia-Africa and Italy-Balkan peninsula while stike slip faulting were determined at the Cephallonia island and the expansion of North Anatolia Fault in the Aegean Sea [8]. Therefore numerous macroseismic observations exist from a wide range of magnitudes and various types of faulting.

The aim of this paper is to perform a preliminary analysis on a catalogue of the macroseismic data which are available for the central and southern Balkan area, which can be used for modeling of the intensity –magnitude (I-M) relation. An effort is made for the proposal of new scaling relations between the macroseismic strength and the magnitude of the earthquakes. The latter can be applied for reliable calibration studies of historical events.

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2 On the theoretical foundations of I-M relationship.

The intensity (Io) is a phenomenological parameter which express the "results" of an earthquake, whereas the magnitude (M) is defined in terms of quantitative measurements.

The practical interest of finding an empirical law to transform macroseismic intensity into magnitude arises mainly from the fact that, often, the historical seismicity of a region is known principally from a catalog of intensities. In principle, if there were a lot of observations recording historical earthquakes, we can use regression analysis [9] to reveal the I-M relationship.

The most widespread approach consists in fitting the (I, M) pairs with a straight line obtained by least-squares minimization. The common practice is to fit the data points (I_k, M_k) with a straight line M= α+β I, (1) where α and β are obtained by minimizing the error function $E_M(\alpha,\beta) = \sum(a + \beta I_\kappa - M_\kappa)^2$. An alternative view used to fit the data points (M_k,I_k) with a straight line I=γ+δM (2), where γ and δ are obtained by minimizing the error function $E_I(\gamma,\delta) = \sum(\gamma + \delta M_\kappa - I_\kappa)^2$. In general, the regression lines given by equations (1) and (2) do not inter-related. At first sight, this result seems paradoxical, since the diversity stems solely from our interpretation of a variable as the independent one. Hence, it may be preferable to have a straight line that fits, in some sense, the data set and remains invariant when the roles of the two variables are interchanged. This line, called the orthogonal regression line, is obtained by minimizing the sum of the squares of the distances from the data points to the regression line. Its equation is given as:

$$I = <I> + 2\,\frac{R}{S_o + \sqrt{S_o^2 - 4R^2}}\,(M - <M>)$$

where the <I> and <M> are the average of I and M values respectively, R is the correlation coefficient, and $S_o = ({}^{\sigma_M}/_{\sigma_I}) - ({}^{\sigma_I}/_{\sigma_M})$ with σ_M and σ_I , denoting the standard deviations of M and I, respectively [10]. However, this method is not always applied to real data since requires a positive slope which is valid when $\sigma_M > \sigma_I$ which is not always the physical case in the analysis of macroseimic data .

In general seismotectonic structures along with the local geological conditions are very complex and the I-M relationship is strongly non-linear. Thus we search for a function I=Φ(M) that gives intensity as a function of magnitude. Magnitude is from the mathematical point of view an in principle unbounded variable, whereas intensity is bounded by definition, since 0<Io<12 and thus we search for a function I= Φ(M), which is bounded, over the whole range of possible magnitudes. Therefore, an approach is to use instead of a straight line a sigmoid function (i.e., an increasing function with two horizontal asymptotes and a single inflection point) to express the intensity-magnitude relationship [9, 11]. Since the present approach is statistical, an appropriate choice might be the normal cumulative distribution function multiplied by the average intensity Ic, (i.e., Ic=6), namely, $I = I_c(1 + Erf(\frac{M-\mu}{\sqrt{2\sigma}}))$ (3), where Erf denotes the error function. Note that only two free parameters (μ and σ) are to be estimated, as in the case of a regression line. These parameters are computed by minimizing the least-squares error using the observed intensity values, $E_I(\mu,\sigma) = \sum(\Phi(M_\kappa) - I_\kappa)^2$. Equation (3) may be inverted, giving the inverse sigmoid curve

$$M = \mu + \sqrt{2}\,\sigma\,Erf^{-1}(\frac{I}{I_c} - 1) \quad (4),$$

which implies that function M depends nonlinearly on the independent variable I, The inverse error function can be defined in terms of the Maclaurin series $Erf^{-1}(z) = \sum_{k=0}^{\infty}\frac{c_k}{2k+1}(\frac{\sqrt{\pi}}{2}z)^{2k+1}$, where $c_0 = 1$ and $c_k = \sum_{m=0}^{k-1}\frac{c_m\,c_{k-1-m}}{(m+1)(2m+1)} = \left\{1, 1, \frac{7}{6}, \frac{127}{90}, \frac{4369}{2520}, \ldots.\right\}$. Thus the series expansion of Erf^{-1} is

$$Erf^{-1} = \frac{1}{2}\sqrt{\pi}\,(z + \frac{\pi}{12}z^3 + \frac{7\pi^2}{480}z^5 + \frac{127\pi^7}{40320}z^7 + \ldots$$

which in a first approximation, keeping only the first term, equation (4) leads to a linear M-I relation as proposed by equation (1).

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3 A preliminary analysis of Macroseismic data from the Balkan area

The macroseismic data used in the present study are observed macroseismic intensities of shallow earthquakes, extracted from [12], which occurred in the Balkan area during the 20th century and for which there is a satisfactory number of macroseismic intensities.[PP]. The total number of the used macroseismic sets is 284. The magnitude range of these earthquakes is between 4.1 and 7.7 and the maximum intensity varies between 4.2 and 10.5.

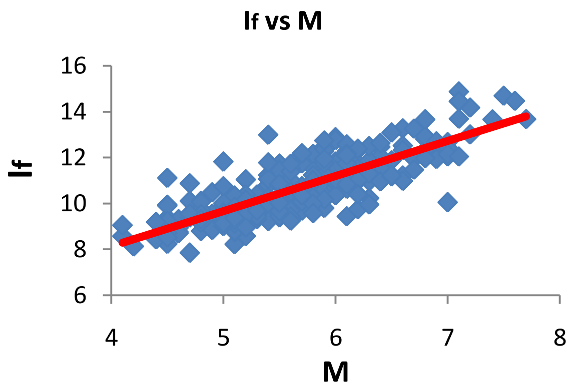

Fig. 1. Plot of the focal Intensity I_f versus magnitude M for earthquake events located in Balkan area. A linear fitting is plotted for comparison.

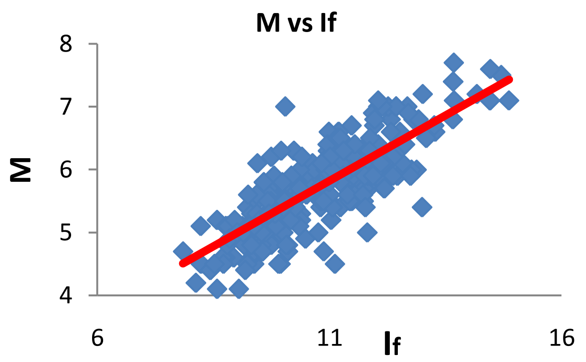

Fig. 2. Plot of the magnitude M versus focal Intensity I_f for earthquake events located in Balkan area. A linear fitting is plotted for comparison.

In [12] presented two categories of the data reanalysed here. The first one extracted from the published macroseismic maps for the Balkan area [6, 13, 14] and were used mainly for earthquakes occurred in the countries of the Balkan area excluding Greece. The second data category includes the values of the macroseismic intensity for every earthquake at the particular sites where they are observed for the area of Greece. The data sources were the bulletins of the National Observatory of Athens along with the macroseimic information which appears in [8] and originally presented in table 1 of [12]. We note that in [12] for earthquakes affecting the other Balkan countries additional information from the ISC and national bulletins were used. In the present analysis the intensity observations used were in the Modified Mercali (MM) intensity scale. In those cases where a different intensity scale was used for the original observations all intensity values were converted to MM using the relations proposed in [13]. For the magnitude we used the moment magnitude, M , as reported in [12], where calculated either explicitly from waveform modeling, when this magnitude was available, or implicitly converted from local or surface wave magnitude, using an appropriate relation for the area [15] which holds for a wide range of earthquake size. Since the epicentral intensity is strongly affected by the local geological conditions it is not representative of the source strength in addition to the fact that the seismic energy is attenuated differently, depending on earthquake's focal depth, h. For this reason, in our analysis of I-M reletionship we used the 'focal' intensity, I_f , as determined in [12] where the epicentral intensity Io corrected for focus to surface attenuation.

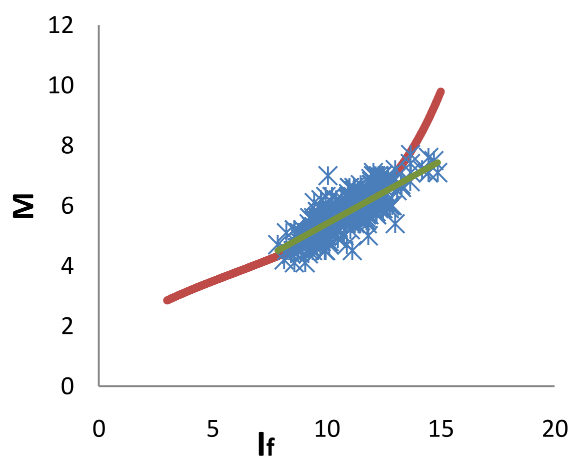

Fig. 3. Plot of the magnitude M versus focal Intensity I_f for earthquake events located in Balkan area. A linear fitting (in green) is plotted for comparison with the non-linear one as expressed from equation (4) (see text).

Figures 1 and 2 present the M-I_f and I_f-M plots using the aforementioned earthquake observation from the Balkan area, along with their linear regressions. In Figure 3 the (I_f , M) pair is given along with the fitting function proposed in equation (4) is given. It appears from each of the figures that, for intermediate intensities, the less-steep sigmoid curve almost coincides with the least-squares regression line corresponding to equation (1), while for higher magnitudes, the non-linear curve seems to fits the data better than the straight line.

4 Concluding remarks

Based on the complexity of seismic effect and the strong dependence of the intensity on geotectonic conditions a straight line cannot adequately represent the intensity- magnitude relationship throughout the full range of theoretically possible values taking into account that the intensity is, by definition, a bounded variable where $0<Io<12$.

In the present work using observational data from the Balkan area we test the applicability of the inverse sigmoid curve to model I_f-M relationship, since it fulfills the boundedness and monotonicity requirements imposed by the physical definitions of the involved quantities and depends on two parameters only, as a straight line.

Our analysis on the Balkan's data set indicates that for intermediate values of intensity, the inverse sigmoid curve given by equations (4) are almost tangent to the corresponding linear fit, while for high intensities, these curves give higher magnitudes than the corresponding regression lines suggested that equation (4) appears to be a good representation of magnitude versus intensity in the whole range of intensities. The latter proposed function could be applied well not only for interpolation purposes but also for extrapolation to quantify the effects of historical earthquakes for which no instrumental data are available.

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