Frequency Limited Model Reduction Technique for a Multivariable Activated Sludge Process and its PI-controller Application

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Abstract: - The high order of a two-input-two-output activated sludge system is reduced using Gawronski and Juang’s frequency limited reduction technique. The manipulated variables are the concentration of nitrate and dissolved oxygen (DO). A reduced fourth order system is obtained within the frequency band of 0.7 rad/s to 1.0 rad/s. The reduced system is compared with a fourth order activated sludge process obtained from Enn’s frequency weighted model reduction technique. Relative gain array (RGA) analysis is done to identify the interactions in the multivariable system hence a multi-loop control is applied. A PI-controller is utilized with optimized parameters gained from Particle Swarm Optimization (PSO) technique. Results obtained are shown in sigma plot with an improved performance in terms of steady-state error.

Key-Words: - Model reduction technique, multi-loop control, PI-controller, Particle Swarm Optimization, multivariable system.

1 Introduction

In a multivariable system, particularly an activated sludge process, problem arises due to its high complexity of the dynamic system. In an activated sludge process presented in this paper, two variables are controlled, which are concentration of nitrate and concentration of dissolved oxygen. Meticulous manipulation of such model is essential to obtain promising results. Since the complex system is hard to control in the control system environment, a model reduction technique will be applied and the characteristics of the original high order system is still retained.

Model order reduction (MOR) is a technique which reduces a high order system to a lower and more manageable system without altering its input-output characteristics. The reduction technique is required mainly due to cost reduction hence improving the overall output of a particular system or plant. Model order reduction was first introduced by Moore [1] and numerous extension follows [2, 3, 4, 5]. Enn’s technique [2] in particular is one of the well-known extensions whereby a filter is applied to the system which acts as an input weight. In Enn’s frequency weighted model reduction technique, the reduction process is focused on a pre-determined frequency band. Gawronski and Juang’s [5] on the other hand, reduces a high order system using frequency limited reduction technique. A modification of [5] was done in [6] and [7] for a single-input-single-output system and implementation of non-linear optimization approach respectively. In [5, 6, 7], the high order system is reduced within a frequency band, whereby in this study, a range of 0.7 rad/s and 1.0 rad/s is selected, since disturbance in the original activated sludge system is observed to be approximately in this range. With a pre-determined frequency band, a more desirable result is obtained, with lower errors compared to the original system.

Since the system is a multi-input-multi-output process, interactions generally occur. The occurrence of system interactions can be analyzed using relative gain array (RGA) analysis. The interactions of the obtained fourth order system are identified and the values are found to be in acceptable range, thus multi-loop control can be implemented [8] for this system. Multi-loop control is then applied using PI-controllers with tuned parameters obtained using Particle Swarm Optimization (PSO) technique. Results obtained show an improved performance of the multi-loop...
control system in the frequency domain in terms of steady-state error.

2 Activated Sludge Process

Being one of the most commonly used biological method, an activated sludge process is used to treat domestic and industrial wastewater [9,10]. The treatment system is divided into two parts, which are the aeration tank as well as the settling tank. In the aeration tank, a biodegradation process is performed, while in the settling tank, a sedimentation process is carried out, as shown in Figure 1.

![Figure 1: Schematic diagram of activated sludge process](image)

In the biodegradation process, concentrated wastewater with organic matter is aerated in the aeration tank. The wastewater is then mixed with biological microorganisms to form a biological floc. In the aeration tank, a large quantity of air or oxygen is pumped whereby from the development of the biological floc, organic content of sewage is reduced. The floc will settle to the bottom tank, known as sludge and returned to the aeration tank, while the treated water is flowed to the next stage to be treated further. At the end of this process, high quality effluent is produced within reasonable operating and maintenance cost.

The manipulated variables discussed in this paper are the concentration of nitrate and the concentration of dissolved oxygen which are involved in the biodegradation process. Biodegradation can be divided into two stages which are nitrification and denitrification. In nitrification, a high concentration of dissolved oxygen (DO) is essential, while DO level is elevated, denitrification process in the anoxic tanks may be prevented. Therefore, the concentration of DO level needs to be controlled for an efficient wastewater treatment [11]. On the other hand, in the denitrification process, presence of nitrate will complete the process. Hence, these two important parameters will be focused on in this study for the activated sludge process.

3 Gawronski and Juang’s Frequency Limited Reduction Technique

In Gawronski and Juang’s frequency limited technique [5,6,7], the original system is reduced utilizing the controllability gramian $P(\Omega)$ and observability gramian $Q(\Omega)$ of the original system within a specified frequency band $\Omega = [\omega_1, \omega_2]$.

The gramians, defined in equation (1) are manipulated.

$$
P(\Omega) = \int_0^\infty e^{\Omega \tau} BB^T e^{\tau - \Omega} d\tau = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\Omega)BB^T R^*(\Omega)d\Omega
$$

$$
Q(\Omega) = \int_0^\infty e^{\Omega \tau} C^T Ce^{\tau - \Omega} d\tau = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\Omega)C^T CR(\Omega)d\Omega
$$

(1)

where

$$
S(\Omega) = \frac{1}{2\pi} \ln((j\Omega I - A)(j\Omega I - A)^{-1})
$$

$S^*(\Omega)$ is the conjugate of $S(\Omega)$.

The original system $G(s) = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is transformed using a similarity transformation $R$ obtained from the factorization of these gramians, in a particular frequency band as shown in equation (2).

$$
H = P_{\Omega}^{-1} = Q_{\Omega}^{-1} = VT_2U^T
$$

(2)

where $V$ and $U$ are unitary matrices while $T_2$ is a positive definite diagonal matrix. The original system is then transformed using $R$ which is obtained from equation (3)

$$
R = P_{\Omega}^{-1}U(T_2)^{-1/2}R^{-1} = T_2^{1/2}U(P_{\Omega})^{-1}
$$

(3)

The transformed original system $G_T(s)$ is obtained in equation (4) and truncated which yields a reduced order system $G_r(s)$.

$$
G_T(s) = \begin{bmatrix} R^{-1}AR & R^{-1}B \\ CR & D \end{bmatrix}, G_r(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}
$$

(4)
4 Relative Gain Array

Relative Gain Array (RGA) is widely applied for the multi-loop structure design [8]. RGA provides information regarding the deduction of best pairing of controlled and manipulated variables as the control performance of the multi-loop system is ensured with the best selection of control loop pairing. In addition, RGA calculation yields the measurement of process interactions, hence provides the best view of loop interaction in multiple-input multiple-output (MIMO) system. This analysis is easy to conduct as it only requires knowledge of steady-state gains but not the overall process dynamics. For a two-input two-output (TITO) system, the RGA is defined as

\[ RGA = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \]  

(5)

where the relative steady state gain, \( \lambda_{ij} \) (dimensionless) relates the \( i^{th} \) controlled variable and \( j^{th} \) manipulated variable.

\[
\lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}} \\
\lambda_{12} = 1 - \lambda_{11} = \lambda_{21} \\
RGA = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix} \quad \text{where} \quad \lambda = \lambda_{11}
\]

Each row and column for RGA will sum to unity at the end of the calculation. Recommended controller pairing corresponds to the \( \lambda_{ij} \) which have the largest positive values closest to one. The MIMO system can be concluded to have the least or no interaction among the loops if the relative gain array yielded identity matrix. This will favour the tuning of selected appropriate controllers, as each controller may be tuned as single-loop controller due to least or no interaction in the system.

5 Multi-loop Control

A Proportional – Integral - Derivative (PID) controller is still one of the widely used controllers in the process industry, even though there is a continual advanced in control engineering [12]. The general function of a PID controller is to maintain the output of a given system and for the system to reach zero steady state error [13]. Due to its simple and reliable nature, PID controller remains one of the most popular controllers at present. The main objective of PID implementation in a system is to improve the system’s output performance through the reduction of settling time, overshoot, as well as ensuring a zero steady-state error.

The implementation of a PID controller depicts a necessity to tune the involved parameters i.e. the proportional (\( K_p \)), integral (\( K_i \)) and derivative (\( K_d \)) gains in order to obtain desirable output. Heuristic approach is commonly used, although it took an amount of time before the optimized parameters are obtained. However, due to its simplicity, this technique is the most commonly used in the industry.

In this study, the effectiveness of a PID controller to the activated sludge process is shown through the application of a multi-loop control. A multi-loop PID-controllers or known as decentralized PID controllers, as defined in [8], acts in a multi-loop fashion, are tuned mainly on a single-loop basis. In this study, a PI-controller is applied to the reduced order system obtained from Gawronski and Juang’s technique using PSO. In figure 2, two controllers, i.e. \( G_{c1} \) and \( G_{c2} \) are applied to the reduced system \( G_r(s) \).

![Figure 2: Multi-loop control of an activated sludge process](image)

6 Optimization

In this paper, particle swarm optimization (PSO) is used to optimize the parameters of the multi-loop PID controllers. The cost function used is the integral time square error (ITSE). These are briefly explained in the following sections.

6.1 Particle Swarm Optimization

PSO is widely applicable in the engineering field due to its flexibility to handle numerous parameters simultaneously. Based on swarm intelligence, the technique was inspired from the social interaction and animal behavior of fish schooling and bird flocking, in search of their food. From this basic concept, PSO has the advantages of easy
implementation and high computational efficiency [14].

In PSO, the swarm is initialized with a population of random solution, which is known as particles. Information is exchanged between particles when the swarm takes off. Each particle has its own velocity with its own flying experience as well as its neighbors’s flying experience. From this concept, PSO is formulated. To explain, in PSO, each particle is treated as a particular point in a D-dimensional space. The \( i \)th particle is represented as \( X_i = [\mathbf{x}_{i1}, \mathbf{x}_{i2}, \ldots, \mathbf{x}_{iD}] \). The best ever visited position, known as \( pbest \) is represented as \( X_i^{pbest} = [\mathbf{x}_{i1}^{pbest}, \mathbf{x}_{i2}^{pbest}, \ldots, \mathbf{x}_{iD}^{pbest}] \). The index of the best particle among all particles in the population known as \( gbest \) is represented as \( X_i^{gbest} = [\mathbf{x}_{i1}^{gbest}, \mathbf{x}_{i2}^{gbest}, \ldots, \mathbf{x}_{iD}^{gbest}] \). The velocity for the particle \( i \) is represented as \( \mathbf{v}_i = [\mathbf{v}_{i1}, \mathbf{v}_{i2}, \ldots, \mathbf{v}_{iD}] \). The velocity and position of the particles are updated according to the following equations:

\[
\begin{align*}
v_{i}^{n+1} &= w \cdot v_{i}^{n} + c_1 \cdot \text{rand} \cdot (x_{i}^{pbest} - x_{i}^{n}) \\
&+ c_2 \cdot \text{rand} \cdot (x_{i}^{gbest} - x_{i}^{n}) \\
x_{i}^{n+1} &= x_{i}^{n} + v_{i}^{n+1}
\end{align*}
\]  

(6)

where \( c_1 \) and \( c_2 \) are positive constant, \( \text{rand}(\cdot) \) is random function between 0 and 1 and \( n \) number of iterations. To balance between the global search and local search capability, inertia weight, \( w \) is introduced into the equation [14].

6.2 Integral Time Square Error (ITSE)

In this study, the objective function used to represent the error requirement, i.e. to make the error zero, is integral time square error (ITSE). ITSE is used to quantitatively measure the performance of the designed PID controller. The ITSE equation used for the objective function is given in the following equation

\[
ITSE = \int_{t_0}^{t_e} t e^2(t) dt
\]  

(7)

with \( e(t) \) is the controlled system output – reference input. To obtain the optimum parameters for the proportional, integral and derivative gain, ITSE is utilized.

7 Results and Discussion

The original system of the activated sludge process is a high 90th order system. Using Gawronski and Juang’s frequency limited reduction technique, a reduced fourth order system is obtained in the frequency band of 0.7 rad/s to 1.0 rad/s. The reason of this action is an attempt to eliminate the irregularities shown in the original system between that particular frequency bands. The results obtained are compared with a fourth order system from Enn’s frequency weighted model reduction technique [15] and the original 90th order system, process as shown in figure 1. It can be observed that Gawronski and Juang’s fourth order system follows the original system closely and compares well with Enn’s fourth order from [15].

The application of PSO on the fourth order Gawronski and Juang’s system yields an absolute

\[
\text{RGA of } A = \begin{bmatrix} 1.3276 & 0.7438 \\ 0.7438 & 1.3276 \end{bmatrix}
\]

which recommends pairing of input 1 to output 1, and input 2 to output 2 with interaction between input 1 and output 2, as well as input 2 to output 1. This is conveniently depicted in figure 2 in the previous section.

The system is then run for optimized PI-controller parameters using PSO and the following sigma plot is obtained as shown in figure 4. It can be observed that the reduced fourth order system yields better performance when a PI-controller is applied. From frequency response analysis, the steady-state error for both nitrate and DO with PI-control is approximately 0.5. In comparison, the open loop control for both nitrate and DO are much bigger (>100) than the system with PI-control. Due to space restrictions, results of the system with PID-controllers are not presented. Further tuning of the system will yield better results, with lower steady-state error.

8 Conclusion

In this paper, a high 90th order system of an activated sludge process is reduced to a low 4th order system using Gawronski and Juang’s frequency limited reduction technique. Utilizing this technique, the system is run between a frequency band of 0.7 rad/s and 1.0 rad/s. The results obtained are compared with Enn’s reduced 4th order system. The 4th order Gawronski and Juang system is then run for PI-controller using PSO as the optimization technique. Result obtained shows an improved performance of the reduced order system. A further improvement can be done by tuning the parameters of the controller to obtain better system performance.
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References:
Figure 3: Reduced fourth order system compared to 90th order original system

Figure 4: Sigma plots for open loop vs optimized PI-controller for DO and nitrate