Design Optimization for Low Power
Asynchronous Motors Used in Medical Equipments

VLAD ION, ENACHE SORIN, ENACHE MONICA
Faculty of Electrical Engineering
University of Craiova
Decebal Street, no. 107
ROMANIA
ivlad@em.ucv.ro, senache@em.ucv.ro, menache@em.ucv.ro

Abstract: - Owing to the direct and indirect interaction with people, electrical motors used in medical equipments must satisfy the highest standards of safety, reliability and precision. It is noticed that in the present economic and technical context, the interference between the optimal design afferent to the steady state and the dynamic design is necessary. All these performances can be achieved with the help of the electronic computer on the basis of some adequate programs, which are to lead to an economic fabrication and operation of the medical equipment driven by asynchronous motors.

Key Words: Small asynchronous motors, optimization, simulation.

1 Introduction
The concept of medical equipment is wide enough; it practically includes all the equipments serving the medical act [2], [3], [6], [11]. As a consequence, the motor operation in its rated parameters is very important. Standardized electrical motors can be used to drive a large range of medical equipments of general use; we can mention a few of them: medical centrifugal machines, therapy of breathing pathways cleaning, equipment for mobility and mobility therapy, patients lifting and patients’ beds lifting, equipment generating and concentrating oxygen, surgery instruments (saws, cutters).

In the standardized motors range the asynchronous motor is the most used one, especially driven by three phase alternating current. The asynchronous motors used in medical equipments are manufactured in a very large range of powers (from a few watts up to a few kW); for medical uses they are usually supplied by voltages lower than 380V and they use the standard frequency of 50 Hz.

An example of asynchronous motor used in a medical equipment is presented in the paper written by S. Halabeya [4]: an active biomedical device for testing and training muscular functions of the patients for the most important joints and members: ankles, knees, shoulders, elbows. As we expect, the system must fulfil extremely high safety standards owing to the direct interaction with patients. This paper aims at answering the present and future requirements regarding the construction modernization of the low power asynchronous motors used in medical equipments, by increasing the operation performances [1], [2], [3], [8], [9], by decreasing the costs etc.

2 Optimal design of low power asynchronous motors

2.1 Mathematical model
The mathematical model used in the design of the synchronous motor with cage rotor [8-9], contains computation relations as equalities and inequalities and some of them can be tables with standardized values, magnetization curves etc. These relations might be found in the specialty literature.

If there are certain restrictions for the asynchronous motor, they are verified while the project is carried out and if they are not fulfilled the mathematical model is resumed for other values assigned to the variables.

2.1.1 Objective function and restrictions
For the optimal design of the very low power asynchronous motors [1], [3], [5], [7] used for driving medical equipments there is considered the criterion

\[ f(x) = C_e = \text{min.} \quad \text{minimum exploitation cost,} \]

\[ C_e = C_{ca} + C_{cr} = N_{ou} T_{ri} c_{el,a} \Sigma \rho + N_{ou} T_{ri} c_{el,r} \Sigma q \]  

(1)

where: \( C_{ca} \) – cost of the active electrical energy lost inside the machine, \( C_{cr} \) – cost of the consumed reactive energy, \( c_{el,a}, c_{el,r} \) – costs of one kWh of active.
electrical energy and reactive electrical energy, respectively; $T_{in}$ – time of investment recovery; $N_{ou}$ – number of annual operation hours; $\Sigma p$, $\Sigma q$ – losses of active and reactive power at rated load operation. The restrictive conditions imposed by the medical active and reactive power at rated load operation.

The variable comply with certain restrictions imposed by the motor heating (maximum values), respectively for concentrating the searching range (minimum values):

$$x_i = \{A, B, J_1, J_{2b}, B_{j1}, D, \delta, \beta_{c1}\}$$

$$x_{\text{min}} \leq x_i \leq x_{\text{max}}$$

(5)

The exploring method we used involves the objective function computation in the network nodes and the determination of that optimum point where $f(\vec{x}) = C_e = \text{minim}$ is obtained.

This method of numerical search [8-9] is known in literature and it has the advantage of an ordered search by a program established. The main stages are:

a) the searching step is established on each direction with relation of the form:

$$\Delta A = \frac{A_{\text{max}} - A_{\text{min}}}{n_A} \quad \Delta D = \frac{D_{\text{max}} - D_{\text{min}}}{n_D} \ldots$$

(6)

where, $n_A$, $n_B$, $n_{J_1}$, $n_{J_{2b}}$, $n_{B_{j1}}$, $n_D$, $n_\delta$, $n_{\beta_{c1}}$ is the number of points on each searching direction, established by the precision we want;

b) there are established the points "spatial nodes": $P_1$, $P_2$, $P_3$,...,$P_N$ in which the value of the objective function will be computed by relations as:

$$P_k(x_k) = P_k(A_{k1}, B_{k2}, J_{1k3}, J_{2bk4}, B_{j1k5}, D_{k6}, \delta, k_7, \beta_{c1k8})$$

(7)

$$J_{1k3} = A_{\text{min}} + k_1 \cdot \Delta A \quad B_{k2} = B_{\text{min}} + k_2 \cdot \Delta B$$

$$J_{1k3} = J_{1k3} + k_3 \cdot \Delta J_1 \ldots$$

(8)

d) passing from a searching point $k$ to another one $p$, is made by modifying a single variable.

e) as a result of the objective function evaluation in all the network nodes, a global optimum is obtained.

The total number of searching points is:

$$N = (n_A + 1)(n_B + 1)(n_{J_1} + 1) \ldots (n_{\beta_{c1}} + 1)$$

(11)

3. Simulations and results

For exemplification, we will redesign the cage rotor asynchronous motor used for driving an active biomedical device [4], [6] for testing and training the patients’ muscle functions at important joints: limbs,
ankles, knees, hips. The results obtained will be compared with the results of the existing motor of the analyzed motor, which is rated as follows: \( P_1=1 \text{ kW} \), \( \eta_1=1500 \text{ rpm} \) – synchronism speed; \( s_0=3.8\% \) – rated slip.

The restrictive conditions imposed by the medical equipment to this motor are:

\( C_e \leq C_{em} = 900 \text{ E} \),
\( m_\text{g} \geq m_\text{g,mi} = 1.2 \),
\( m_\text{g} \geq m_\text{g,mi} = 2.0 \),
\( i_p \leq i_p = 4.50 \),
\( D_{c\text{max}} \leq D_{c\text{max}} = 190 \text{ mm} \),
\( L_e \leq L_{e,\text{max}} = 290 \text{ mm} \).

All the quantities afferent to the existing motor obtained by redesign, will be considered as reference quantities and they are noted by index "m". For example: \( \cos\theta_m=0.636 \), \( n_m=0.788 \), \( M_{\text{max},m}=2.29*M_N \), \( M_{p,m}=1.74*M_N \), \( I_{p,m}=3.67*I_N \), \( D_{em}=138 \text{ mm} \), \( L_{em}=223 \text{ mm} \).

The costs (fabrication, exploitation and total) have been computed on the basis of the known documentation: \( T_1=4 \text{ years} \) – time of the investment recovery; \( N_{\text{cor}}=240*8=1920 \text{ hours/year-number of annual operation hours}; \( c_{cu}=12 \text{ E/kg} \) – cost of one kilo of copper; \( c_{fe}=0.95 \text{ E/kg} \) – cost of one kilo of iron (siliceous sheet); \( c_{\text{uel},r}=0.036 \text{ E/kVARh} \) – cost of one kVARh of reactive electrical energy; \( c_{\text{uel},e}=0.132 \text{ E/kWh} \) – cost of one kWh of active electrical energy.

For the existing motor redesigned the following costs have resulted: \( C_{1m}=141.6 \text{ E} \), \( C_{em}=734 \text{ E} \), \( C_{1m}=876 \text{ E} \), electromagnetic stresses: \( A_m=230 \text{ A/cm} \), \( B_{em}=0.694 \text{ T} \), \( J_{1m}=5.40 \text{ A/mm}^2 \), \( J_{2b,m}=3.51 \text{ A/mm}^2 \), \( B_{1m}=1.47 \text{ T} \), and main dimensions: \( D_{m}=80 \text{ mm} \), \( \delta_m=0.30 \text{ mm} \), \( \beta_{1m}=0.508 \).

In this paper we propose an example of industrial application which helps us at defining a set of optimum configurations for the asynchronous motor we have analyzed where the optimization criterion is \( C_{e}=\text{minimum} \).

Carrying out an efficient optimization program, which provides the optimum solution in a short time, is a requirement of the electrical machines building factory that has the purpose to shorten the duration of the product assimilation, the investments efficiency and to gain the sale market.

In order to answer this requirement we aim at reducing the variables number, by selecting those variables which have weight on the criterion and on the restrictive conditions imposed.

Further on we emphasize the importance of one variable on the optimization criterion established \( C_{e}=\text{minimum} \), by studies carried out by each variable \( x=A_{i}, B_{j}, J_{2b}, B_{1j}, \Delta, \delta, \beta_{1j} \). The graphics presented are plotted in per unit and for reference there are used relations as:

\[
C_e = \frac{C_{e,\text{var} m}}{C_{e, m}} \quad \text{and} \quad C_{ea} = \frac{C_{e,\text{var} m}}{C_{e, a \text{ m}}}, \quad (12)
\]

\( C_{e,\text{var} m}, C_{e, a \text{ m}} \) – exploitation cost and cost of the active electrical energy consumption for the variant of motor analyzed; \( C_{e, m}, C_{e, a \text{ m}} \) – exploitation cost and cost of the active electrical energy consumption for the existing variant of motor etc.

### 3.1. Optimization by one variable

There has been made a study considering the electromagnetic stresses as variables (\( A, B, J_1, J_{2b}, B_{1j} \)), where there have been considered limits \(-40\%\), respectively \(+15\%\) relatively to the reference values given and we present the most significant results.

**Variable B – air-gap magnetic induction**

The analysis aims at identifying the optimum value of the variable \( B \) – air-gap magnetic induction relatively to the criterion established \( f(x)=C_e=\text{minimum exploitation cost} \) and, for this value, to see how the quantities taken into account modify: \( c_{e} – \text{exploitation cost}, c_{1} – \text{total cost}, \), \( c_{ea}, c_{er} – \text{cost of the active and reactive electrical energy in relative units.} \)

\[
\begin{align*}
&c_{1}, c_{e}, c_{ea}, c_{er} \text{ [f.u.]} \\
&\text{Fig. 1. Variation curves of the exploitation costs relatively to the variable B – air-gap magnetic induction.}
\end{align*}
\]

The decrease of the air-gap magnetic induction down to the optimum value \( B_r=0.60 \text{ T} \) (fig. 1) emphasizes the facts below:

**Advantages**: the exploitation cost, the cost of the reactive electrical energy decrease;

**Drawbacks**: the cost of the active electrical energy increases.

Further on there is presented the case when the variables are the important constructive dimensions (\( D, \delta, \beta_{1j} \)), where there have been considered limits \(-40\%\), respectively \(+40\%\) relatively to the reference values; we present the most significant values.

**Variable D – machine diameter**

The analysis aims at indentifying the optimum value of the variable \( D \) – machine diameter relatively to the criterion established \( f(x)=C_e=\text{minimum exploitation cost.} \)
The increase of the machine diameter up to the optimum value \( D_o = 100 \text{ mm} \), (fig. 2) means:
- **Advantages**: the total cost, the exploitation cost, the reactive electrical energy cost decrease;
- **Drawbacks**: the cost of the active electrical energy increases.

**Variable \( \delta \) – machine air-gap**

The analysis aims at identifying the optimum value of the variable \( \delta \) – machine air-gap relatively to the criterion established \( f(x) = C_e = \text{minimum} \).

The decrease of the machine air-gap down to the optimum value \( \delta_o = 0.2 \text{ mm} \), (fig. 3) involves:
- **Advantages**: the total cost, the exploitation cost and the cost of the reactive electrical energy decrease;
- **Drawbacks**: the cost of the active electrical energy increases.

Further on these costs are presented separately relatively to all the main variables expressed in per unit as below:

\[
x = \frac{A}{A_m}, \quad x = \frac{B}{B_m}, \quad x = \frac{J_1}{J_{1m}}, \quad x = \frac{D}{D_m}, \ldots, \quad (13)
\]

\( A, B, J_1, D, \ldots \) – current load, air-gap magnetic induction, stator current density, machine diameter, ..., 
\( A_m, B_m, J_{1m}, D_m, \ldots \) – reference values for the motor analyzed: \( A_m = 230 \text{ A/cm}; \ B_m = 0.694 \text{ T}; \ J_{1m} = 5.40 \text{ A/mm}^2; \ B_{1lm} = 1.47 \text{ T}; \ D_m = 80 \text{ mm}; \ \delta_m = 0.30 \text{ mm}; \ \beta_{c1m} = 0.508. \)

Analyzing fig. 4.a we notice the optimum values of the electromagnetic stresses relatively to the criterion \( C_e = \text{minimum} \), and in figure 4.b we see the optimum constructive dimensions. Example: \( C_e = \text{minimum} \) for \( B = 0.9 \times B_m = 0.9 \times 0.694 = 0.624 \text{ T} \), etc.

In fig. 4.c and fig. 4.d we see the main variables which modify significantly the costs of the active and reactive electrical energy (components of \( C_e \)).
3.2 Optimization by two variables

In order to determine the influence of the combination of two variables on the objective function \[8\cdot10\], the program of optimal design has been run. This research is efficient owing to the fact that, many times, the designer has finite possibilities to choose the number of variables of which values might be modified, so that the objective function is fulfilled. On all the graphics we have presented the value of the objective function resulted from classical design is plotted with brown ball and the optimized value is plotted with red ball. The research has been carried out for several combinations of variables.

Variables: \(D\) – machine diameter and \(\delta\) – air-gap

The study carried out by a single variable emphasized that the quantities \(D\) and \(\delta\) may be considered as main variables with high weight relatively to the criterion enunciated: \(f(x)=C_e\) – minimum exploitation cost. It has been seen that these variables has an important weight on the exploitation cost at the motor analyzed. The range of variation for these variables is \((0.7\div1.2)\cdot D\), respectively \((0.7\div1.2)\cdot \delta\).

Analyzing figure 5 it is noticed that the optimum values which lead to the objective function accomplishment \(C_e\) –minimum are \(\delta=0.2\) mm –minimum air-gap and \(D=100\) mm –diameter larger by 20% than the diameter of the existing machine. In these conditions, the exploitation cost has a decrease by 4.35% in comparison with the existing machine.

![Fig. 5. Spatial curve of the exploitation cost versus \(D\) –machine diameter and \(\delta\) -air-gap.](image)

Variables: \(J_1\) –current density and \(B_{j1}\) –stator yoke magnetic induction

Next main variables with high weight relatively to the criterion enunciated are \(J_1\) and \(B_{j1}\). The variation range has been also kept for these variables, \((0.7\div1.2)\cdot J_1\), respectively \((0.7\div1.2)\cdot B_{j1}\).

The optimum of the objective function \(C_e\) –minimum (fig. 6), is obtained for minimum electromagnetic stresses, resulting a decrease of the exploitation cost by 6.87%, but the total cost has a slight increase of 1.71%.

![Fig. 6. Spatial curves for \(c_e\), \(c_t\) – exploitation cost and total cost versus \(J_1\) –current density and \(B_{j1}\) – stator yoke magnetic induction.](image)

3 Conclusions

The study and the simulations carried out had as a main objective to identify the main variables in case of optimization \(f(\vec{x})=C_e\)=minimum, to reduce the variables number and, finally the computation effort necessary for the optimization. Thus, by means of this program, the optimum solution can be provided to the customer in a very short time.

The final results of the optimization for the asynchronous motor we have analyzed are filled in the table 1. There can be seen the comparison between \(V_m\) –the real variant of existing motor and \(V_o\) –the optimized variant. On this occasion there have been emphasized other quantities important in design, some of them imposed even by customer: \(m_p\), \(m_m\) –starting torque and the maximum one; \(D_e\), \(L_e\) –overall dimensions; \(C_t\), \(C_f\), \(C_e\) –total cost, fabrication cost and exploitation cost.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Variant</th>
<th>(I_p) (r.u.)</th>
<th>(m_p) (r.u.)</th>
<th>(m_m) (r.u.)</th>
<th>(D_e) (mm)</th>
<th>(L_e) (mm)</th>
<th>(C_t) (E)</th>
<th>(C_f) (E)</th>
<th>(C_e) (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values imposed</td>
<td>(\leq 4.5)</td>
<td>(\geq 1.2)</td>
<td>(\geq 2.0)</td>
<td>(\leq 190)</td>
<td>(\leq 290)</td>
<td>(\leq 900)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(V_m) Real var.</td>
<td>3.67</td>
<td>1.74</td>
<td>2.29</td>
<td>138</td>
<td>223</td>
<td>876</td>
<td>141</td>
<td>734</td>
<td></td>
</tr>
<tr>
<td>(V_o) Opt. var.</td>
<td>4.12</td>
<td>1.37</td>
<td>2.08</td>
<td>176</td>
<td>260</td>
<td>818</td>
<td>240</td>
<td>578</td>
<td></td>
</tr>
</tbody>
</table>

The optimum machine has resulted for the following variables: \(A_{s}=171\) A/cm; \(B_{s}=0.498\) T; \(J_{1,o}=3.46\) A/mm²; \(J_{2b,o}=3.80\) A/mm²; \(B_{j1,o}=1.12\) T, \(D_e=105\) mm, \(\delta_o=0.21\) mm; \(\beta_{c1_o}=0.439\).
For the optimum values of the variables in figure 7 there have been plotted all the important quantities taken into consideration.

Analyzing figure 7 (where the values of the optimized machine are related to the real one), we notice the advantages: the exploitation cost $C_e$, the cost of the active and reactive electrical energy, $C_{ea}$ and $C_{er}$, the total cost $C_t$ decrease. At the same time we also see the drawbacks occurred: the starting torque $m_p$ and the maximum torque $m_m$ decrease, the starting current $I_p$ increases, the fabrication cost $C_f$ increases, the overall dimensions $D_e, L_e$ increase, the motor weight $m$ increases, the iron and copper consumptions, $m_{Fe}$ and $m_{Cu}$ increase.

It is noticed that at the final optimum variant, by a correct choice of the electromagnetic stresses and of the constructive dimensions, there has resulted a decrease of the exploitation cost by 7.11% in comparison with the reference value, keeping at required limits the total cost, the starting torque, the maximum torque and the overall dimensions.

References