The Level Crossing Rate of the Ratio of Product of Two k-μ Random Variables and k-μ Random Variable

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Abstract: - In this paper, the average level crossing rate of the ratio of product of two k-μ random variables and k-μ random variable will be analyzed. The joint probability density function of the ratio of product of two k-μ random variables and k-μ random variable and its first derivative will be calculated. By using this expression, the average level crossing rate of the ratio of product of two k-μ random variables and k-μ random variable will be obtained. This scenario presents an interference limited k-μ multipath fading line-of-sight environment. The ratio of product of two k-μ random variables and k-μ random variable represent signal-to-interference envelope ratio.

Key-Words: - average level crossing rate, cochannel interference, first derivative, joint probability density function, k-μ fading, signal-to-interference envelope ratio

1 Introduction

The ratio of random variables is important in performance analysis of wireless communication system operating over fading channels. The statistics of the ratio of random variables enables evaluation of the outage probability, bit error probability and the capacity of wireless communication system. The signal envelope and cochannel interference envelope experience small and large scale variation in the presence of fading. Short term fading is result of multipath propagation due to reflection, refraction, diffraction and scattering of radio wave. Long term fading is result of large obstacles and large deviation in terrain profile between transmitter and receiver [1, 2].

The cochannel interference is interfering signal at the same frequency. The signals from two or more channels of different location and at the same frequency interfere. In interference limited environment level of cochannel interference is sufficiently high as compared with thermal noise, so the noise can be ignored in performance analysis. In this channel the ratio of signal and cochannel interference envelope is important performance measure of communication system [3].

In this paper, the ratio of product of two k-μ random variables and k-μ random variable is considered. The probability density function of the ratio of product of two α-μ random variables and α-μ random variable is calculated in [4]. By using this result, the bit error probability and the outage probability of wireless communication system over composite multipath α-μ fading in the presence of cochannel interference subjected to α-μ fading can be evaluated [5].

The product of two random variables in the numerator of the ratio can represent desired signal envelope affected simultaneously to two multipath fading. The random variable in denominator of the ratio can represent cochannel interference envelope subjected to multipath fading. In paper [6] the ratio of random variable and product of two random variables is analyzed. The probability density function and cumulative distribution function of ratio of α-μ random variables and the product of two α-μ random variables are evaluated. The random variable in numerator of the ratio can represent desired signal envelope subjected to multipath fading. The product of two random variables in the denominator of the ratio can represent cochannel interference envelope which suffer simultaneously two multipath fading. The expression for probability density function can be used for evaluation of the bit error probability, the outage probability and the capacity of wireless communication system operating over multipath

ISBN: 978-960-474-328-5

fading channel in the presence of cochannel interference, subjected simultaneously to two multipath fading.

In paper [7] the ratio of the product of two random variables is considered. The probability density function and cumulative distribution function of ratio of the product of two α-μ random variables are calculated. The product of two random variables in the numerator of the ratio can represent desired signal envelope subjected simultaneously to two multipath fading. The product of two random variables in the denominator of the ratio can represent cochannel interference envelope affected simultaneously to two multipath fading. The expression for probability density function is used for evaluation of the bit error probability, outage probability and capacity of wireless communication system operating over composite multipath fading environment in the presence of cochannel interference subjected simultaneously to two multipath α-μ fading.

The second order statistics of selection macro-diversity system operating over Gamma shadowed k-μ fading channels are determined in [8].

In this paper the ratio of product of two k-μ random variables and k-μ random variable is considered. The level crossing rate of the ratio of product of two k-μ random variables and k-μ random variable is calculated. The product of two k-μ random variables in numerator of the ratio can represent desired signal envelope subjected simultaneously to two k-μ multipath fading. The random variable in denominator of the ratio can represent cochannel interference envelope affected to multipath k-μ fading. The expression for level crossing rate of the ratio of random variables can be used for calculation of average fade duration of wireless communication system operating over composite, interference limited, multipath k-μ fading environment in the presence of cochannel interferences subjected to k-μ multipath fading.

This paper is organized as follows. In section II the ratio of product of two k-μ random variables and k-μ random variable is considered. In section III the expression for calculation the level crossing rate is derived. The section IV concludes the work.

2 Ratio of Product of Two k-μ Random Variables and k-μ Random Variable

In this section ratio of product of two k-μ random variables and k-μ random variable is analyzed. The k-μ distribution describes small scale variation of signal envelope in interference limited linear line-of-site multipath fading environment [9]. The k-μ random variable has two parameters. The parameter k is related with dominant component envelope of fading. The parameter μ is associated with the number of clusters of scattering wave. The k-μ distribution is general distribution and Rayleigh, Rice, Nakagami-m distributions can be derived from k-μ distribution as special cases. By setting $k=0$, k-μ distribution reduces to Nakagami-m distribution. For $\mu=1$ and $k=0$, from k-μ distribution can be derived Rayleigh distribution. Rice distribution for n is obtained from k-μ distribution for $\mu=1$ as special case.

If random variables x, y and z follow k-μ distribution, then it is:

Random variables $x_1, x_2, \ \ , x_{2\mu}, y_1, y_2, \ \ , y_{2\mu}, z_1, z_2, \ \ , z_{2\mu}$ have Gaussian distributions:

$$x = x_1^2 + x_2^2 + \cdots + x_{2\mu}^2 \qquad (1)$$

$$y = y_1^2 + y_2^2 + \cdots + y_{2\mu}^2 \qquad (1)$$

$$z = z_1^2 + z_2^2 + \cdots + z_{2\mu}^2 \qquad (1)$$

Random variables $x_1, x_2, \ \ , x_{2\mu}, y_1, y_2, \ \ , y_{2\mu}, z_1, z_2, \ \ , z_{2\mu}$ have Gaussian distributions:

$$p_{x_i}(x_i) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x_i - A_1)^2}{2\sigma_1^2}}, \ i = 1,2,\dots,2\mu \qquad (4)$$

$$p_{y_j}(y_j) = \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(y_j - A_2)^2}{2\sigma_2^2}}, \ j = 1,2,\dots,2\mu \qquad (5)$$

$$p_{z_k}(z_k) = \frac{1}{\sqrt{2\pi}\sigma_3} e^{-\frac{(z_k - A_3)^2}{2\sigma_3^2}}, \ k = 1,2,\dots,2\mu \qquad (6)$$

The probability density function of k-μ random variables x, y and z are:

ISBN: 978-960-474-328-5

$$p_x(x) = \frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa)\,\Omega_1} \left(\frac{x}{\Omega_1}\right)^{\mu}$$

$$e^{-\mu(1+\kappa)\left(\frac{x}{\Omega_1}\right)^2} I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\,\frac{x}{\Omega_1}\right), x \geq 0 \tag{7}$$

$$p_y(y) = \frac{2\mu(1+\kappa)^{\mu+1}}{\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa)\,\Omega_2} \left(\frac{y}{\Omega_2}\right)^{\mu}$$

$$e^{-\mu(1+\kappa)\left(\frac{y}{\Omega_2}\right)^2} I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\,\frac{y}{\Omega_2}\right), y \geq 0 \tag{8}$$

$$p_z(z) = \frac{2\mu(1+\kappa)^{\mu+1}}{\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa)\,\Omega_3} \left(\frac{z}{\Omega_3}\right)^{\mu} e^{-\mu(1+\kappa)\left(\frac{z}{\Omega_3}\right)^2}$$

$$I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\,\frac{z}{\Omega_3}\right), z \geq 0 \tag{9}$$

The ratio of the product of two k-μ random variables x and y, and k-μ random variable z is:

$$w = \frac{xy}{z}, \tag{10}$$

Then:

$$x = \frac{wz}{y}, \tag{11}$$

The first derivative of w is:

$$\dot{w} = \frac{\dot{x}y}{z} + \frac{x\dot{y}}{z} - \frac{xy\dot{z}}{z^2} \tag{12}$$

The first derivatives of k-μ random variables x, y, and z are:

$$\dot{x} = 2x_1\dot{x}_1 + 2x_2\dot{x}_2 + \cdots + 2x_{2\mu}\dot{x}_{2\mu} \tag{13}$$

$$\dot{y} = 2y_1\dot{y}_1 + 2y_2\dot{y}_2 + \cdots + 2y_{2\mu}\dot{y}_{2\mu} \tag{14}$$

$$\dot{z} = 2z_1\dot{z}_1 + 2z_2\dot{z}_2 + \cdots + 2z_{2\mu}\dot{z}_{2\mu} \tag{15}$$

The linear transformation of Gaussian random variables is Gaussian random variable. Therefore, random variables $\dot{x}$, $\dot{y}$ and $\dot{z}$ have conditional Gaussian probability density functions. The average values and variances of $\dot{x}_1$, $\dot{x}_2$, … $\dot{x}_{2\mu}$ are:

$$\overline{\dot{x}_1} = \overline{\dot{x}_2} = \cdots = \overline{\dot{x}_{2\mu}} = 0 \tag{16}$$

$$\sigma^2_{\dot{x}_1} = \sigma^2_{\dot{x}_2} = \sigma^2_{\dot{x}_{2\mu}} = \pi^2 2\sigma_1^2 f_m^2 = f_1^2 \tag{17}$$

The average values and variances of $\dot{y}_1$, $\dot{y}_2$, …, $\dot{y}_{2\mu}$ are:

$$\overline{\dot{y}_1} = \overline{\dot{y}_2} = \cdots = \overline{\dot{y}_{2\mu}} = 0 \tag{18}$$

$$\sigma^2_{\dot{y}_1} = \sigma^2_{\dot{y}_2} = \sigma^2_{\dot{y}_{2\mu}} = \pi^2 2\sigma_2^2 f_m^2 = f_2^2 \tag{19}$$

The average values and variances of $\dot{z}_1$, $\dot{z}_2$, …, $\dot{z}_{2\mu}$ are:

$$\overline{\dot{z}_1} = \overline{\dot{z}_2} = \cdots = \overline{\dot{z}_{2\mu}} = 0 \tag{20}$$

$$\sigma^2_{\dot{z}_1} = \sigma^2_{\dot{z}_2} = \sigma^2_{\dot{z}_{2\mu}} = \pi^2 2\sigma_3^2 f_m^2 = f_3^2 \tag{21}$$

The random variable $\dot{w}$ is:

$$\dot{w} = \frac{2y}{z}(x_1\dot{x}_1 + x_2\dot{x}_2 + \cdots + x_{2\mu}\dot{x}_{2\mu}) +$$

$$+ \frac{2x}{z}(y_1\dot{y}_1 + y_2\dot{y}_2 + \cdots + y_{2\mu}\dot{y}_{2\mu}) -$$

$$- \frac{2xy}{z^2}(z_1\dot{z}_1 + z_2\dot{z}_2 + \cdots + z_{2\mu}\dot{z}_{2\mu}) \tag{22}$$

The random variable $\dot{w}$ follows conditional Gaussian distribution. The main value of $\dot{w}$ is zero. The variance of $\dot{w}$ is:

ISBN: 978-960-474-328-5

$$\sigma_{\dot{w}}^2 = \frac{yy^2}{z^2}\left(x_1^2\sigma_{\dot{x}_1}^2 + x_2^2\sigma_{\dot{x}_2}^2 + \cdots + x_{2\mu}^2\sigma_{\dot{x}_{2\mu}}^2\right) +$$

$$+\frac{yx^2}{z^2}\left(y_1^2\sigma_{\dot{y}_1}^2 + y_2^2\sigma_{\dot{y}_2}^2 + \cdots + y_{2\mu}^2\sigma_{\dot{y}_{2\mu}}^2\right) +$$

$$+\frac{yx^2y^2}{z^4}\left(z_1^2\sigma_{\dot{z}_1}^2 + z_2^2\sigma_{\dot{z}_2}^2 + \cdots + z_{2\mu}^2\sigma_{\dot{z}_{2\mu}}^2\right) =$$

$$= \frac{yy^2}{z^2}xf_1^2 + \frac{yx^2}{z^2}yf_2^2 + \frac{yx^2y^2}{z^3}zf_3^2 =$$

$$= \frac{yxy}{z^2}\left(xf_1^2 + yf_2^2 + \frac{xy}{z}f_3^2\right) =$$

$$= \frac{yxy}{z^2}\left(yf_1^2 + \frac{wz}{y}f_2^2 + \frac{y}{z}f_3^2\frac{wz}{y}\right) =$$

$$= \frac{yxy}{z^2y}\left(y^2f_1^2 + wzf_2^2 + wyf_3^2\right) \qquad (23)$$

The conditional Gaussian distribution of $\dot{w}$ is:

$$p_{\dot{z}}(\dot{w}/wyz) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{z}}}e^{-\frac{\dot{z}^2}{2\sigma_{\dot{z}}^2}} =$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{z}{2\sqrt{x}\sqrt{y^2f_1^2 + wzf_2^2 + wyf_3^2}}$$

$$e^{-\frac{\dot{w}^2z^2}{8x\left(y^2f_1^2 + wzf_2^2 + wyf_3^2\right)}} \qquad (24)$$

Using previously obtained expression, the joint probability of ratio of two k-μ random variables and k-μ random variable and there's first derivatives can be evaluated. Also, the probability density function of the first derivative of the ratio can be determined.

2 The Level Crossing Rate of the Ratio of Product of Two k-μ Random Variables and k-μ Random Variable

The joint probability density function of w, $\dot{w}$, y and z is:

$$p_{w\dot{w}yz}(w\dot{w}yz) = p_{\dot{w}}(\dot{w}/wyz) \cdot p_{wyz}(wyz) =$$

$$= p_{\dot{w}}(\dot{w}/wyz) \cdot p_y(y) \cdot p_z(z) \cdot p_w(w/yz) \qquad (25)$$

The conditional probability density function of w can be derived using transformation method:

$$p_w(w/yz) = \left|\frac{dx}{dw}\right| p_x\left(\frac{wz}{y}\right) \qquad (26)$$

where is:

$$\frac{dx}{dw} = \frac{z}{y} \qquad (27)$$

By substituting previous expression in the expression for joint probability density function (JPDF) of w, $\dot{w}$, y and z, it can be written:

$$p_{w\dot{w}yz}(w\dot{w}yz) = p_{\dot{w}}(\dot{w}/wyz) \cdot p_y(y) \cdot$$

$$\cdot p_z(z) \cdot \frac{z}{y} p_x\left(\frac{wz}{y}\right) \qquad (28)$$

The joint probability density function of w and $\dot{w}$ is:

$$p_{w\dot{w}}(w\dot{w}) = \int_0^\infty dy \int_0^\infty dz\, p_{w\dot{w}yz}(w\dot{w}yz) =$$

$$= \int_0^\infty dy \int_0^\infty dz \frac{z}{y} p_{\dot{w}}(\dot{w}/wyz) p_y(y) p_z(z) p_x\left(\frac{wz}{y}\right) \qquad (29)$$

The average level crossing rate of w can be calculated as the average value of the first derivative of w:

$$N_w = \int_0^\infty d\dot{w}\, \dot{w} p_{w\dot{w}}(w\dot{w}) = \int_0^\infty d\dot{w}\, \dot{w}$$

$$\int_0^\infty dy \int_0^\infty dz \frac{z}{y} p_{\dot{w}}(\dot{w}/wyz) p_y(y) p_z(z) p_x\left(\frac{wz}{y}\right) =$$

$$= \int_0^\infty dy \int_0^\infty dz \frac{z}{y} p_x\left(\frac{wz}{y}\right) p_y(y) p_z(z)$$

$$\left(\int_0^\infty d\dot{w}\, \dot{w} p_{\dot{w}}(\dot{w}/wyz) \right) =$$

$$= \int_0^\infty dy \int_0^\infty dz \frac{z}{y} p_x\left(\frac{wz}{y}\right) p_y(y) p_z(z) \frac{\sigma_{\dot{w}}}{\sqrt{2\pi}} =$$

$$= \int_0^\infty dy \int_0^\infty dz \frac{\sqrt{2}^2}{y\sqrt{2\pi}} \frac{2\sqrt{w}}{\sqrt{2y}}$$

$$\sqrt{y^2 f_1^2 + wz f_2^2 + wy f_3^2} p_x\left(\frac{wz}{y}\right) p_y(y) p_z(z) \tag{30}$$

The level crossing rate of the ratio of product of two k-μ random variables and k-μ random variable is presented in Fig. 1.

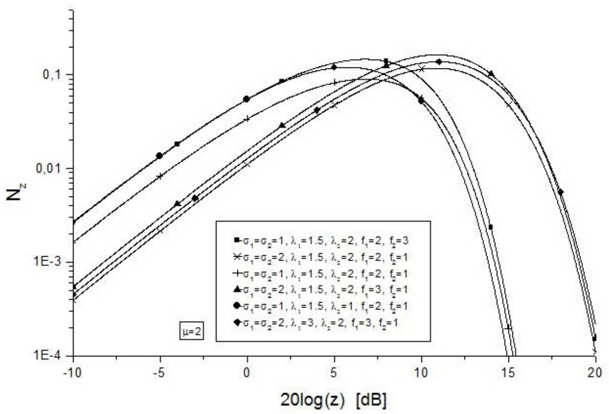

Fig. 1. The level crossing rate of the ratio of product of two k-μ random variables and k-μ random variable

From this figure it can be seen the influence of different fading parameters.

4 Conclusion

In this paper, the ratio of product of two k-μ random variables and k-μ random variable is determined. The product of two k-μ random variables in numerator of the ratio represents desired signal envelope subjected simultaneously to two independent multipath k-μ fading. The k-μ random variable in denominator of the ratio represents cochannel interference envelope affected to multipath k-μ fading. In interference limited k-μ multipath fading line-of-sight environment, the ratio of product of two k-μ random variables and k-μ random variable represent signal-to-interference envelopes ratio and can be used for evaluation system performance as the outage probability, the bit error probability and the system capacity. The outage probability, the bit error probability and the system capacity are the first order system performance.

In this paper the second order system performance such as average level crossing rate of the ratio of product of two k-μ random variables and k-μ random variable is analyzed. The joint probability density function of the ratio of product of two k-μ random variables and k-μ random variable and its first derivative is calculated. Using this expression, the average level crossing rate of the ratio of product of two k-μ random variables and k-μ random variable is obtained.

This result can be used for evaluation the average fade duration of wireless communication system operating over composite multipath k-μ fading line-of-sight environment in the presence of cochannel interference which suffer multipath k-μ fading. The average fade duration can be obtained as the ratio of the outage probability and the average level crossing rate. It is also an important system measure.

Acknowledgment

This work has been funded by the Serbian Ministry for Science under the projects III-44006, TR-33035 and TR-32052.

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