A clustering first – route second method for the solution of many-to-many Dial a Ride problem

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Abstract: A clustering first-route second method to solve the many-to-many Dial a Ride (DAR) problem has been proposed. The clustering phase is the main novelty of the method, taking into account both space and time variables to realize the clusters.

The procedure has been tested on a real network during the organization of the mobility of students between some offices and Departments of “Roma Tre” University (Rome, Italy), obtaining good results in terms of waiting time and number of served users. The computational times (few seconds) promise the achievement of an on line service.

Key-Words: - Dial a ride problem, cluster method, clustering first-route second, Greedy Algorithm, Pickup and delivery problem

1 Introduction

The Dial a Ride (DAR) systems belong to the demand responsive transit systems, usually adopted for supplying areas of low ridership and/or low population density (low demand value, [6]).

The problem of identifying a path for one/more vehicles involved in a DAR system, and more generally the Pickup and delivery problem from which DAR derives, has been widely discussed in literature; for an accurate review see Diana and Dessouky, 2004 [5]. The DAR is a NP-hard problem and the earliest relevant publications date back to 1980’s [12]. Its solution changes if we are in off-line or on-line contexts, if single vehicle or more vehicles are considered, if capacity constraint and/or constraints on the time windows of the generic i-th request are considered [4].

The adopted algorithms are generally heuristic algorithms. The “Tabu search” metaheuristic methods have been used either to solve off-line DAR problems [3] or on-line ones [1]; also Simulated Annealing ([10], [13]), Genetic Algorithms and Evolutionary algorithms ([8], [2]) have been widely adopted.

Healy and Moll (1995, [7]) have proposed an extension of classic “local search” algorithm in order to obtain a better quality of solutions. Same idea has been recently followed in [9] with the development of an evolutionary Local Search (ELS) method.

Diana and Dessouky (2004, [5]) developed a “regret” insertion heuristic method computationally more complex than a simple insertion heuristic one, but with a better quality of solutions with respect to a “local search” algorithm.

In the following paragraphs the analytical formulation of the DAR problem is reported and the new proposed method for the many-to-many case is explained; finally, the results of a real application are shown.

2 Problem definition

The analytical formulation is reported below.

Given a graph G=[N,A], with N and A respectively equal to the set of nodes and links, once defined the following variables:
\( \mathbb{z}_{i}^{k} = \begin{cases} 1 & \text{if the } i \text{-call request is satisfied by the } k \text{-vehicle} \\ 0 & \text{otherwise} \end{cases} \)

\( \mathbb{x}_{i,j}^{k} = \begin{cases} 1 & \text{if the } k \text{-vehicle travels from } i \text{-point to } j \text{-point} \\ 0 & \text{otherwise} \end{cases} \)

\( T_{i}^{k} = \text{starting time of } k \text{-vehicle from } i \text{-point} \)

where:

\( i \) = destination of the \( i \)-call request

\( i' \) = origin of the \( i \)-call request

\( K \) = set of vehicles

\( V \) = set of call requests

\( N^+ \) = origins (pick up points)

\( N^- \) = destinations (delivery points)

\( N^+ U N^- = N \) = nodes to be served

the Objective function (OF) to be minimized is:

\[
\min f(x, T, z) \tag{1}
\]

subject to the following constrains:

\[
\sum_{k \in K} z_i^k = 1 \quad \forall i \in V \tag{2}
\]

\[
\sum_{j \in V} x_{i,j}^k = \sum_{j \in V} x_{j,i}^k = z_i^k = \sum_{j \in V} x_{i,j}^k = \sum_{j \in V} x_{j,i}^k 
\forall i \in V \quad \forall k \in K
\tag{3}
\]

\[
T_{0}^{k} = 0 \quad \forall k \in K
\tag{4}
\]

\[
T_{i}^{k} \leq T_{i'}^{k} \quad \forall i \in V
\tag{5}
\]

\[
x_{i,j}^{k} \geq 1 \Rightarrow T_{i}^{k} + t_{i,j} \leq T_{j}^{k} \quad \forall i, j \in N \quad \forall k \in K
\tag{6}
\]

\[
a_i \leq T_{i}^{k} \leq b_i \quad \forall i \in V \quad \forall k \in K
\tag{7}
\]

\[
x_{i,j}^{k} \leq 0.1; \quad z_i^k \leq 0.1; \quad T_{i}^{k} \geq 0 \quad \forall i, j \in N \quad \forall k \in K
\tag{8}
\]

### 2.1 Solution method

In order to solve the D.A.R. many-to-many problem, a cluster first – route second heuristic method is adopted.

#### 2.1.1 Clustering phase

The total service time \( T \) is divided into \( n_t \) intervals with \( i = 1, \ldots, n_t \), where \( t \) is identified as the time needed on average by a vehicle to visit all the nodes; the transport demand (Origin-Destination OD movements) is distributed into different clusters, according to the time interval of the service request submission: trips with service request within \( n_1 \), trips with service request between \( n_1 \) and \( n_2 \) and so on (time criteria).

For each cluster it must be verified that there are not simultaneously OD movements in opposite directions \((s_{ij} \text{ and } s_{ji})\), because it is assumed that the same path cannot serve both the trips from \( i \) to \( j \) and from \( j \) to \( i \); if such condition is not verified, it is necessary to separate clusters into sub clusters on the basis of directionality criteria (space criteria):

1. vectors connecting origin to destination of each movement are plotted;
2. then these are shown in a Cartesian system with the base of each vector fixed in the origin of the axes;
3. all the movements falling, with this construction, in the first and fourth quadrant or second and third quadrant must belong to the same matrix (to the same cluster).

For the matrices relating to the first interval \( n_1 \), the starting nodes are then chosen according to the following criteria:

- the starting node is the one with the highest number of generated movements, and in case of nodes with equal number of generated movements, the one with the lower number of attracted movements;
- the starting nodes of clusters relative to the same time interval must be different, if some of them are the same, they must be diversified even choosing nodes with a number of lower number of generated movements.

Once defined the starting node for a generic cluster in \( n_1 \), all movements with destination in the starting node cannot be served by the path, as it is not closed; so they are included in another existing cluster relative to the same time interval \( n_1 \).

Then, the paths for all the clusters relative to the first time interval \( n_1 \) are identified through the application of the routing algorithm. Computing the paths, the end nodes are identified and they will be the starting nodes relative to the clusters of the second time interval \( n_2 \).
The choice of which end node is better suited to be the starting node for each cluster of the following time interval depends on the criteria previously reported. Also in this case the movements with destination in the starting node will be included into another existing cluster relative to the same time interval.

The starting nodes assignment process continues until all time intervals $n_i$, previously identified, are completed.

### 2.1.1 Routing phase
The general scheme of the second stage of the proposed algorithm is the following:

1. a starting path is defined through the resolution of a Travelling Salesman Problem (TSP) and the O.F. value is computed (9);
2. from the previous starting path, the sequence of a $k$ number of nodes is modified, thus creating a new path;
3. if the new path verifies the constraints, the O.F. value is computed and compared with the previous one;
4. if this O.F. value is smaller than the previous one, the new path is kept, otherwise the previous path is kept;
5. a new modification of the nodes sequence is carried out (see point 2.) and the process continues until a fixed number of iterations.

The heuristic “Greedy Algorithm” is adopted in order to solve the TSP, with an additional constrain concerning the exclusion, in the path, of the nodes without DAR service request.

The O.F. value is computed as follows [11]:

$$F.O. = A + \mu B$$

(9)

With A equal to the path travel time of vehicle (or vehicles) and B as the average on board time:

$$A = \sum_{i=1}^{V} t_i; \quad B = \frac{\sum_{i=1}^{U} \sum_{u=1}^{V} T_{uu}^v \cdot q_{uu}^v}{\sum_{i=1}^{U} \sum_{u=1}^{V} q_{uu}^v} \quad v = 1...V, \ u = 1...U$$

(10)

where $t_i$ represents the total path travel time of vehicle $v$, $T_{uu}^v$ the $u$ users’ travel time on board of the vehicle $v$ and $q_{uu}^v$ the number of users of group $u$ on the vehicle $v$; the $\mu$ term is a weight coefficient.

The O.F. is computed for the path derived from the solution of the TSP and it is set as the minimum value in the initialization phase of the algorithm, then for each new generated path, the following criteria have to be verified:

1. the served users are equal or greater than the served users from the previous path;
2. $B \leq \alpha T$ direct connection, i.e. the average on board time has to be lower or equal than $\alpha$ times the time to serve the same number of users with a direct origin-destination connection.

At the end of the process the optimal path will be the one with the minimum value of the O.F. with respect to the fixed constrains.

### 3 Case study
The proposed method has been applied to supply the students mobility demand between some offices and Departments of the “Roma Tre” University (Rome, Italy), considering a many-to-many DAR service during the off-peak hours of the day.

The network consists of 6 nodes, representing 6 main points of the University located between the “Trastevere” rail station and the district of EUR in the Southern of Rome.

The input data of the model are:

- Origin/Destination (OD) demand matrices (for both the morning and the afternoon off-peak hours);
- Travel time values between the OD pairs (generated using an assignment model);
- Time in which each user wants to access the DAR service.

Starting from these data, the cluster method has been applied, generating the following 6 clusters (Table1):

<table>
<thead>
<tr>
<th>Morning off-peak hour</th>
<th>Afternoon off-peak hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00-11:30</td>
<td>15:00-15:30</td>
</tr>
<tr>
<td>11:30-12:00</td>
<td>15:30-16:00</td>
</tr>
<tr>
<td>Cluster A</td>
<td>Cluster B</td>
</tr>
<tr>
<td>Cluster C</td>
<td>Cluster D</td>
</tr>
<tr>
<td>Cluster D*</td>
<td>Cluster D*</td>
</tr>
</tbody>
</table>

Then, applying the routing process to each cluster (Table 2, Table 3):
Table 2 - Routing results (morning off-peak hour)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>No. served points</th>
<th>Travel time (minutes)</th>
<th>Average on board travel time (minutes)</th>
<th>O.F. (minutes)</th>
<th>% served users</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>23</td>
<td>8</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>21</td>
<td>7</td>
<td>28</td>
<td>82%</td>
</tr>
<tr>
<td>B*</td>
<td>2</td>
<td>9</td>
<td>9</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

The obtained results have been then compared with what happens if the same OD demand is supplied by a traditional bus service (called "Unibus" line) or by a taxi service.

Table 3 - Routing results (afternoon off-peak hour)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>No. served points</th>
<th>Travel time (minutes)</th>
<th>Average on board travel time (minutes)</th>
<th>O.F. (minutes)</th>
<th>% served users</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>6</td>
<td>19</td>
<td>6</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>21</td>
<td>7</td>
<td>28</td>
<td>96%</td>
</tr>
<tr>
<td>D*</td>
<td>2</td>
<td>6</td>
<td>6</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

The comparison with the Unibus service (Fig.1-Fig.2) shows that the proposed model generates paths with high effectiveness and efficiency through low values of the average on board time and the total traveled distances.

Fig. 2 – Comparison of average on board travel times between DAR and Unibus

The comparison with the taxi service (Tab.4-Tab.5) shows that in the morning off-peak hour the difference of the average on board time between the two systems is very low, while the difference in terms of total traveled time is around 10%. In the afternoon off-peak hour, since the OD demand is five times higher than the morning OD demand, total traveled time for the DAR service is greater than 50% respect to the taxi service; however the differences in terms of average on board times remain less than 50% and this suggests the goodness of the generated paths.

Table 4 - Comparison D.A.R. - Taxi (morning off-peak hour)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Average on board time (minutes)</th>
<th>Average on board time (minutes)</th>
<th>Total travel time (minutes)</th>
<th>Total travel time (minutes)</th>
<th>Increment of the Total travel time [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D.A.R.</td>
<td>TAXI</td>
<td>D.A.R.</td>
<td>TAXI</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>8</td>
<td>7.5</td>
<td>124</td>
<td>112</td>
<td>+11</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>6</td>
<td>61</td>
<td>55</td>
<td>+11</td>
</tr>
<tr>
<td>B*</td>
<td>9</td>
<td>9</td>
<td>27</td>
<td>27</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 5 - Comparison D.A.R. - Taxi (afternoon off-peak hour)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Average on board time (minutes)</th>
<th>Average on board time (minutes)</th>
<th>Total travel time (minutes)</th>
<th>Total travel time (minutes)</th>
<th>Increment of the total travel time [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D.A.R.</td>
<td>TAXI</td>
<td>D.A.R.</td>
<td>TAXI</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>4</td>
<td>90</td>
<td>58</td>
<td>+55</td>
</tr>
<tr>
<td>D</td>
<td>7</td>
<td>4</td>
<td>237</td>
<td>144</td>
<td>+65</td>
</tr>
<tr>
<td>D*</td>
<td>6</td>
<td>6</td>
<td>30</td>
<td>30</td>
<td>-</td>
</tr>
</tbody>
</table>

The final step of the study has been to apply the method to the same network assuming an increase in demand (in particular the demand has been doubled) and distributing randomly the increase between the different OD pairs. In this last case, the results showed a proportion of served users equal to about 70%, and this is probably a result of an imbalance of the distribution of the trips between the clusters related to the same time interval.

4 Conclusion

The proposed model has proved to be very effective in relation to the generated paths on the adopted test network: the results have been optimal in terms of on board time, of total traveled distances and of users served, especially if compared with a standard bus service. The difference in terms of on board time, with respect to a taxi service, showed the "convenience" that the generated paths of the DAR would be able to offer to their potential users. Finally the computational time of the algorithm are about few seconds, promising factor for the realization of an on-line DAR service.

However, some issues remain to be explored: firstly, it is appropriate to improve some steps of the clustering procedure, which is considered the most innovative element of the proposed method. In relation to this procedure, a revision is needed in order to achieve a "balance" of the OD demand between the different clusters. A more distributed OD demand between the clusters allows an increase in the percentage of users served. Further analyses should be made in relation to the geometric criterion of subdivision of the trips in the opposite direction as it has not been tested in any phase of the routing algorithm.

With regard to the constraints, it must be emphasized the fact that the model actuality, once reached the capacity, exits from the process. The future development involves the insertion of the capacity constraint as an active constraint in the procedure, together with the maximum size of the vehicle fleet. The last step of the development of the model consists in including the users of the "last minute". At the moment the model is able to serve only the users within a certain timeframe prior to the start of the service, these users are advised of the pick-up time and this factor is a strict constraint for all users that requests to access to the service while the vehicle is already moving. The complete implementation and study of all these elements will allow the implementation of the model in order to create a commercial tool.

References:


