MHD effect on convective boundary layer flow of a viscoelastic fluid embedded in porous medium with Newtonian heating

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Abstract: - A study has been carried out to investigate the effect of magnetic field presence on the mixed convection boundary layer flow of viscoelastic fluid past a sphere with Newtonian heating embedded in a porous medium. The governing non-similar partial differential equations are transformed into dimensionless forms and then solved numerically using the Keller-Box method. Results on the effect of the viscoelastic parameter K, magnetic parameter M, and mixed convection parameter λ on the velocity and temperature of the fluid flow have been shown graphically and discussed briefly. Numerical results of skin friction as well as wall temperature are also presented in tabular form. The comparison of results obtained in the present work with the existing publication is found in excellent agreement.

Key-Words: - Mixed convection Newtonian heating Viscoelastic Keller box porous medium

1 Introduction

During these recent years, the interest of flows in viscoelastic fluids has grown considerably because of their applications in engineering and several industrial-manufacturing processes involving petroleum drilling, manufacturing of foods and papers. On the other hand, this type of fluid also has some applications involving atomization such as paints, coating, inks, jet fuels and possible to reduce frictional drag on the hulls of ships and submarines. Based on our literature survey, there has been extensive research done on viscoelastic fluid in last few decades [1-14].

Magnetohydrodynamics (MHD is a branch of science that deals with the motion of electrically conducting fluids under the presence of magnetic field. This situation is essentially one of mutual interaction between the fluid velocity field and the magnetic field; the motion affects the magnetic field and the magnetic field affects the motion. Thus, the term MHD attempts to convey this relationship [15]. Investigations on MHD flow problems in non-Newtonian fluids focusing on viscoelastic fluids have attracted a number of researchers. For example, Abel et al. [16] concentrate on buoyancy force and thermal radiation effects in MHD boundary layer viscoelastic fluid flow over continuously moving stretching surface while Cortell [17] investigates the effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet. Effect of the variable viscosity and thermal conductivity on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet has been investigated by Salem [18].

The heat transfer in porous medium is a process involving its application in a broad spectrum of disciplines ranging from chemical engineering to geophysics. Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. Nandeppanavar et al. [19] have studied the flow and heat transfer characteristics of a viscoelastic fluid in a porous medium over an impermeable stretching sheet with viscous dissipation using the power series method (Kummer's function) while Hayat et al. [20] have conducted an investigation on heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid.

Motivated by studies conducted and mentioned above, this present paper aims to study the effects of magnetic field presence on mixed convection boundary layer flow in a viscoelastic fluid past a sphere with Newtonian heating embedded in porous medium. The governing boundary layer equations are first transformed into a system of dimensionless equations using the dimensionless variables, and then attain non-similar equations by using similarity transformation. Later, a finite difference scheme called the Keller-Box method is applied in order to solve the problem.

2 Governing Equations

Consider a steady mixed convection boundary layer flow about an isothermal sphere of radius *a*, where it is placed in a viscoelastic fluid with uniform ambient velocity U_∞. Figure 1 illustrates the physical model and coordinate system of the problem. For the case of Newtonian heating, the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature as proposed by Merkin [21],

$$\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} = -h_s T_w, \tag{1}$$

where T_w is the unknown local surface temperature and h_s is a coefficient of proportionality for the surface heat flux.

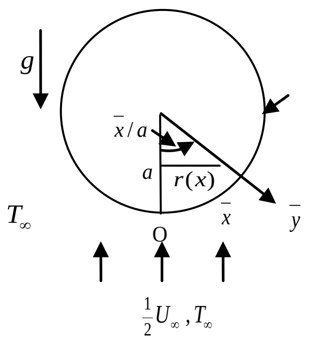

Figure 1: Physical model and coordinate system

If $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, and T is the fluid temperature, then the boundary layer equations can be written as follow:

Continuity equation:

$$\frac{\partial}{\partial \bar{x}}\left(\bar{r}\ \bar{u}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{r}\ \bar{v}\right) = 0, \tag{2}$$

Momentum equation

$$\begin{aligned} \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} &= \bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \\ &+ \frac{k_0}{\rho}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right] \\ &- \left(\frac{\sigma B_0}{\rho} + \frac{\upsilon}{K^*}\right)\left(\bar{u} - \bar{u}_e\right) \\ &+ g\,\beta(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right), \end{aligned} \tag{3}$$

Energy equation:

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{4}$$

subject to the boundary conditions

$$\begin{aligned} &\bar{u} = \bar{v} = 0, \quad \frac{\partial T}{\partial \bar{y}} = -h_s T \quad \text{on} \quad \bar{y} = 0, \\ &\bar{u} = \bar{u}_e(\bar{x}), \ \ \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \ \ T = T_\infty, \ \ \text{as} \ \ \bar{y} \to \infty. \end{aligned} \tag{5}$$

where ρ, g, β, μ, k_0, and α are the density, gravitational acceleration, coefficient of thermal expansion, dynamic viscosity, vortex viscosity and thermal diffusivity of the fluid, respectively. Meanwhile, $\bar{u}_e(\bar{x})$ is the local free stream velocity outside the boundary layer and $\bar{r}(\bar{x})$ is the radial distance from the symmetrical axis to the surface of the sphere which are given by

$$\bar{r}(\bar{x}) = a\sin\left(\frac{\bar{x}}{a}\right), \quad \bar{u}_e(\bar{x}) = \frac{3}{2}U_\infty\sin\left(\frac{\bar{x}}{a}\right) \tag{6}$$

Then, the following dimensionless variables are introduced.

$$\begin{aligned} &x = \frac{\bar{x}}{a},\ y = \mathrm{Re}^{1/2}\left(\frac{\bar{y}}{a}\right), r(x) = \frac{\bar{r}(\bar{x})}{a}, \\ &u = \frac{\bar{u}}{U_\infty}, v = \mathrm{Re}^{1/2}\left(\frac{\bar{v}}{U_\infty}\right), u_e(x) = \frac{\bar{u}_e(\bar{x})}{U_\infty}. \\ &\theta = \mathrm{Re}^{1/2}\left(T - T_\infty\right)/\left(q_w a/k\right), \end{aligned} \tag{7}$$

where $\mathrm{Re}=\frac{U_\infty a}{\upsilon}$ is the Reynolds number.

By substituting Equation (7) into Equations (2)-(4), the following dimensionless equations are obtained.

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0, \quad (8)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{\partial^2 u}{\partial y^2} +\mathrm{K}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right] -(\mathrm{M+P})(u-u_e)+\lambda\theta\sin(x), \quad (9)$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}, \quad (10)$$

where K is the viscoelastic parameter, M is the magnetic parameter, P is the porosity and λ is the mixed convection parameter which are denoted as

$$\mathrm{K}=\frac{U_\infty k_0}{a\rho\upsilon}, \quad \lambda=\frac{\mathrm{Gr}}{\mathrm{Re}^{5/2}}, \quad M=\frac{\sigma B_0 a}{\rho U_\infty}, \quad P=\frac{\upsilon}{K^*}\frac{a}{U_\infty}. \quad (11)$$

with $\mathrm{Gr}=g\beta(T_w-T_\infty)a^3/\upsilon^3$ being the Grashof number. It is worth mentioning that $\lambda>0$ is for assisting flow and $\lambda<0$ is for opposing flow, respectively.

The boundary conditions (5) now become

$$u=v=0, \frac{\partial T}{\partial y}=-\gamma(1+\theta) \quad \text{on} \quad y=0, \qquad u_e=\frac{3}{2}\sin x, \quad \frac{\partial u}{\partial y}=0, \quad \theta=0, \quad \text{as} \quad y\to\infty. \quad (12)$$

3 Solution Procedures

In order to solve Equations (8)-(10), it is assumed that as $x\to 0$, $\frac{\sin x}{x}\to 1$.
Furthermore, an appropriate transformation called similarity transformation is introduced in the following form:

$$\psi=xr(x)f(x,y), \quad \theta=\theta(x,y), \quad (13)$$

where ψ is the stream function defined as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y}, v=-\frac{1}{r}\frac{\partial\psi}{\partial x}, \quad (14)$$

Which is fully satisfied Equation (8). By applying the similarity variables (13) into Equations (9)-(10), the following non-similar equations are obtained.

$$\frac{\partial^3 f}{\partial y^3}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{9}{4}\frac{\sin x\cos x}{x} +\left(1+x\frac{\cos x}{\sin x}\right)f\frac{\partial^2 f}{\partial y^2}+ \mathrm{K}\left[\begin{array}{l}2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-\left(1+\frac{x}{\sin x}\cos x\right)\\ \left(f\frac{\partial^4 f}{\partial y^4}+\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right)\end{array}\right] -(M+P)\left(\frac{\partial f}{\partial y}-\frac{\sin x}{x}\right)+\lambda\theta\frac{\sin x}{x} =x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) +\mathrm{K}x\left[\begin{array}{l}\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4}-\frac{\partial^3 f}{\partial y^3}\frac{\partial^2 f}{\partial x\partial y}\\ -\frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^4}+\left(\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^2\partial x}\right)\end{array}\right], \quad (15)$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}+\left(1+\frac{x}{\sin x}\cos x\right)f\frac{\partial\theta}{\partial y} =x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right), \quad (16)$$

and the boundary conditions (12) become

$$f=0, \quad \frac{\partial f}{\partial y}=0, \quad \frac{\partial\theta}{\partial y}=-\gamma(1+\theta), \quad as \ y=0, \qquad \frac{\partial f}{\partial y}\to\frac{3}{2}\frac{\sin x}{x}, \quad \frac{\partial^2 f}{\partial y^2}\to 0, \theta\to 0, \quad y\to\infty. \quad (17)$$

It is noticed that at the lower stagnation point of the sphere ($x \approx 0$), Equations (15)-(16) are reduced to

$$f''' + 2ff'' - f'^2 + \frac{9}{4} - (\mathrm{M+P})(f' - 1) + \lambda\theta + 2\mathrm{K}(f'f''' - ff'''' - f''^2) = 0, \quad (18)$$

$$\frac{1}{\Pr}\theta'' + 2f\theta' = 0, \quad (19)$$

with the boundary conditions

$$f(0) = f'(0) = 0, \quad \theta'(0) = -\gamma(1+\theta(0)),$$
$$f'(\infty) \to \frac{3}{2}, \quad f''(\infty) \to 0, \quad \theta(\infty) \to 0. \quad (20)$$

where prime denotes differentiation with respect to *y*.

4 Results and Discussion

The system of partial differential equations (15) and (16) subjects to the boundary conditions (17), and the system of ordinary differential equations (18) and (19) subjects to the boundary conditions (20) are solved numerically using the Keller-Box method: which is very well described in the book by Cebeci and Bradshow [22], for some values of viscoelastic parameter K, mixed convection parameter λ, and magnetic parameter M for the value of Prandtl number 7. In order to ensure the accuracy and convergence of the numerical solution, the step size Δy has been optimized and the results presented are independent of the step sizes at least up to the six decimal places.

The convergence criterions are based on the relative difference between the current and previous iteration values of velocity and temperature gradients at wall. The present results for values $f''(0)$ and $\theta(0)$ are compared with those of Salleh *et al*. [23] in order to validate the numerical results obtained. The comparison shows that the numerical solutions (see Table 1) obtained by the present authors concurs very well with those of existing publication. It can be concluded that the numerical method works efficiently for present problem and the authors are confident that the results presented are accurate.

Table 2 shows the numerical values of skin friction $f''(0)$ and wall temperature $\theta(0)$ for the fixed values of mixed convection parameter $\lambda = 0.5$, Prandtl number Pr = 7, magnetic parameter M = 1.0, porosity parameter P = 0.5 and various values of viscoelastic parameter K, respectively. It can be seen from the table that as the viscoelastic parameter K increases, the values of skin friction are decreased while wall temperature are increased.

Numerical values of skin friction $f''(0)$ and wall temperature $\theta(0)$ for various values of mixed convection parameter λ at Pr = 7, M = 1.0, P = 0.5 and K = 0.2 are shown in Table 3. It is found that as λ increases, it leads to increasing skin friction and reducing wall temperature. This behavior can also be found in Salleh *et al*. [23] for the viscous fluid case.

The velocity and temperature profiles near lower stagnation point ($x = 0$) for some values of mixed convection parameter λ when Pr = 7, K = 0.2, M = 1.0 and P = 0.5 are given in Figs. 2 and 3. It is found that the fluid velocity is increasing while the temperature profile decreasing as the mixed convection parameter, λ is increased.

The effect of viscoelastic parameter on velocity and temperature profile at the lower stagnation point at Pr = 7.0, M = 1.0, P = 0.5 and $\lambda = 0.5$ are illustrated in Figs. 4 and 5. Based on Fig. 4, it is noticed that the fluid velocity is decreasing when the value of viscoelastic parameter K is increased. Moreover, the fluid velocity is lower for a viscoelastic fluid to be compared to the fluid velocity for a Newtonian fluid (K = 0) and it can be said that the thickness of the velocity boundary layer for a viscoelastic fluid is higher than for a Newtonian fluid. Further, Fig. 5 shows that by increasing the value of viscoelastic parameter K may lead to lower fluid temperature up to one point (y = 1.3) and lead to higher fluid temperature for other y. This behavior reflects the coupling of the energy equation to the momentum equation through the temperature dependent body forces.

The velocity and temperature profiles for different values of magnetic parameter M are displayed in Figs. 6 and 7. It can be observed that both velocity and temperature of the flow are decreasing as the value of magnetic parameter is increased.

5 Conclusion

In this paper, the problem of steady MHD mixed convection boundary layer flow of a viscoelastic fluid past a sphere with Newtonian heating embedded in porous medium has been investigated numerically. The governing boundary layer equations are transformed into a dimensionless form and the resulting nonlinear systems of partial differential equations are solved numerically using the Keller-Box method. This study has revealed how the mixed convection λ, viscoelastic parameter K as well as magnetic parameter M affect the fluid flow significantly. From the results obtained, it is revealed that

- An increase on the values of mixed convection parameter λ may lead to higher fluid velocity and reduce the fluid temperature.
- As viscoelastic parameter K is increased, both fluid velocity and temperature are decreased.
- Both velocity and temperature of the flow are reduced when the Prandtl number Pr is increased.

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APPENDIX

Table 1: Comparison of $f''(0)$ and $\theta(0)$ for various values of λ when K = 0.0, Pr = 7, M = 0.0 and P = 0.0

λ	Pr = 7.0 Salleh *et al.* [23]		Present results	
	$f''(0)$	$\theta(0)$	$f''(0)$	$\theta(0)$
0.05	2.4235	1.1174	2.423580	1.115154
0.10	2.4365	1.1141	2.436594	1.112504
0.50	2.5378	1.0933	2.537790	1.092512
1.00	2.6579	1.0709	2.657596	1.070125
2.00	2.8808	1.0326	2.880535	1.031973
3.00	3.0854	1.0009	3.085097	1.000291
4.00	3.2760	0.9739	3.275562	0.973278
5.00	3.4552	0.9503	3.454659	0.949787
6.00	3.6249	0.9296	3.624330	0.929047
7.00	3.7867	0.9110	3.786018	0.910512
8.00	3.9415	0.8943	3.940828	0.893782
9.00	4.0904	0.8790	4.089630	0.878558
10.00	4.2340	0.8651	4.233122	0.864606
20.00	5.4682	0.7679	5.466887	0.767471

Table 2: Values of $f''(0)$ and $\theta(0)$ for various values of K when $\lambda =$ 0.5, Pr = 7, M = 1.0 and P = 0.5

K	$f''(0)$	$\theta(0)$
0.1	2.321080	1.203917
0.5	1.657244	1.529397
1.0	1.320725	1.843319
2.0	1.019421	2.371827
3.0	0.870813	2.851849
4.0	0.779390	3.316204
5.0	0.716645	3.778204
6.0	0.670675	4.244801
7.0	0.635511	4.720131
8.0	0.607788	5.206843
9.0	0.585441	5.706693
10.0	0.567123	6.220847

Table 3: Values of $f''(0)$ and $\theta(0)$ for variousvalues of λ when K = 0.2, Pr = 7, M = 1.0 and P = 0.5

λ	$f''(0)$	$\theta(0)$
0.05	1.554009	1.580540
0.10	1.657244	1.529397
0.50	1.573778	1.570378
1.00	1.753193	1.485792
2.00	1.924419	1.415965
3.00	2.075756	1.361417
4.00	2.212711	1.316891
5.00	2.338641	1.279430
6.00	2.455773	1.247205
7.00	2.565671	1.219009
8.00	2.669487	1.194004
9.00	2.768096	1.171585
10.00	2.862183	1.151301
20.00	3.638658	1.015844

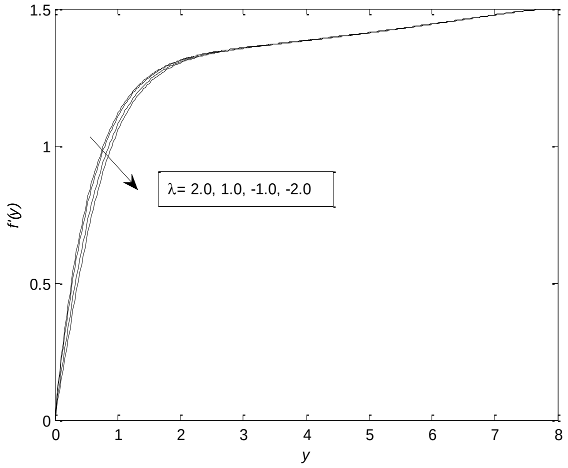

Figure 2: Velocity profile for Pr = 7, K = 0.2, M = 1.0, P = 0.5 and various values of λ

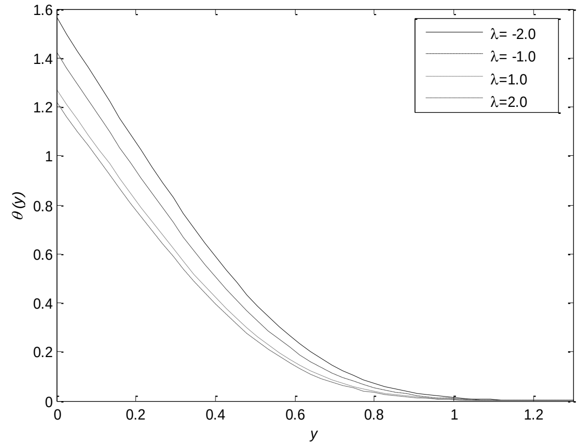

Figure 3: Temperature profile for Pr = 7, K = 0.2, M = 1.0, P = 0.5 and various values of λ

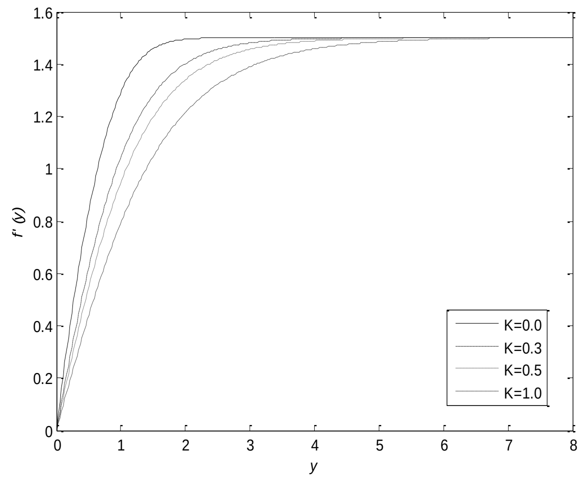

Figure 4 Velocity profile for Pr = 7, λ = 0.5, M = 1.0, P = 0.5 and various values of K

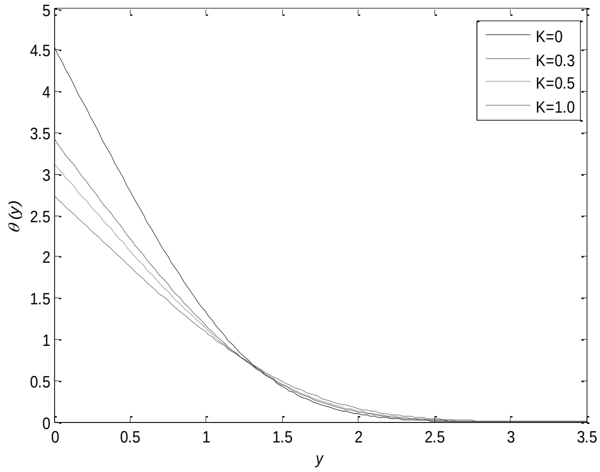

Figure 5: Temperature profile for Pr = 7, λ = 0.5, M = 1.0, P = 0.5 and various values of K

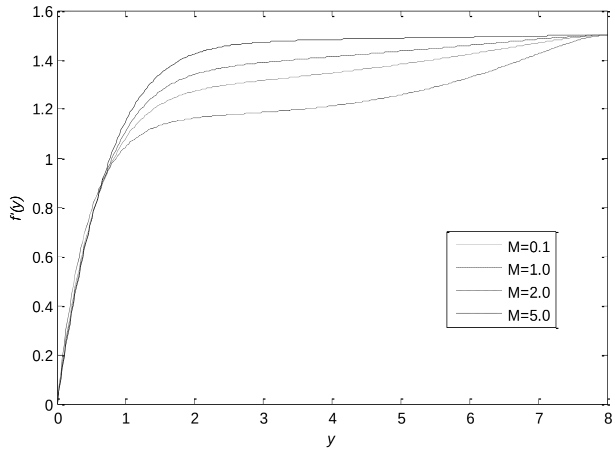

Figure 6: Velocity profile for Pr = 7, λ = 0.5, K = 0.2, P = 0.5 and various values of M

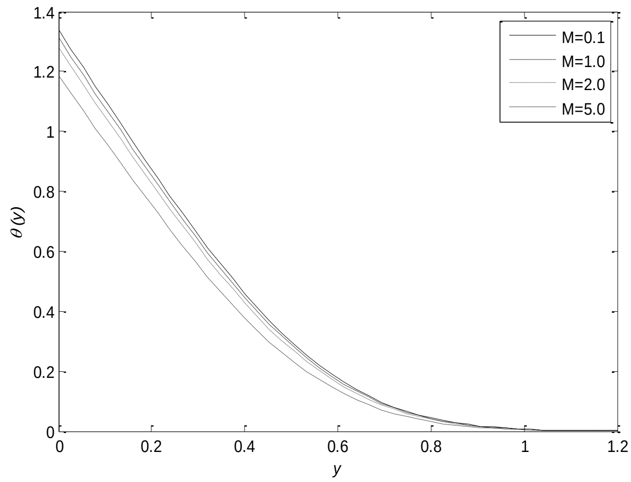

Figure 7 Temperature profile for Pr = 7, λ = 0.5, K = 0.2, P = 0.5 and various values of M