Mathematical Model for the Determination of the Non-Stationary Coefficient of Heat Transfer in Mine Works

DAN CODRUT PETRILEAN
Department of Mechanical Engineering Industry and Transport
University of Petroşani
Str.Universitatii, no.52, 332006 Petroşani
ROMANIA
dcpetrilean@yahoo.com

Abstract: The paper deals with the issue of determining the non-stationary heat transfer depending on the coefficient of convection and on the dimensionless temperature of the wall of the mine work. The equation which expresses the heat transfer between air and rock massif is solved using numerical integration technique. The criterial form of the calculation relation for the dimensionless temperature of wall of the mine was determined using the Bessel functions.

Key words: numerical integration technique, mathematical model, the variation of the coefficient of heat transfer, dimensionless temperature, heat transfer, thermal conduction, thermal convection, Bi criterion, Fo criterion.

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Nomenclator:

c – Specific mass heat at constant pressure (kJ/(kg/centigrade^{-1}));
ρ – density of the rock (kg/m^{-3});
λ – Coefficient of conduction of the rock (W·m^{-1}·centigrade^{-1});
α – coefficient of convection (W·m^{-2}· centigrade^{-1});
x, z, y – coordinates;
τ – time, (s).
r – radius of the cylinder of the hydro-energetic work, (m);
r_0 – radius of the work, (m);
φ – angle of direction of the cylindrical coordinates;
y – current coordinate measured along the excavation, (m);
T – Dimensionless temperature of the mine work
t_a – air temperature, (centigrade);
t_r – Temperature of the rocks (centigrade);
t – Temperature of the wall of the mine work, variable in time (centigrade);
Fo, Bi – Fo and *Bi* criterion;
A and B – constants;
h - Relative coefficient of heat transfer;
e – Euler number
k – non-stationary coefficient heat transfer;
$I(x)$ and $K(x)$ - Bessel functions with a purely imaginary argument.
a – Coefficient of thermal diffusion, (m^{2}s^{-1}).

1 Introduction

The conditions of thermal comfort are realised within the limits of a minimum number of requirements which are set out by physiological and psychological labour instructions. Three basic factors compete for the realisation of normal conditions for this kind of works, namely: air temperature;air humidity;air speed.

The paper has considered the temperature as the other factors, namely speed and humidity, being easily obtained through different measuring procedures.

The problems for heat transfer between air and the rock massif were dealt with by [1], [4], [5], [8], [9], [10].

The new elements brought forward and studied by the paper are the following:

- conceiving a model for the variation of the non-stationary coefficient of heat transfer considering the dimensionless temperature and the coefficient of convection of the rocks, without studying the

influence of pneumatic installations and underground machineries;
- creating the graphs which present the dependency of the non-stationary coefficient of heat transfer on the Fo invariant for different known values of Bi = ct invariant, often met in the practice of mine works.

2 Mathematical model overview

The non-stationary complex process of the energy transfer within the rock massif is represented by the differential equation of thermal conduction which was written without any interior heat sources:

$$c \cdot \rho \frac{\partial t(x, y, z, \tau)}{\partial \tau} = \mathrm{div}[\lambda\ \mathrm{grad}(x, y, z, \tau)] \quad (1)$$

Admitting the continuous, uniform, isotropic body, the physical constancies of which are independent of the temperature, the form of equation (1) in cylindrical coordinates is the following:

$$\frac{\partial t(r, \varphi, y, \tau)}{\partial \tau} = a \left(\begin{array}{l} \frac{\partial^2 t(r, \varphi, y, \tau)}{\partial r^2} + \frac{1}{r} \frac{\partial t(r, \varphi, y, \tau)}{\partial r} + \\ + \frac{1}{r^2} \frac{\partial^2 t(r, \varphi, y, \tau)}{\partial \varphi^2} + \frac{\partial^2 t(r, \varphi, y, \tau)}{\partial y^2} \end{array} \right) \quad (2)$$

If the temperature is symmetrical to the axis of the mine work, then:

$$\frac{\partial^2 t(r, \varphi, y, \tau)}{\partial \varphi^2} = 0 \quad (3)$$

The gradient of the temperature in a radial direction is larger than in the direction of the axis of the mine work and in consequence:

$$\frac{\partial^2 t(r, \varphi, y, \tau)}{\partial y^2} = 0 \quad (4)$$

Therefore, it may be written:

$$\frac{\partial T}{\partial \tau} = a \left[\frac{\partial^2 T}{\partial \tau^2} + \frac{1}{r} \frac{\partial T(r, \tau)}{\partial \tau} \right] \quad (5)$$

In a dimensionless form, depending on variables r and τ, the equation becomes[2]:

$$\frac{\partial T}{\partial \tau} = a \left[\frac{\partial^2 T}{\partial \tau^2} + \frac{1}{r} \frac{\partial T(r, \tau)}{\partial r} \right] \quad (6)$$

where:

$$T = \frac{t - t_a}{t_r - t_a} \quad (7)$$

The independent variables receive values within the following ranges: $r_0 \le r \le \infty$; $0 \le \tau < \infty$, with the condition that $r > r_0$. 2 situations may be observed:
- For $\tau = 0$; $t = t_r$, it results $T = 1$;
- For $\tau > 0$ and $r \to \infty$; $t \to t_r$, but $T = 1$.

For $r = r_0$ the energy transfer on the surface of the walls is the following:

$$-\lambda \frac{\partial T}{\partial r} + aT = 0 \quad (8)$$

Using Laplace's method the Bessel type equation ($\nu = 0$) is obtained with the following form:

$$x^2 y'' + xy' - x^2 y = 0 \quad (9)$$

with the following general solution:

$$y = AI_0(x) + BK_0(x) \quad (10)$$

After having gone through a series of steps, the dimensionless temperature of the wall of the work results and has a criterial form, if the work passes from a cylindrical form to a spherical one [9]:

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$$T_p = 1 - \frac{Bi}{Bi + r}\frac{r_0}{r}\left\{\begin{array}{l} \mathrm{erfc}\dfrac{\frac{r}{r_0}-1}{2\sqrt{Fo}} - e^{\left(\frac{r}{r_0}-1\right)(Bi+1)}\cdot \\ \cdot\, e^{Fo(Bi+1)^2}\cdot \mathrm{erfc}\left[\begin{array}{l}\sqrt{Fo}(Bi+1)+ \\ + \dfrac{\frac{r}{r_0}-1}{2\sqrt{Fo}}\end{array}\right]\end{array}\right\} \tag{11}$$

For $r = r_0$, equation (11) becomes:

$$T_p = 1 - \frac{Bi}{Bi + r}\left[1 - e^{Fo(Bi+1)^2}\cdot \mathrm{erfc}\sqrt{Fo}(Bi+1)\right] \tag{12}$$

With the notations:

$$\mathrm{erfc}\ x = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx$$

$$x = \sqrt{Fo}(Bi+1) \tag{13}$$

$$f(x) = 1 - e^{x^2}\cdot \mathrm{erfc}\ x$$

$$T_p = 1 - \frac{Bi}{Bi+1}\cdot f(x) \tag{14}$$

Where it has been written: $Bi = \frac{ar_0}{\lambda}$; $Fo = \frac{a\tau}{r_0^2}$.

The coefficient of thermal conduction for the analysed case is comprised within the following interval[4], $\lambda = (1.3 \ldots 1.8)\,\frac{W}{mK}$.

Considering a variable operation, the heat flux is a complex function which has the following form:

$$Q = f(\tau, r, \varphi, y, \lambda, c, \rho, t_a, t_r, S) \tag{15}$$

The thermo-physical constancies of rocks may in their turn depend on the coordinates of the temperatures, and the temperatures are functions of time. The heat flux is known from the following relation:

$$Q = a(t_r - t_a)S \tag{16}$$

In order to simplify and quantify the thermal calculations, make it easier for graph developers, the processing of experimental data and the modelling of thermal exhausts, the final formulas may be reduced to a usual form for the thermal calculation which have the following form[7]:

$$Q = k(t_r - t_a)S \tag{17}$$

or

$$k = a\cdot\frac{t - t_a}{t_r - t_a} = a\cdot T_p \tag{18}$$

3 Case study

The problem of determining the variation of the non-stationary heat transfer depending on the dimensionless temperature of the wall of the mine work as well as the specific values of the coefficient of convection is brought forward. As the form of the relations obtained for the dimensionless temperature is a criterial one, the solution may be one obtained considering the range of Bi and Fo criteria specific to the mining conditions. The dimensionless temperature of the wall of the mine work is expressed depending on the Fo and Bi dimensionless criteria, namely particular values specific to mine works.

In study, the value matrix of the dimensionless temperature will be determined for constant Bi values and variable Fo ones. Considering he specificity of the mine works, the value range of Bi and Fo adimensional criteria was considered to be: Bi = 1÷7; Fo = 0.001÷0.015[9],[10]. The value of the coefficient of thermal diffusion involved in the Fo criterion is known from the speciality literature, namely $a = (0{,}32 \div 1{,}65)10^{-6}\ m^2/s$[8].

Considering the notation $x = \sqrt{Fo}(Bi+1)$ and attributing the variation vectors i, j to the corresponding Bi and Fo dimensionless criteria, the value matrix of the dimensionless parameter x is obtained, presented therefore in Figure 1:

x =

	0	1	2	3	4	5	6
0	0.063	0.095	0.126	0.158	0.19	0.221	0.253
1	0.089	0.134	0.179	0.224	0.268	0.313	0.358
2	0.11	0.164	0.219	0.274	0.329	0.383	0.438
3	0.126	0.19	0.253	0.316	0.379	0.443	0.506
4	0.141	0.212	0.283	0.354	0.424	0.495	0.566
5	0.155	0.232	0.31	0.387	0.465	0.542	0.62
6	0.167	0.251	0.335	0.418	0.502	0.586	0.669
7	0.179	0.268	0.358	0.447	0.537	0.626	0.716
8	0.19	0.285	0.379	0.474	0.569	0.664	0.759
9	0.2	0.3	0.4	0.5	0.6	0.7	0.8
10	0.245	0.367	0.49	0.612	0.735	0.857	0.98

Fig. 1 Value matrix for function x

Developing and solving the following equation:

$$\operatorname{erfc} x = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$$

the numerical solutions for the error function erfc x with the matrix form as presented in figure 2 are therefore obtained:

erfcx =

0.929	0.893	0.858	0.823	0.788	0.754	0.72
0.899	0.849	0.8	0.752	0.704	0.657	0.612
0.877	0.816	0.756	0.698	0.641	0.586	0.533
0.858	0.788	0.72	0.654	0.59	0.529	0.471
0.841	0.764	0.689	0.616	0.547	0.481	0.42
0.827	0.742	0.661	0.583	0.509	0.44	0.376
0.813	0.722	0.635	0.552	0.475	0.403	0.338
0.8	0.704	0.612	0.525	0.444	0.371	0.306
0.788	0.687	0.59	0.5	0.417	0.342	0.277
0.777	0.671	0.57	0.477	0.392	0.317	0.252
0.729	0.602	0.486	0.382	0.293	0.221	0.168

Fig. 2 Value matrix for the error function

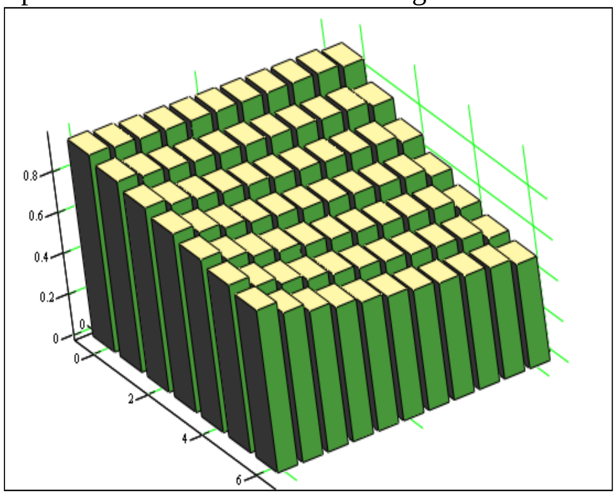
Considering that equation (12) is correct and the condition $r = r_0$, then the numerical value matrix for the dimensionless temperature of the wall of the hydro-energetic work is determined, taking into account as well the number of values of the error function and the range of the specific Bi and Fo dimensionless criteria. Thus, the values for the dimensionless temperature are presented in Figure 3:

$T_{.p}$ =

0.966	0.953	0.944	0.936	0.929	0.923	0.918	0.913	0.909	0.904	0.887
0.934	0.91	0.892	0.878	0.866	0.856	0.846	0.838	0.83	0.823	0.793
0.904	0.87	0.845	0.826	0.81	0.795	0.783	0.772	0.761	0.752	0.713
0.875	0.832	0.802	0.778	0.758	0.741	0.726	0.713	0.701	0.69	0.644
0.848	0.797	0.762	0.735	0.712	0.693	0.676	0.661	0.647	0.635	0.586
0.822	0.764	0.725	0.695	0.67	0.648	0.63	0.614	0.599	0.586	0.538
0.797	0.733	0.691	0.658	0.631	0.608	0.589	0.572	0.557	0.544	0.51

Fig. 3 Value matrix for the dimensionless temperature of the wall

A more suggestive presentation of the numerical values in Figure 3 may be observed using a 3D representation such as the one in Figure 4:



$T_{.p}$

Fig. 4. 3D representation of the value matrix of the dimensionless temperature

The variation of the dimensionless temperature of the wall of the mine work depending on the Fo criterion, and constant Bi values is highlighted in Figure 5:

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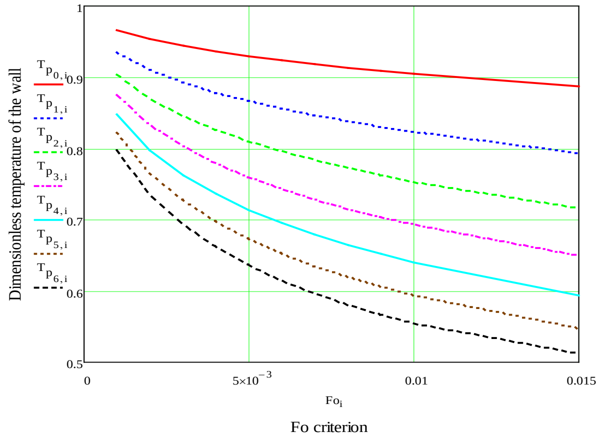

Fig. 5 The dependency of the dimensionless temperature of the wall of the mine work on a variable Fo (Fo = 0.001, 0.002, 0.003, 0.004, 0.005, 0,006, 0,007, 0,008, 0,009, 0,01, 0,015) and a constant Bi (Bi = 1, 2, 3, 4, 5, 6, 7)

Relation (18) is taken into consideration for the determination of the non-stationary coefficient of heat transfer, considering therefore a variation of the coefficient of thermal convection within the limits $\alpha = 12.5 \div 15\ W/\mathrm{m}^2\mathrm{K})$, with a pace of 0.5 points. The value matrix for the dimensionless temperature of the wall of the mine work was determined and presented in Figure 3, while Figure 4 presents its 3D representation. Taking in to account both the graph for the determination of the dimensionless temperature as well as the values of the coefficient of convection specific to the mine work, Figures 6, 7, 8, 9, 10, and 11 present the variation graphs of the non-stationary coefficient of heat transfer depending on the Fo and Bi criteria.

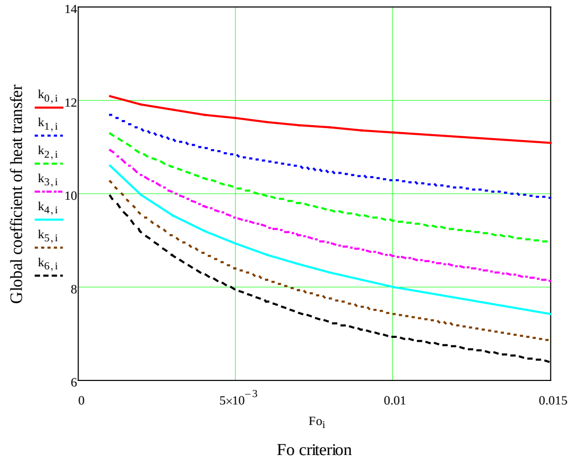

Fig. 6 The dependency of the non-stationary coefficients of heat transfer on the Fo and Bi criteria for α = 12,5 W/(m^{2}K)

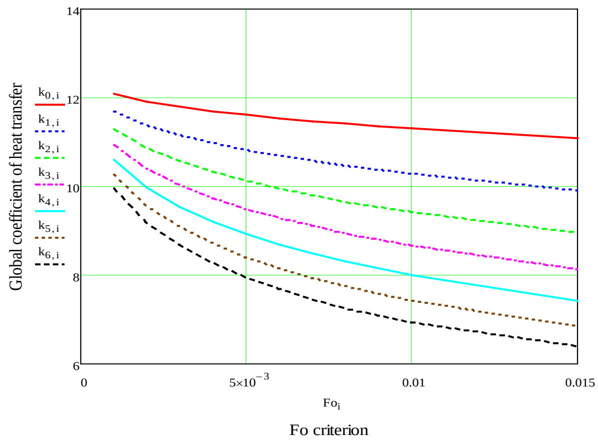

Fig. 7 The dependency of the non-stationary coefficients of heat transfer on the Fo and Bi criteria for α = 13 W/(m^{2}K)

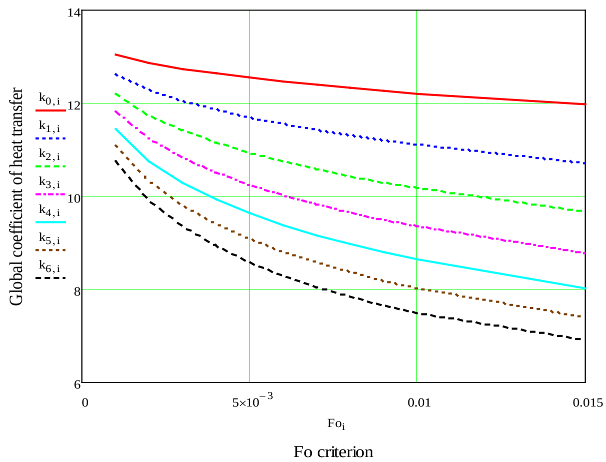

Fig. 8 The dependency of the non-stationary

coefficients of heat transfer on the Fo and Bi criteria for α = 13,5 W/(m^{2}K)

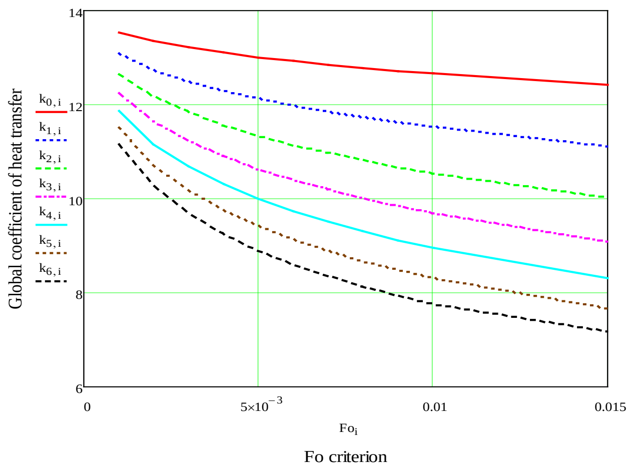

Fig. 9 The dependency of the non-stationary coefficients of heat transfer on the Fo and Bi criteria for α = 14 W/(m^{2}K)

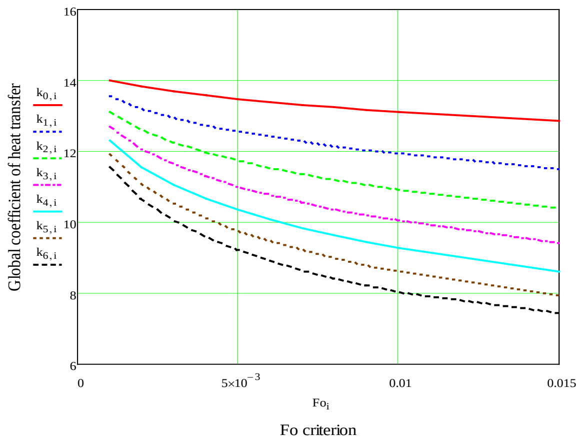

Fig. 10 The dependency of the non-stationary coefficients of heat transfer on the Fo and Bi criteria for α = 14,5 W/(m2K)

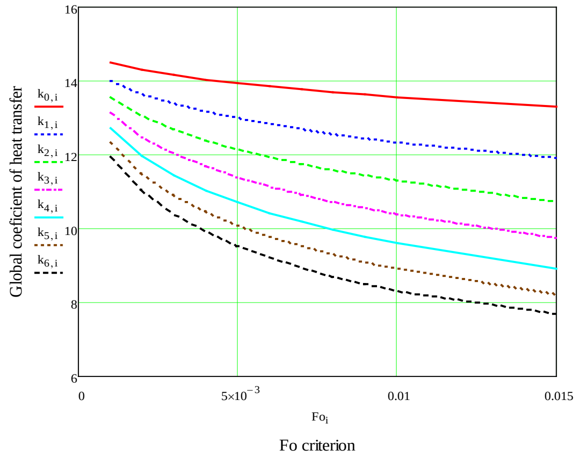

Fig. 11 The dependency of the non-stationary coefficients of heat transfer on the Fo and Bi criteria for α = 15 W/(m2K)

4 Conclusions

The paper is meant to establish a mathematical model which is able to anticipate, depending on the geo-mining conditions, the variation of the non-stationary heat transfer based on the determination of the distribution of the dimensionless temperature of the wall of the mine, as a result of the air-massif rock heat exchange.

The importance to determine the Bi = ct. curves for concrete conditions of mine works represent the most important problem in establishing the dimensionless temperature, needed for the quantification of the variation module of the non-stationary coefficient of heat transfer along the mine work.

The graph which represents the dependency of the dimensionless temperature of the wall of the mine work on the Fo and Bi invariants was created.

The graphs expressing the dependency of the non-stationary heat transfer which depends on the Fo and Bi invariants as well as on the coefficient of convection of the rocks. Taking into account the value matrixes obtained and on the graphs created, the following conclusions may be drawn:

1. T(r, Bi, Fo*)* values confirm the validity of the relation of variable heat transfer (12), as its real domain is comprised within the range 0÷1, as it may be observed in figures 3, 4, and 5.

2. The dimensionless temperature of the wall decreases in relation to the Fo and Bi criteria, as it

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may be observed from the value matrix presented in Figure 3 and respectively Figure 5.
3. Together with the increase in value of the Bi criterion the dimensionless temperature presents a significant decrease, as it may be observed from the value matrix presented in Figure 3 and respectively Figure 5.
4. The non-stationary coefficient of heat transfer presents a logarithmic variation dependant on the Fo and Bi criteria, as it may be observed in figures 6, 7, 8, 9, 10, and 11.
5. The non-stationary coefficient of heat transfer decreases as the coefficient of thermal convection increases and it may be observed in figures 6, 7, 8, 9, 10, and 11.
6. Based on the graphs presented in Figure 5, starting from the abscissa of known values of Fo invariant and intersecting the Bi curve, the value of the dimensionless temperature is obtained on the ordinate.
7. Based on the graphs presented in figures 6, 7, 8, 9, 10, and 11, knowing the coefficient of thermal convection of the rocks, starting on the abscissa from
known values of Fo invariant and intersecting the Bi curve, the value of the searched non-stationary heat transfer is obtained on the ordinate.

The study for the determination of the non-stationary coefficient of heat transfer was based on the determination of the variation of the dimensionless temperature dependant on the Fo and Bi criteria, namely a variable Fo and a constant Bi.

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