LQR Controller Design for a SAS System with Magnetorheological Damper

K.M.I.U.Ranaweera, K.A.C.Senevirathne,M.K.Weldeab, H.R.Karimi
Department of Engineering, Faculty of Engineering and Science
University of Agder, 4879 Grimstad, Norway, E-mail: Hamid.r.karimi@uia.no

Abstract-This paperinvestigates the behavior of theMagnetorheological(MR) damper in the SAS system. An LQR controller is designed and implemented in Matlab/Simulink with Dahl model representation of MR damper and the vibrations of beamsof the system are discussed.The controller is implemented together with the experimental SAS setup and the effect of the controller parameters to the vibration response of the SAS system is analyzed.

I. INTRODUCTION

In most of the applications under civil, mechanical and electrical engineering environments are commonly affected by many kinds of undesirable vibrations that are harmful to the complete system. To eliminate or reduce the vibration level by isolating the system from the vibration sources, the vibration suspension systems are used.

One of the most common applications of suspension system is in automobile industry to isolate the vehicle body and its occupants from the irregularities of the road surface and to avoid the tires tend to lift off the ground when it passes over a bump or a dip, allowing the passengers to a comfortable and a safe journey

One of the better suspension systems currently using for the vibration isolation is the Semi Active Suspension (SAS) Systems with a rotational magneto-rheological brake (MR brake) which creates braking torque by changing the viscosity of the MR fluid inside the brake according to the applied current.This paper is to discuss about the vibration suppression of the SAS system by designing an LQR controller to control the input current to the MR rotary brake.

II. VIBRATION ISOLATION SYSTEMS

There are different types of undesirable vibrations may occurs at different applications such as cars and carriages, motors and machine tools, oil and gas platforms, buildings and constructions in a zone of seismic activity, laboratory tables (especially optical) setups, etc. For most of the cases, requirements and process of the vibration isolation system is identical and mainly these systems can be categorized into passive dampers, semi-active dampers and active dampers

The Passive vibration control involves an inherent compromise between low-frequency and high-frequency vibration isolation [1]. Passive suspension system consists of an energy dissipating element, which is the damper, and an energy-storing element, which is the spring. Since these two elements cannot add energy to the system this kind of suspension systems are called passive.Passive suspension systems are subject to various tradeoffs when they are excited across a large frequency bandwidth.

Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel [2]. The additional forces to change spring and damper coefficients are determined by a feedback control. But this is too expensive for wide spread commercial use because of their complexity and large power requirements. Also, a failure of the force actuator could make the vehicle very unstable and therefore dangerous to drive.

Semi active dampers are capable of changing their damping characteristics by using a small amount of external power [3]. Semi active suspensions are less complex, more reliable, and cheaper than active suspensions. They are becoming more and more popular for commercial vehicles.

III. MR DAMPER

The MR damper is a type of semi-active damper where the flow of MR fluid is controlled by varying the supplied current and hence changes the level of damping. The advantage of MR dampers over the conventional dampers are the simple construction, compromise between high frequency isolation and natural frequency isolation, offer semi-active control, use very little power and have very quick response.

A. MR Fluid

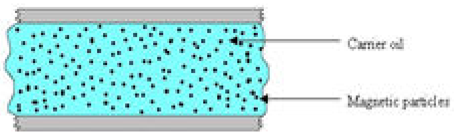


Fig 1: MR Fluid without Magnetic Field Applied

MR Fluid displays Newtonian-like behavior when the Fluid exposed to a magnetic field [5], the ferrous particles that are dispersed throughout the fluid form magnetic dipoles. These magnetic dipoles align themselves along lines of magnetic flux, as shown in Fig 2.

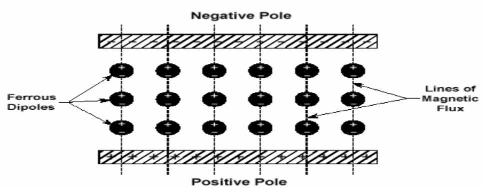


Fig 2: Polarized MR Fluid Particles under Applied Magnetic Field

Typically, MR fluids can be used in four different modes (squeeze mode, valve mode, shear mode and pinch mode) [4], all of which can be applied to MR damper design depending on the damper's intended use. Some of the MR dampers are linear and some are rotary dampers, but for the SAS systems typically used the MR rotary damper

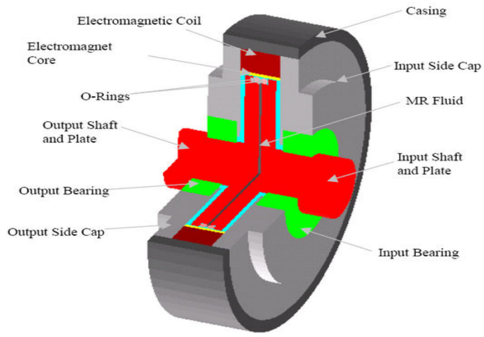


Fig 3: MR Rotary Damper

IV. MATHEMATICAL MODEL OF THE SEMI-ACTIVE SUSPENSION SYSTEM

The laboratory setup shown in Fig 4represents a physical system for the quarter car model, which is the most common application to analyze the vertical dynamics in a SAS system.

This system is driven by a DC motor with gear coupled to an eccentric small wheel. The suspended car wheel rolls due to the small wheel rotation and oscillates up and down due to the small wheel eccentricity.

By using dynamic mathematical model simulation, it is possible to analyze the behavior and performance of systems consisting of rigid or flexible parts undergoing displacement motions.

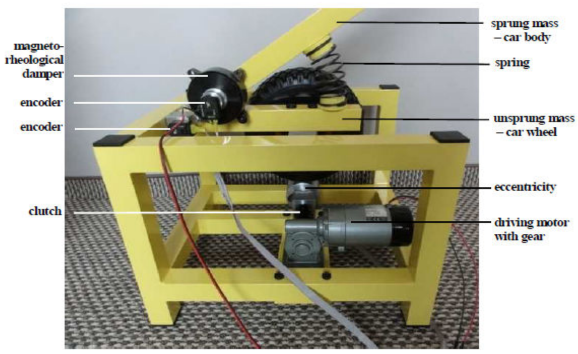


Fig 4: The Laboratory setup of SAS system

A. Geometrical View of the SAS System

The Fig 5 shows the geometrical structure of the SAS system. There are two main types of torques affecting to the MR brake, one is from the upper beam and the other one is from the lower beam. This torques can be represented by two specific dynamic equations.

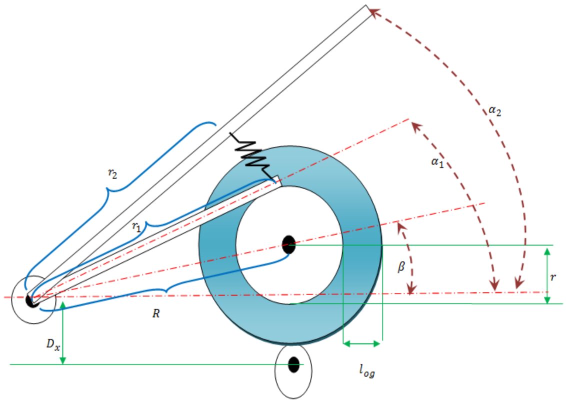


Fig5: Geometrical Structure of the SAS System

B. The Momentums Acting on the SAS System

The moment of Inertia of the lower beam

$$T_{Unsprung\ Mass} = J_1 \frac{d^2\alpha_1}{dt^2}$$

The moment of Inertia of the upper beam

$$T_{Sprung\ Mass} = J_2 \frac{d^2\alpha_2}{dt^2}$$

$J_1 (0.1273\ kgm^{-2}) and\ J_2 (1.1453\ kgm^{-2})$are moment of inertias of the lower beam and upper beam with respect to its axis rotation.

α_1-The angle between the upper beam and horizontal line
α_2-The angle between the lower beam and horizontal line

Viscous Friction Damping Torque (Kinetic energy of the MR rotary brake) affecting on lower beam and the upper beam;

$$T_{Viscous} = k_1 \frac{d\alpha_1}{dt} T_{Viscous} = k_2 \frac{d\alpha_2}{dt}$$

$k_1 (1\ Ns/m)\ and\ k_2 (0.3525\ Ns/m)$are the viscous friction coefficients of the MR fluid effecting to the lower beam and the upper beam.

Potential energy of the lower beam and the upper beam;

$$T_{Gravity\ lower} = m_1 g_1 r_1\ Cos(\beta - \alpha_1)$$
$$T_{Gravity\ Upper} = m_2 g_2 r_2\ Cos\alpha_2$$

β-The angle between the line drawn through the centers of the tire and MR damper and the horizontal line

Kinetic energy of the spring;

$$T_{Spring} = r_2 K_s [l_{os} - \sqrt{(r_1 Cos\alpha_1 - r_2 Cos\alpha_2)^2 + (r_2 Sin\alpha_2 - r_1 Sin\alpha_1)^2}]$$

ISBN: 978-1-61804-164-7

K_s -The elasticity coefficient of the spring ($3333Nm^{-1}$)
l_{os} -The length of the no load spring ($0.125m$)
l_o -The length of the loaded spring

r_1 ($0.2712m$) and r_2 ($0.1867m$) are the distances between the spring mount and the beams (the lower and upper beam)

Elasticity Spring torque of the tire;

$$T_{Tire\_Elasticity} = k_g RCos(\beta - \alpha_1)(l_{og} + RSin(\beta - \alpha_1) + r - D_x + U_{kin})$$

k_g-The elasticity coefficient of the tire $16000Nm^{-1}$
U_{kin} -The input Eccentricity
x -The displacement of elasticity of the tire
D_x -The distance between the beam pivot and the pivot of the eccentric ($0.2\ m$)
R -The distance between the MR damper and the center of the tire ($0.2256\ m$)

The Damping torque of the tire

$$T_{Tire,Damping} = f_g\left(\frac{d(D_x - u_{kin})}{dt} - \frac{d\alpha_1}{dt} RCos(\beta - \alpha_1)\right)$$

C. *Torque Analysis for the Beams*

The Fig6 represents, all the torques affecting on the upper beam.

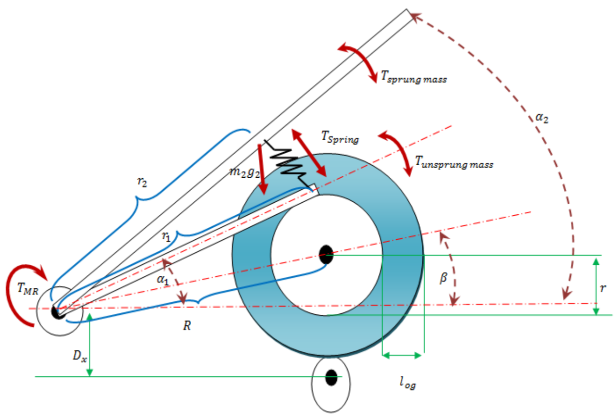


Fig 6: Torques acting on the upper beam of the SAS System

Applying

$$\sum T_{sum} = J\dot{\omega}$$

For the upper beam;

$$J_2\frac{d^2\alpha_2}{dt^2} + m_2g_2r_2\ Cos\alpha_2 - r_2K_s\left(l_{os} - \sqrt{(r_1Cos\alpha_1 - r_2Cos\alpha_2)^2 + (r_2Sin\alpha_2 - r_1Sin\alpha_1)^2}\right) + k_2\frac{d\alpha_2}{dt} = -C\left(\frac{d\alpha_1}{dt} - \frac{d\alpha_2}{dt}\right) = -\left(\frac{d\alpha_1}{dt} - \frac{d\alpha_2}{dt}\right)M_{MR}(i) \qquad (1)$$

Where, C is the damping coefficient of MR Damper

For the lower beam;

$$J_1\frac{d^2\alpha_1}{dt^2} + m_1g_1r_1\ Cos(\beta - \alpha_1) + r_1K_s\left(l_{os} - \sqrt{(r_1Cos\alpha_1 - r_2Cos\alpha_2)^2 + (r_2Sin\alpha_2 - r_1Sin\alpha_1)^2}\right) + k_1\frac{d\alpha_1}{dt} - k_gRCos(\beta - \alpha_1)\left(l_{og} + RSin(\beta - \alpha_1) + r - D_x + U_{kin}\right) - f_g\left(\frac{d(D_x - u_{kin})}{dt} - \frac{d\alpha_1}{dt}RCos(\beta - \alpha_1)\right) = -C\left(\frac{d\alpha_1}{dt} - \frac{d\alpha_2}{dt}\right) = -\left(\frac{d\alpha_1}{dt} - \frac{d\alpha_2}{dt}\right)M_{MR}(i) \qquad (2)$$

V. LINEARIZATION OF SAS MODEL

Linearization is a linear approximation of the nonlinear system that is valid in a small region around the operating point by pre defining the inputs and outputs to represent it in a state space model [6].

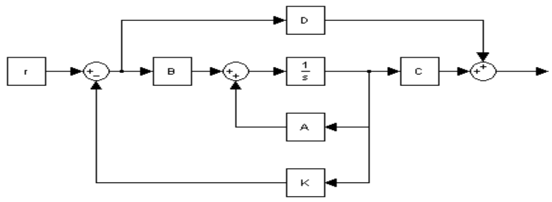


Fig 7: Typical state space model with feedback

$$\dot{x}(t) = (A - BK)x(t) + Br$$
$$y(t) = (C - DK)x(t) + Dr$$

When D is zero, $y(t) = Cx(t)$.

The SAS system is a SIMO (Single Input Multi Output) system and the torque to the system is taken as the input and the angle and velocity of the body and wheel are taken as the outputs as shown in the Fig 8.
For the SAS system, the *Input vector*

$$u = [MR\ Damper\ Torque\ ]$$

And the *Output Vector*

$$y = \begin{bmatrix} Wheel\ \_\ Angle \\ Body\ \_\ Angle \\ Wheel\ \_\ Velocity \\ Body\ \_\ Velocity \end{bmatrix}$$

Linearized Model Parameters:

$$A = 10^3\begin{bmatrix} 0 & 0 & 0.0010 & 0 \\ 0 & 0 & 0 & 0.0010 \\ -6.1721 & 1.1918 & -0.1434 & 0 \\ 0.0912 & -0.0856 & 0 & -0.0003 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.8731 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

ISBN: 978-1-61804-164-7

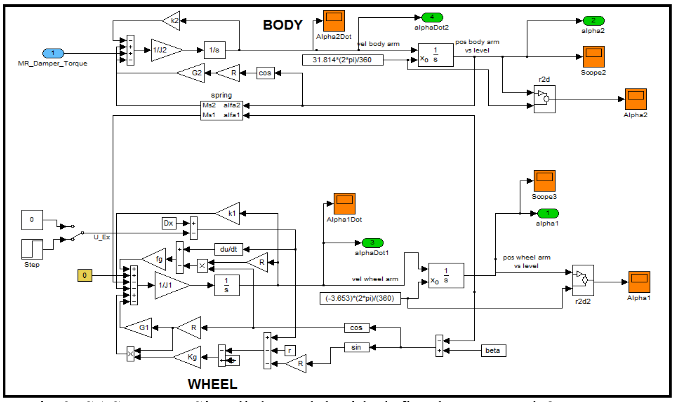


Fig 8: SAS system Simulink model with defined Inputs and Outputs

VI. LQR CONTROLLER DESIGN

Linear Quadratic Regulator (LQR) Controller is one of the controller methods of controlling Multi Input Multi output (MIMO) plants, which are characterized by transfer matrices instead of transfer functions. In LQR controller the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic function,

$$J(u) = \int_0^\infty (x^T Q x + u^T R u) dt$$

Qbea non-negative symmetric definite defined as $Q = C^T C$ and the R be a positive definite. The feedback gain matrix of the system [7], K is given by

$$K = R^{-1} B^T S$$

Where, $\boldsymbol{S}$ (a symmetric matrix) is the solution of the ***Algebraic Riccati Equation,***

$$0 = SA + A^T S - SBR^{-1} B^T S + Q$$

The output of the controller gain matrix K is torque which is an input to the upper beam. The Dahl model is used to represent the MR damper in this simulink model. Hence the torque output of the LQR controller should be converted to a current using the Dahl model equations.

$$T_{MR} = K_x(i)\dot{\theta} + K_y(i)z$$

Where; $\dot{z} = \alpha(\dot{\theta} - |\dot{\theta}| z)$

Where θ is the angle, K_x is the damping coefficients which depend linearly on the current *(i)* and the *z* is the hysteretic variable. Parameters K_y and α control the shape of the hysteresis curve.

The current dependent parameters K_x and K_y follows the below equations.

$$K_x = K_a + K_b i$$

$$K_y = K_1 + K_2 i$$

Torque output from the Dahl model;

$$T_{MR,Controller} = (K_a + K_b i)\dot{\theta} + (K_1 + K_2 i)z$$

So the controlled current to be sent to the MR damper is(Fig 9),

$$i = \frac{T_{MR,C} - K_a \dot{\theta} - K_1 z}{K_b \dot{\theta} + K_2 z}$$

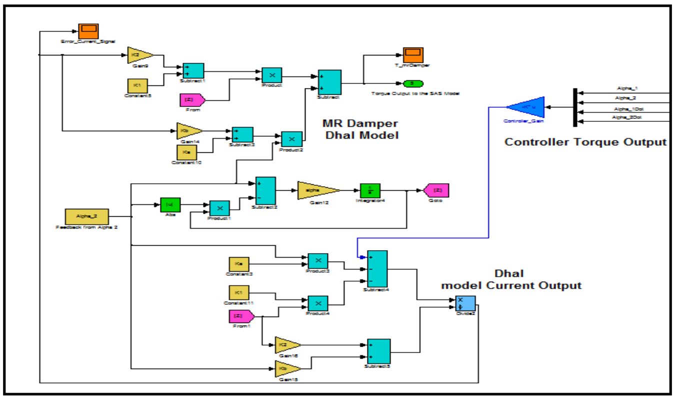


Fig 9: Controller setup with MR Damper (Dahl model) in Simulink Model

A. Simulation Results for the Theoretical Linearized Model

The effect of the controller parameter R on vibration response of the theoretical model is shown in Fig 10.

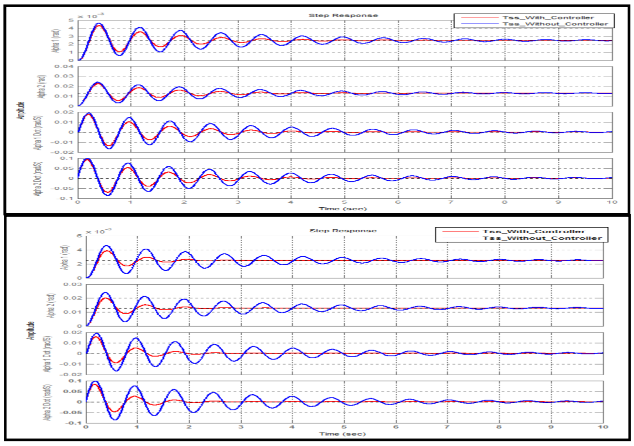
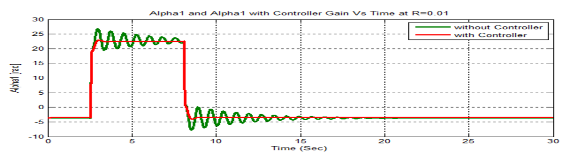
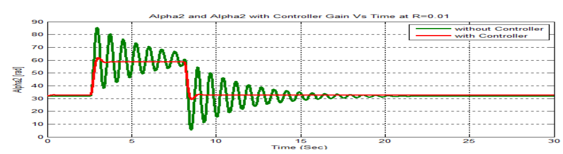


Fig 10: Linearized system Response with and Without Controller at R=0.5 and R=0.1

B. Simulation Results for the Theoretical SAS Model

The effect of controller on Alpha_1 and Alpha_2 vibration responses for a pulse and step excitationsare illustrates in the below Fig 11 and Fig 12 for R=0.01:





ISBN: 978-1-61804-164-7

Fig 11: Vibration response of Alpha_1 and Alpha_2 for a pulse excitation with and without Controller

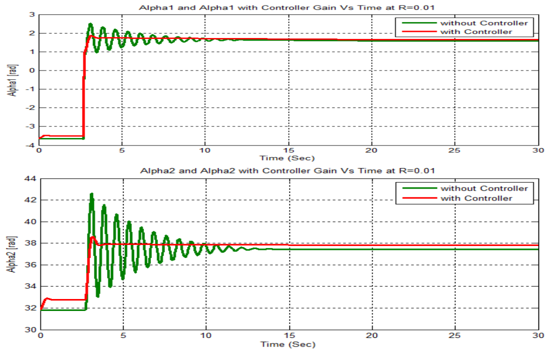


Fig 12: Vibration response of Alpha_1 and Alpha_2 for astep excitation with and without Controller

Alpha 1 and Alpha 2 behavior comparison with Different R values for pulse excitation:

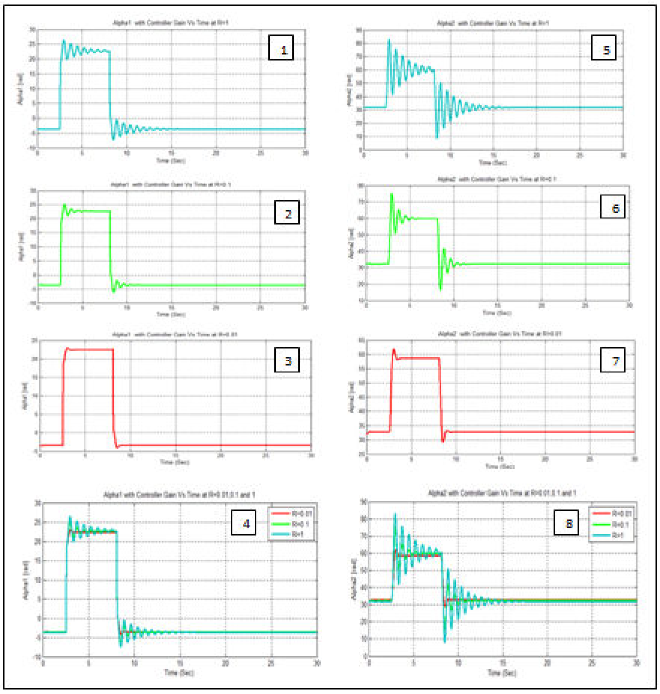


Fig 13: Alpha1 [1,2,3,4,] and Alpha2 [5,6,7,8] plots for R=1, 0.1,0.01 for Pulse Excitation

According to the figures above, it can be clearly figure out that the SAS system stabilizes immediately by reducing the vibrations with the decrease of R values.

C. *Simulation Results for the Laboratory SAS Setupwith LQR Controller*

Vibration response analysis was done for the LQR controller by connecting it to the real time experimental setup for the Semi Active Suspension system. The Simulink model of the LQR controller and RTWT model of the SAS system, which communicates real time with experimental setup, is as shown in Fig 14.

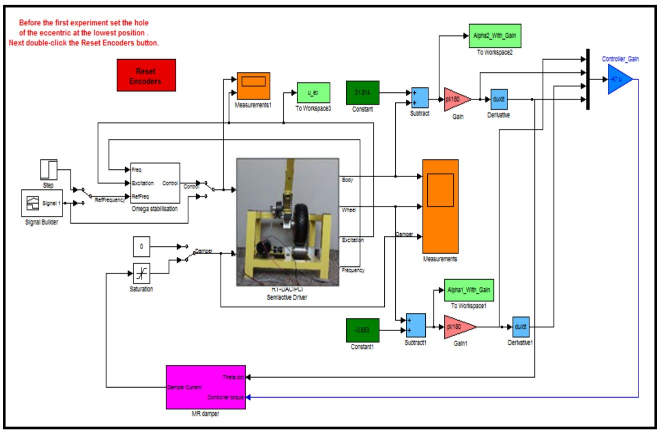


Fig 14: Real time experimental Simulink model for the SAS system with LQR controller

Comparison of Alpha2 response without controller and with LQR controller for R=0.01

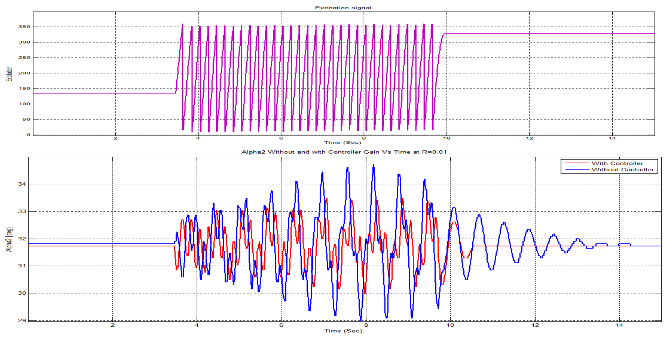


Fig. 15: Excitation Signal and Alpha2 (Body angle) responses without and with LQR controller for R=0.01 for a pulse excitation

The vibration Response results obtained, while suppressing the upper beam spring for a small constant angle setting and releasing it

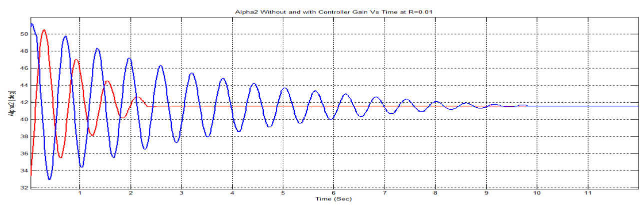


Fig 16: Alpha2 Relative to the horizontal axis with and without LQR controller for R= 0.01

VII. Conclusion

The purpose of this paper was to analyze vibration responses based on an LQR techniquefor a semi active suspension system, which consisting a nonlinear device, the MR damper.The MR damper was modeled using the Dahl Model to study all the controller techniques.
In this paper the results were analyzed for different excitations including step and pulse, using the Simulink model of the SAS system and as well as the experimental SAS setup. An acceptable level of vibration control was obtained from all these controllers while optimizing the controller gains.

REFERENCES

[1] T. Ram Mohan Rao, G. VenkataRao, k.SreenivasaRao& A. Purushottam , "Analysis Of Passive And Semi Active Controlled Suspension Systems For Ride Comfort In An Omnibus Passing Over A Speed Bump" International Journal of Research and Reviews in Applied Sciences, 5 (1), 7-17, October 2010.

[2] Mohamad, Maziah and Mailah, Musa and Muhaimin, Abdul Halim, "Vibration Control of Mechanical Suspension System Using Active Force Control". In: Proceedings the 1st International Conference on Natural Resources Engineering and Technology INRET2006, 24-25 July 2006, Marriot, Putrajaya, Malaysia.

[3] NimaEslaminasab, "Development of a Semi-active Intelligent Suspension System for Heavy Vehicles". PhD thesis, The University of Waterloo, URI: http://hdl.handle.net/10012/3658.

[4] James Poynor, "Innovative Designs for Magneto-Rheological Dampers". Master Thesis, Advanced Vehicle Dynamics Laboratory, Virginia Polytechnic Institute and State University, 2001.

[5] Magnetorheological fluid from Wikipedia (http://en.wikipedia.org/wiki/Magnetorheological_fluid)

[6] AshaDharan, Silje Helene Olsen Storhaug, Hamid Reza Karimi"LQG Control of a Semi-active Suspension System equipped with MR rotary brake" Proceedings of the 11th WSEAS international conference on Instrumentation, Measurement, Circuits and Systems, 176-181, 2012.

[8] Jingqing Han, "From PID to Active Disturbance Rejection Control", IEEE transactions on industrial electronics, vol. 56, no. 3, 2009

[9] Peter J. Gawthrop"Self-tuning PID Control Structures", IEE Colloquium on Getting the Best Our of PID in Machine Control (Digest No.: 1996/287),1-4, 1996.