Backstepping Control Design for a Semiactive Suspension System with MR Rotary Brake

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Abstract
The purpose of this paper to design a semiactive controller for a vehicle suspension system that employs a magnetorheological (MR) damper as the actuator. The controller was developed using backstepping methodology for the laboratory model “Semi Active Suspension System (SAS)” which represents a physical system for a quarter car model, which can be used to analyze the vertical dynamics of a car body due to the road disturbances. First the SAS system was modeled by the mathematical equations and then the backstepping controller was designed to suppress the vibrations of the car body. The dual model was used to model the nonlinear behavior of the MR damper. The mathematical model of the SAS system and the control system were designed in Matlab/Simulink and system performance was evaluated. Finally the controller was connected to the experimental model and the vibration responses were obtained for different disturbances.

I. INTRODUCTION
One of the most common applications of suspension system is the automobile industry to suppress the vibrations of the vehicle body, created by the irregularities of the road surface. A good vibration suspension system will provide comfortable ride for the passengers. Many researchers and academicians nowadays made the research and development regarding the suspension systems in order to achieve a good car ride and handling quality[7]. One of the better suspension systems currently using for the vibration isolation is the Semi Active Suspension (SAS) System. It consists of a magnetorheological damper which creates braking torque by changing the viscosity of the MR fluid inside the brake according to the applied current[5]. By controlling the input current, output torque of the MR damper can be controlled to make the system stable quickly.
In this paper a control mechanism for controlling the torque of the MR damper is developed for the laboratory model “Semi Active Suspension System” which represents the quarter car model by using Backstepping control mechanism.

II. SEMIACTIVE SUSPENSION SYSTEM
The study of vehicle vibration reduction is made by analyzing the dynamics of the Semi-Active Suspension (SAS) System developed by the Polish Company Inteco Limited. SAS laboratory model represents a physical system of a quarter car model, and can be used to analyze the vertical dynamics of the car wheel. As shown in Figure 1, the SAS system consists of an upper beam which represents the car body, a wheel, rotational MR damper and a spring. It is driven by a DC motor with gear coupled to an eccentric small wheel. The suspended car wheel rolls due to the small wheel rotation and oscillates up and down due to the small wheel eccentricity.

Figure 1: Semiactive Suspension System (SAS)

Figure 2 provides the geometrical view of the SAS system. Using this figure the mathematical modeling equations for the upper lever and the lower lever can be derived as follows[4].

The equations of motion of the upper beam are:

\[ J_2 \frac{d^2 \alpha_2}{dt^2} + k_2 \frac{d \alpha_2}{dt} + M_2 \alpha_2 - r_2 k_2 (l_{os} - l_s) = T_{MR}(t) \]
The equations of motion of the lower beam are;

\[ a_t = a_t \]

\[ J_1 \frac{d^2 a_t}{dt^2} + M_1 \cos(\beta - a_t) + r \kappa_s (l_{um} - l_u) + k_1 \frac{da_t}{dt} \]

\[ -k_s R \cos(\beta - a_t) (l_{um} + R \sin(\beta - a_t) + r - D_s + u_a) - f_g \frac{d(D_s - u_a)}{dt} \]

\[ T_{MR} = T_{MR}(i) \]

Where; \( l_u = \sqrt{(r_1 \cos a_1 - r_2 \cos a_2)^2 + (r_2 \sin a_2 - r_1 \sin a_1)^2} \)

\[ J_1, J_2 \] - Moment of inertia of the lower beam and the upper beam with respect to its axis rotation

\[ M_1, M_2 \] - Gravitational moment of the upper and lower beam

\[ k_1, k_2 \] - Viscous friction coefficients of the upper beam and lower beam

\[ k_s \] - The elasticity coefficient of the spring

\[ l_{um} \] - The length of the non-loaded spring

\[ l_u \] - The length of the loaded spring

\[ k_s \] - The elasticity coefficient of the tire

\[ f_g \] - The absorption coefficient of the tire

\[ u_a \] - The kinetic sinusoidal excitation

\[ T_{MR} \] - MR damper torque

### III. BACKSTEPPING CONTROLLER DESIGN

The control methodology must be implemented to decrease the vibrations of the body when the tire is subjected to the external excitation force such as bumps or vibrations. Therefore in backstepping control method we mainly focus on the external excitation suppression of the body[2] so that we use only the mathematical model of the body.

Let’s define;

The viscous friction damping torque, \( M_{d2} = k_a a_2 \)

The gravitational force torque, \( M_{g2} = M_2 \cos a_2 \)

Spring torque, \( M_{s2} = 2k_h (l_{um} - l_u) \)

Then the equation of the upper beam can be written as,

\[ \dot{a}_2 = a_2 \]

\[ \dot{a}_2 = J_2^{-1}(M_{d2} - M_{g2} + M_{s2} + T_{MR}(i)) \]

The equilibrium point of the system can be found when,

\[ (a_2, \dot{a}_2) = (0, 0) \]

It was found as, \( (a_2, \dot{a}_2) = (0.55 \text{ rad}, 0) \) when \( T_{MR}(0) = 0 \).

Defining the new coordinates \( z_1 \) and \( z_2 \) by shifting the origin to the equilibrium point.

\[ (z_1, z_2) = (a_2 - a_{2,eq}, a_2 - a_{2,eq}) = (a_2 - a_{2,eq}, a_2) \]

In the new coordinate system equation of the upper beam becomes,

\[ z_2 = J_2^{-1}(M_{d2} - M_{g2} + M_{s2}) + J_2^{-1}T_{MR} \]

In order to apply the backstepping technique let’s define standard backstepping variables.

\[ e_1 = z_1 \]

\[ e_2 = z_2 + h_1 e_1, h_1 > 0 \]

\[ e_2 = z_2 + h_2 \]

Lyapunov’s direct method of stability is used to determine a rule for the stability of the system. Candidate Lyapunov candidate function[1];

\[ V = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \]

Derivative of \( V \),

\[ \dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 \]

\[ \dot{V} = e_1 \dot{z}_1 + e_2 \dot{z}_2 \]

\[ \dot{V} = e_1 (e_2 - h_1) + e_2 \dot{z}_2 + h_2 \dot{e}_2 - h_1 \dot{e}_1 \]

\[ \dot{V} = -h_2 e_1^2 + h_2 \dot{e}_2 + e_2 \dot{z}_2 \]

\[ \dot{V} = -h_2 e_1^2 \]

\[ \dot{V} = -h_2 e_1^2 + 2e_2 \dot{z}_2 \]

\[ \dot{V} \leq -h_2 e_1^2 - h_2 e_2^2 < 0, \text{ given that } h_1 h_2 > 0 \]

If,

\[ e_1 \left(\dot{a}_2 - a_{2,eq}\right)(1 + h_1 h_2) + J_2^{-1}(M_{d2} - M_{g2} + M_{s2}) + J_2^{-1}T_{MR} + e_2(h_1 + h_2) \leq 0 \]

Then, \( \dot{V} < 0 \)

Also, \( \dot{V} > 0 \) and \( V(0, 0) = 0 \)

Therefore according to the Lyapunov second method of stability the system is stable.

Then,

\[ (e_1, e_2) \rightarrow (0, 0) \]

\[ (z_1, z_2) \rightarrow (0, 0) \]

This implies that, \( (a_2, a_2) \rightarrow (a_{2,eq}, 0) \)

In order to be the system stable let’s define the control law,

\[ e_2 \left(\dot{a}_2 - a_{2,eq}\right)(1 + h_1 h_2) + J_2^{-1}(M_{d2} - M_{g2} + M_{s2}) + J_2^{-1}T_{MR} + e_2(h_1 + h_2) = 0 \]

\[ (a_2 - a_{2,eq})(1 + h_1 h_2) + J_2^{-1}(M_{d2} - M_{g2} + M_{s2}) + J_2^{-1}T_{MR} + e_2(h_1 + h_2) = 0 \]

\[ -J_2^{-1}T_{MR} = (a_2 - a_{2,eq})(1 + h_1 h_2) + J_2^{-1}(M_{d2} - M_{g2} + M_{s2}) + J_2^{-1}T_{MR} + e_2(h_1 + h_2) = 0 \]

\[ -T_{MR} = (a_2 - a_{2,eq})(1 + h_1 h_2) + J_2^{-1}(M_{d2} - M_{g2} + M_{s2}) - J_2^{-1}T_{MR} = e_2(h_1 + h_2) \]

The controller must command the MR damper to produce the torque derived above to suppress the vibrations of the body as soon as possible. The MR damper can be controlled by the input current or voltage. Therefore the appropriate current or voltage must be found to give as the input to the MR damper to produce the required torque.
IV. MR DAMPER

The MR damper is a type of semi-active damper where the flow of MR fluid is controlled by varying the amount of current supplied and hence changes the level of damping [5]. The torque-current relationship of the MR damper is highly nonlinear which is called as hysteresis behavior. Therefore developing a mathematical model for a vibration suspension system is a big challenge if MR damper is used as the vibration suppression device.

To describe the hysteretic torque/velocity and torque/displacement response of MR damper, several models have been proposed in the literature, including the Bingham model, Dhal model, the Bouc-Wen phenomenological model, Lugre Model, neural networks, etc [6],[9]. In this paper Dhal model is selected to model the MR damper. The governing equations of the MR damper are;

\[ T_{MR} = K_x(i)\dot{\theta} + K_y(i)z \]

with

\[ z = \alpha (\theta - |\dot{\theta}|) \]

where \(\theta\) is the angle, \(K_x\) is the damping coefficients which depend linearly on the current \((i)\) and the \(z\) is the hysteretic variable. Parameters \(K_y\) and \(\alpha\) control the shape of the hysteresis curve.

The current dependent parameters \(K_x\) and \(K_y\) described by following equations.

\[ K_x = K_{b1} + K_{b2} i \]
\[ K_y = K_{l1} + K_{l2} i \]

Damper must produce the damping torque of;

\[ T_{MR,c} = -J_2(\alpha_2 - \alpha_2\theta)(1 + h_2\dot{\theta}) + (M_{g2} + M_{g2}) - \alpha_2 J_2(h_1 + h_2) \]

Damper Torque given from the Dhal model;

\[ T_{MR,c} = (K_a + K_b)\dot{\theta} + (K_1 + K_2)zi \]

Therefore the control current that should be sent to the MR damper is,

\[ i = \frac{T_{MR,c} - K_y \dot{\theta} - K_x z}{K_y \dot{\theta} + K_x z} \text{ or } \]
\[ i = \frac{-J_2(\alpha_2 - \alpha_2\theta)(1 + h_2\dot{\theta}) + (M_{g2} + M_{g2}) - \alpha_2 J_2(h_1 + h_2) - K_2 \dot{\theta} - K_x z}{K_2 \dot{\theta} + K_x z} \]

V. SIMULATION RESULTS

The SAS system with the backstepping controller is modeled in the Matlab/Simulink and the Simulink diagram is given in Figure 3.

Figure 3: SAS system with the backstepping controller

The following MR damper parameters for the dhal model are selected for the Simulation.

\(K_1 = 5, K_2 = 1.5, K_a = 0.001, K_b = 0.001, \alpha = 5\)

Backstepping control was applied to the SAS model with the parameters \(h_1 = 10\) and \(h_2 = 10\). Vibration response of the body was obtained for different excitation input and the results are illustrated in the following figures.
According to Figure 4, it can be clearly seen that, when the backstepping controller is present the body stabilizes more quickly by reducing the amplitude of the vibration.

VI. LABORATORY EXPERIMENT

Vibration analysis was done for the experimental set up for the Semi Active Suspension system. The SAS system has been designed to operate with an external, PC-based digital controller. The control computer communicates with the level sensors, values and pump by a dedicated I/O board and the power interface. The I/O board is controlled by the real time software which operates in Matlab/Simulink/RTWT rapid prototyping environment.

The Back stepping controller for vibration suppression was added to the simulink model that communicates with the SAS system, as illustrated in Figure 5.

Different signals were given as the reference rotating frequency of the eccentric wheel to introduce vibrations to the Wheel. Then the vibration response of the upper beam was obtained with and without connecting the Back stepping controller. The results are illustrated in Figure 6.

According to the results obtained from the experiment we can see that, the amplitudes of the vibrations were decreased when the back stepping controller is connected and the body stabilizes more quickly with the controller. In the experimental set up maximum input current for the MR damper is 1.5A. Therefore optimal control cannot be achieved due to the current limitation in the experimental setup as the results obtained with the simulations in Matlab.

VII. CONCLUSION

In this paper we have developed a controller based on the backstepping method for a semi active suspension system. The semi active device is an MR damper and the proposed controller has been tested in Matlab/Simulink and with the real time experimental model. The obtained results have shown a good performance of the backstepping controller in the simulation as well as the laboratory experiment.

REFERENCES
