

Mathematics with Software Mathematica at Riga Technical University

MARIJA ILTINA, ILMARS ILTINS
 Department of Engineering Mathematics
 Riga Technical University
 Meza Street 1/4
 LATVIA
marijai@inbox.lv, iltins@inbox.lv

Abstract: - Teaching of mathematics to the students of the first and second year at Riga Technical University is discussed in the paper, by using software MATHEMATICA. Two different approaches are explored. The first approach refers to the students of the first year who learn linear algebra, analytical geometry and calculus. The second approach is applied when teaching numerical methods to the students of the second year.

Key-Words: - mathematics; MATHEMATICA package; numerical methods, calculus, teaching technologies, laboratory work

1 Introduction

Various types of mathematical areas are taught to students of all departments of Riga Technical University (RTU) by Department of Engineering Mathematics together with Department of Probability Theory and Statistics. Linear algebra, analytical geometry and differential calculus for one and several argument functions are taught during the first term of the first year. Here it should be noted that these areas are not taught as separate subjects, but they are taught as a one subject named “Higher mathematics”. Teaching of calculus continues during the second term that includes integral calculus, differential equations and series. Some integral transforms are also taught in several departments. During the second year theory of probability and mathematical statistics, and one more subject are taught at all the departments, which is called “additional sections of mathematics” and whose content may differ across the departments. Theory of complex variable, field theory and integral transforms are included therein in various proportions at most departments. At Department of Computer Science and Information Technology (CSIT), this course is called “Numerical methods”. It is known that realization of numerical methods is associated with a huge amount of calculations, and it is clear that their summary is very complex if some mathematical software is not applied in the learning process. Similarly application of mathematical software in numerical

methods is complex one without any skill in calculation of the simplest problems. Therefore, students are trained for applying mathematical software, MATHEMATICA 5 in this case, already from the first term of the first year. Lectures take place once per two weeks in a computer class, and students are taught to solve all kinds of problems by using software MATHEMATICA 5 there, which are discussed during lectures and practical classes.

2 Content of Laboratory Work in the First Year

To make teaching of mathematics more attractive and to introduce the latest technologies in the teaching process of mathematics, the Department of Engineering Mathematics proposed to divide 3 classes of practical work as follows in 2006: 2 classes of practical work and 1 class of laboratory work. For such a decision a significant role lied in the understanding that actual examples of mathematical application in engineering can be illustrated only by using software. Here it should be noted that a term at RTU lasts 16 weeks, thus 1 class of laboratory work means 8 laboratory works per term, once in two weeks lasting 2 academic hours each. One decided to apply software **Mathematica 5** for laboratory work, by demonstrating how problems that are discussed during lectures and practical work must be realized in software.

Basic principles how mathematics should be taught, by using software, followed from [1]. Teaching technology was taken over from [2] – [6] to a great extent. All the materials that students require for laboratory work are available at chapter of RTU electronic study aids.

Laboratory work starts with introduction into application of software **Mathematica**. Practically it means that several first classes are devoted to explanation of **Mathematica** basic principles. Previous knowledge in software **Mathematica** or any other software is not needed. We stated that many students had problems to master in a short period of time that:

- All commands in **Mathematica** begin with a capital letter.
- Arguments of all **Mathematica** commands and user-defined functions must be inserted in square brackets instead of any other brackets.
- When defining a user function, one may not write $f[x]=$, or $f(x)=$, as one must write $f[x_]=$.
When one writes $2x$, a space is placed between 2 and x automatically that means multiplication sign, but when one writes $x2$, it is perceived as one symbol.
- Spaces within commands or definitions are inadmissible,
- Entered, but unperformed command or definition has no impact on further actions (not pressed **Shift**→**Enter**).

That is not a complete list of pitfalls.

After introduction actions with matrixes are discussed in details, beginning with input of a matrix: **Input**→**CreateTable/Matrix/Palette**. Calculation of a determinant and a matrix rank, matrix transposition and finding an inverse matrix are explored: command **Det[A]**, **MatrixRank[A]**, **Transpose[A]**, **Inverse[A]**. Then solving of linear equation systems is discussed, by using command **LinearSolve[A,B]** in the event of one solution and **RowReduce[A]** in the event of a wealth of solutions. Here it should be mentioned that students have several home tasks involving these issues and the ones enumerated below, which they do by calculations on paper. Software **Mathematica** is applied to check correctness of their calculations. Among actions with vectors, calculation of a module, scalar multiplications and vector multiplication are reviewed: command **Norm**, **Dot**,

Cross. That helps understanding that there are 2 types of multiplication actions in vector algebra.

A part of mathematical analysis during term 1 includes the following issues:

- Algebraic transformations,
- Borders,
- Derivatives of one and several argument functions,
- Graphs.

Our experience shows that many students of the first year have poor knowledge in algebraic transformations. Therefore such commands like **Expand**, **ExpandAll**, **Simplify**, **FullSimplify**, **TrigExpand**, **Factor**, **Collect** are considered in order to improve knowledge of the students in algebraic transformations.

We esteem that a graphic interpretation of the results can significantly improve understanding of mathematics. Thus, great attention is paid to drawing commands of various graphs. Commands for drawing of two-dimensional graphs in Descartes coordinates, **Plot**, **ParametricPlot** and **ImplicitPlot**, as well as for drawing of graphs in polar coordinates, **PolarPlot** are discussed. Command **ListPlot** is used for drawing of geometrical objects. Geometrical interpretation of two argument- functions is also considered: command **Plot3D**, **ParametricPlot3D**, **ContourPlot**, **DensityPlot**. Knowledge and application of graph drawing commands helps understanding limits (command **Limit**) and unilateral limits (**Direction**→**1**, **Direction** → **-1**).

When discussing derivatives, various kinds of command **D** application are mastered: in one argument function (also given parametrically and implicitly), several argument function, for higher order derivatives. Then determination of extreme and points of inflection by using command **D** and **Solve** is mastered. Commands for determination of extremes are discussed: **Maximize** and **Minimize**. Here finding of line tangent and normal and their depiction in a drawing are also explored.

The major taught issues during the second term are the following:

- Integrals (indefinite, definite, recurrent),

- Ordinary differential equations (ODE),
- Series,
- Laplace transform.

Calculation of indefinite, definite and recurrent integrals is considered in details, by applying command **Integrate**. Then application of integrals for calculating space, volumes, arc length is studied. A geographic interpretation of a problem is topical in those tasks therefore commands **InequalityPlot**, **InequalityPlot3D**, **FilledPlot**, **Solve**, **Reduce** are taught.

When discussing differential equations, command **DSolve** is applied both for general solution and particular solution. Differential equations whose solutions are expressed by special functions as well as analytically insolvable equations are also considered. In the latter command **NDSolve** is used. Particular attention is paid to graphic illustration of the solution. Solving differential equations is also studied, by applying of Laplace transform with commands **LaplaceTransform** and **InverseLaplaceTransform**.

When learning series, the major features of series convergence are implemented in the software. Raising a function into Taylor's series is studied, by using command **Series** and **SeriesTerm**. An extension of function into the Fourier series is implemented. When discussing the latter, extended functions and their partial sum graphs are compared with different number of addends.

Monograph [7] was really useful for drafting study aids mentioned above.

3 Content of Laboratory Work during Term 3

During the third term students master a course "Numerical methods", which anticipates 8 lectures and 8 laboratory works. The course of numerical methods includes the following topics:

- Solution of linear algebraic equation systems with direct and iteration methods,
- Interpolation and approximation,

- Numerical differentiation and integration,
- Solution methods of non-linear equations and their systems,
- Numerical solution methods of ordinary differential equations,
- Notion of poorly determined problems.

Lectures "Numerical methods" are read in a traditional way. Materials of lectures are repeated during laboratory work, by applying software **Mathematica**, with previously drafted examples that illustrate the material of lectures.

During the first laboratory work different types how a matrix is divided into multipliers and their applications are studied. The following methods are taught:

- $A=Q^T R$, where Q is an orthogonal matrix, R is a right triangular matrix, and a corresponding command **QRDecomposition[A]**,
- $A=R^T R$, and corresponding command **CholeskyDecomposition[A]**,
- $A=Q^T R Q$, and adequate command **SchurDecomposition[A]**,
- $A=LR$, where L is a left triangular matrix, and corresponding command is **LUdecomposition[A]**.

The last decomposition into multipliers is studied in details. It is showed how it is used in solving of linear equation systems, finding determinants and inverse matrix.

During the second laboratory work, students master basics of programming. Cycle forming commands **Do**, **While** and **For** are discussed. The following linear equation systems with tridiagonal matrix solution approach are studied.

$$a_i x_{i-1} - b_i x_i + c_i x_{i+1} = d_i, \quad i = 1, 2, \dots, n$$

$$a_1 = c_n = 0$$

$$\xi_{i+1} = \frac{c_i}{d_i - a_i \xi_i}, \eta_{i+1} = \frac{a_i \eta_i - d_i}{d_i - a_i \xi_i}, i = 1, 2, \dots, n$$

$$x_n = \eta_{n+1}$$

$$x_i = \xi_{i+1}x_{i+1} + \eta_{i+1}, \quad i = n, n-1, \dots, 2, 1.$$

A system can be also solved by command **TridiagonalSolve**.

Solution of linear equation system by means of iteration methods is discussed during the third laboratory work. Jacobi method and Gauss-Seidel method are considered. Cases are also shown when these methods do not converge. Iteration method in the following form is discussed in details:

$$x^{k+1} = x^k + \tau(B - Ax^k),$$

where A is a factor matrix, B is the right side, τ is an iteration parameter.

During the fourth laboratory work, various types of interpolation problem are studied. Application of command **InterpolatingPolynomial** is studied:

- Problems occurring when making interpolation with high rank polynomial,
- Interpolation of data set with a set of cubic polynomial, by using command **Interpolation**,
- Interpolation with cubic spline, applying command **SplineFit**,
- Interpolation with a rational part, by using command **RationalInterpolation**,
- Two-dimensional interpolation, by applying command **ListInterpolation**.

All of the above is illustrated with varied graphic materials.

The next laboratory work is devoted to solution of approximation problems. The following problems are studied:

- Data approximation with a given linear combination of functions, by applying command **Fit**,
- Non-linear approximation, by using command **FindFit**,

Having used an interpolation of graphic results, one shows how a linear combination of functions fit for approximation can be obtained. Different and common things for approximation problem are emphasized.

The following issues are discussed during the sixth laboratory work:

- Numerical differentiation,

- Numerical integration with rectangular, trapezium and Simpson's formula ,
- Numerical integration with Gaussian quadrature formulae,
- Integration of the function given in a table, by using command **ListIntegrate**.

When illustrating numerical differentiation with an example, loss of precision is demonstrated, making differentiation with too less interval. Gaussian quadrature formulae

$$\int_a^b f(x)dx = \sum_{k=1}^n w_k f(x_k)$$

are analyzed in details. One shows how weight factor w_k and nodes x_k , can be obtained, by using command **GaussianQuadratureWeights**. Special attention is paid to loss of integration precision if a function to be integrated changes rapidly.

The seventh laboratory work regards methods for solving of non-linear equations and their systems such as:

- Solution of non-linear equations and their systems by Newton's method,
- Application of commands **NSolve** and **FindRoot**.

Particular attention is paid to choice of guess value, and a wide range of graphical material is provided.

The last laboratory work is devoted to numerical solution of differential equations, and it includes:

- Euler's method,
- Command **NDSolve**.

Students are explained how Koshi problem and several boundary problems should be solved.

3 Assessment of Results

Software MATHEMATICA was applied in the teaching process of mathematics for the first time during academic year 2007/2008. In order to evaluate impact of the novelty on the learning process, a student survey was carried out in early 2008 and results of examination session of winter 2006 were compared in paper [8]. The first year students were only interviewed during the second

term. Students were asked two questions, which they could answer with yes or no. 81% of respondents answered affirmatively to a question whether software MATHEMATICA should be used for teaching students. Question whether a student has used courses of software MATHEMATICA that are not directly linked with subject of mathematics was answered affirmatively by 73% of respondents. Another three questions were asked in the survey:

- Whether application of software MATHEMATICA assisted in getting ready for examination,
- Whether application of software MATHEMATICA assisted in understanding basic principles of mathematics better,
- Whether software MATHEMATICA should be also used for other subjects being taught at the Riga Technical University.

Five answers were possible: “definitely disagree”, “disagree”, “unsure”, “agree”, and “definitely agree”. Summarized results in percent from paper [8] is given in table 1.

	disagree	unsure	agree
Q1	38	29	33
Q2	42	39	19
Q3	31	33	36

Table 1. Survey results 2008.

Here answers “definitely disagree” and “disagree” are combined like answers “agree” and “definitely agree” are. Examination results were also compared between 2006 (before introduction of software MATHEMATICA) and 2007. Authors of paper [8] concluded that the results almost did not change. They also concluded that students who passed the examination better answered to the questions “agree” and “definitely agree” more often, whereas the two remaining answers were more often provided by the students who passed the examination worse and mediocre. Besides authors [8] noted that the students were weaker in elementary mathematics in 2007 than the students in 2006 (all students of the first year pass a test in elementary mathematics in the beginning of a term in Riga Technical University in order to determine an entry level of enrolled students in mathematics).

An analogous survey and comparison of examination results in 2006 and 2007 were carried out as regards subject “numerical methods”. Here it should be noted that students learning subject of “numerical methods” in 2007 had not learnt software MATHEMATICA during their first year. We presumed that we would teach basic principles of software MATHEMATICA very fast and then we could tackle “numerical methods”. This presumption was erroneous. We had to return to the simplest things during entire term that caused problems both to students and lecturers. Thus, a survey carried out for 2007 and comparison thereof with results of 2006 cannot be used.

A survey of students concerning numerical methods provided in early 2012 provided the following findings:

- 96% percent of students answered ‘yes’ to the question whether software MATHEMATICA should be taught to students of Riga Technical University,
- 94% percent of students answered ‘yes’ to the question whether students use software MATHEMATICA for solving issues not directly related with mathematics.

Answers of the third term students in percentage to the questions provided in table 1 in the early 2012 are given in table 2.

	disagree	unsure	agree
Q1	6	14	80
Q2	5	14	81
Q3	15	20	65

Table 2. Survey results 2012.

Table 1 and table 2 are not directly comparable. Different students with different learning experience have been surveyed (the first group after term 1, whereas other groups after term 3). Though it follows from table 2 that attitude of the students towards software MATHEMATICA was positive.

4 Conclusion

This article described teaching of mathematics at Riga Technical University, by using software MATHEMATICA. Laboratory work in mathematics

is developed so that their topics are in parallel with lectures and practical work. Findings of the student survey showed that reaction of students to application of software MATHEMATICA was positive and many students used this software for solving other problems as well that were not directly associated with mathematics. Experience of the authors showed that teaching of “numerical method” and obviously other subjects as well by using software MATHEMATICA would consume quite a long time for mastering basics of that software. Therefore students who transfer to this curriculum from other curricula, where software MATHEMATICA is not taught, teaching is ensured together with the first year students as well as entire set of the first term learning aids are delivered. Learning aids are available at the database of electronic learning aids of Riga Technical University, and library [9], [10], [11]. The authors wish to thank the Latvian Council of Science for financial support under the Project No. 2005/0127/VPDI/ESF/PIAA/04/APK/3.2.3.2./0021/0007.

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