Dynamic and kinematics parameters optimisation of vertical transport installations

FLORIN DUMITRU POPESCU
Department of Mechanical Engineering, Industrial Engineering and Transportation
University of Petroşani
Str. Universităţii, Nr.20, Petroşani
ROMÂNIA
fpopescu@gmail.com

Abstract: - The acceleration and deceleration periods during each race of a vertical transport installation may be considered as periods of transitional processes where kinematics and dynamic measures variations take place (acceleration, speed and forces) as well as some electric measures (actuating motor's current).
Considering T as the duration of the process, for the minimisation of the installed power, the minimisation of the energy dissipated in the system is imposed during the transitional period: $W = D_1\left(\int_0^T i^2(t)\,dt\right) \to \min(!)$ where i(t) is the intensity of the current absorbed by the actuating motor. According to the same considerations this integral is equivalent to: $W = D_2\left(\int_0^T a^2(t)\,dt\right) \to \min(!)$ where a(t) is the acceleration of the system, considered here as a command value. Mainly, in order to minimise the above integrals, values as small as possible need to be adopted for current i(t) as well as for acceleration a(t). On the other hand, if the acceleration drops drastically it may lead to exceeding the imposed value of process T, implying as well a compromising solution.

Key-Words : Acceleration, Tachograms, Speed, Torque, Power

1 Kinematics parameters optimisation

Two constant acceleration phase tachograms are used in the case of reduced transport systems and are characterised by the lack of a constant speed period.

1.1 Constant acceleration tachograms

Therefore, the process is composed of only two periods of time: the acceleration phase t1 and the deceleration phase t2 (figure 1).

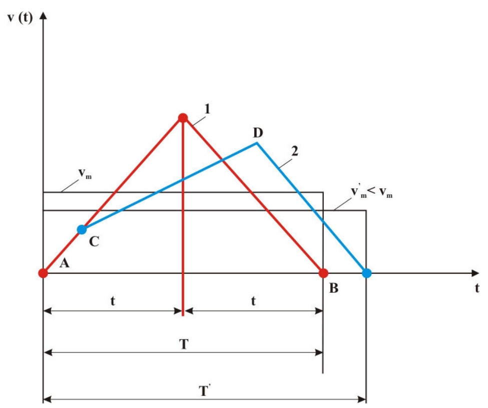

Fig.1 Speed variation trajectories for the two phase tachogram

The discovery of a law of variations is imposed either for i(t) or for a(t), for which the transition of the system from the point of balance A to the point of balance B to be realised in the shortest period of time possible. According to figure 1, for the transition in time of the system from point A to B, trajectory 1 needs to be followed. The speed of movement needs to be maximum:

$$V_m T = H \qquad (1)$$

where, H is the distance undergone. If H is constant and T = min, then: $V_m = V_{m\,max}$.

In order for the average speed to have a maximum value, the acceleration is imposed to be maximum a_{max}. It is also valid for the deceleration period t_2. If on one part of the trajectory (for instance CD), the acceleration is smaller than the maximum admitted one, the average speed decreases therefore increasing the period of the transitional process T′ (line 2). In this case, for the optimum process, the acceleration is a staircase function:

$$\left.\begin{aligned} a(t) &= a_m; \quad 0 < t < \frac{T}{2} \\ a(t) &= -a_m; \quad \frac{T}{2} < t < T \end{aligned}\right\} \qquad (2)$$

The law variation of speed and space is obtained by integrating the equations of movement variation considering the equations (2):

$$v(t)=\int_0^T a(t)\,dt \tag{3}$$

$$h(t)=\int_0^T v(t)\,dt \tag{4}$$

Therefore:

$$\left.\begin{aligned} v(t)&=a_m t\\ h(t)&=\frac{1}{2}a_m t^2\end{aligned}\right\}\quad 0<t<\frac{T}{2} \tag{5}$$

$$\left.\begin{aligned} v(t)&=a_m(T-t)\\ h(t)&=a_m\left(Tt-\frac{T^2}{2}-\frac{T^2}{4}\right)\end{aligned}\right\}\quad \frac{T}{2}<t<T \tag{6}$$

For the determination of the period of time T, the limit condition is used $h(T)=H$. In this way $h(T)=a_m\left(T^2-\frac{T^2}{2}-\frac{T^2}{4}\right)=a_m\frac{T^2}{4}$ and:

$$T=2\sqrt{\frac{H}{a_m}} \tag{7}$$

For $t=\frac{T}{2}$, speed v reaches the maximum value:

$$V_{max}=a_m\frac{T}{2}=a_m\sqrt{\frac{H}{a_m}} \tag{8}$$

1.2 Variable acceleration tachograms

The continuous variation of the acceleration will be replaced with a variation in steps within the same phase of the trajectory for a period of time T of the process (figure 2), in a finite number of equal intervals with a duration of:

$$\tau=\frac{T}{n} \tag{9}$$

It is supposed that acceleration a (as a command value) is constant within each sub-interval, with values comprised between a_1, a_2, ... ,a_n. By divide the acceleration, the following may be written:

$$W^*=\sum_{i=1}^{n}a_i^2\tau \tag{10}$$

depending on n variables.

Out of all the staircase functions a(t) the chosen one is that for which the minimum of the W^* sum is obtained and simultaneously ensuring the compliance with the limit conditions:

$$\left.\begin{aligned} v(0)=0;\quad v(T)=v_k=0\\ h(0)=0;\quad h(T)=h_k=H\end{aligned}\right\} \tag{11}$$

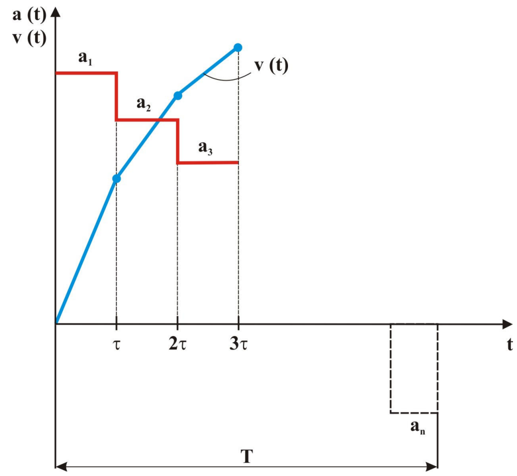

Fig.2 Variation in steps of the acceleration within the same phase

Permanently decreasing the duration of intervals τ, as a result of a limit transition, a continuous dependence a(t) will be obtained which minimises the integral W. Therefore, this is the optimum command condition.

The speed v given by relation (3), considering the initial condition $v(0)=0$, varies according to a dotted line (figure 2), consisting of parts of lines the coordinates of which t = 0, t = τ, t = 2τ, ..., t = T are:

$$\left.\begin{aligned} &v_0=v(0)=0;\\ &v_1=v(\tau)=a_1\tau;\\ &v_2=v(2\tau)=(a_1+a_2)\tau;\\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ &v_n=v(n\tau)=\sum_{i=1}^{n}a_i\tau=0\end{aligned}\right\} \tag{12}$$

The movement h will be composed of sections of parabola. Considering the initial condition $h(0)=0$, based on relation (12) h_i ordinates in points t = 0, t = τ, t = 2τ, ..., t = T are obtained.

$$\left.\begin{aligned} &h_0=h(0)=0;\\ &h_1=h(\tau)=\frac{\tau}{2}v_1=\frac{\tau^2}{2}a_1;\\ &h_2=h(2\tau)=\frac{\tau}{2}v_2=\tau^2a_1;\\ &h_3=h(3\tau)=\frac{\tau}{2}v_3=\tau^2(2a_1+a);\\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ &h_n=h(T)=\frac{\tau}{2}v_n+\tau^2\sum_{k=1}^{i-1}(n-k)a_k=h_k\end{aligned}\right\} \tag{13}$$

ISBN: 978-1-61804-162-3

In order to determine the conditioned extreme of sum W^* considering the relations (12) and (13), it is sufficient enough to determine the unconditioned extreme of the auxiliary function V:

$$V = W^* + \lambda_1 v_k(a_i) + \lambda_2 h_k(a_i) \tag{14}$$

where λ_1 and λ_2 undetermined Lagrange multipliers, determining the limit conditions.

Therefore,

$$\begin{aligned} V = \tau\sum_{i=1}^{n} a_i^2 + \lambda_1\tau\sum_{i=1}^{n} a_i + \lambda_2\frac{\tau^2}{2}\sum_{i=1}^{n} a_i + \\ +\lambda_2\tau^2\sum_{i=1}^{n-1}(n-i)a_i \end{aligned} \tag{15}$$

The conditions needed for the extremes, is expressed by the system $\frac{\partial V}{\partial a_i} = 0;\quad i = 1,\ldots,n$.

Considering the expression (15), the following are obtained:

$$\frac{\partial V}{\partial a_i} = 2\tau a_i + \lambda_1\tau + \frac{\lambda_2}{2}\tau^2 + \lambda_2\tau^2(n-i) = 0 \tag{16}$$

From where:

$$a_i = -\frac{\lambda_1}{2} - \frac{\lambda_2}{4}\tau - \frac{\lambda_2}{2}\tau(n-i) \tag{17}$$

If for T = ct, the duration of the interval τ decreases unlimited, and the number of intervals n tends towards infinity, then a_i passes into a(t), and τ_i in t. Considering $n \cdot \tau = T$, it results:

$$a(t) = -\frac{\lambda_1}{2} - \frac{\lambda_2}{2}(T-t) \tag{18}$$

In order to determine the Lagrange multipliers the following limit conditions are applied:

For $t = 0; a(0) = a_a$; and $t = t_a; a(t_a) = 0$, where:

a_a - is the initial value and the largest of the command measure (acceleration); considering an optimum process it varies linearly from $+a_a$ to $-a_a$;

t_a - is the moment the acceleration passes through the neutral.

The following equation system results applying these conditions for expression (18):

$$\left.\begin{aligned} a_a &= -\frac{\lambda_1}{2} - \frac{\lambda_2}{2}T \\ 0 &= -\frac{\lambda_1}{2} - \frac{\lambda_2}{2}(T - t_a) \end{aligned}\right\}$$

Solving the system according the unknown λ_1 and λ_2, it results: $\lambda_1 = 2a_a;\quad \lambda_2 = -\frac{2a_a}{t_a}$.

Considering that $T = 2t_a$ and $a(T) = -a_a$, equation (18) becomes $a(t) = -a_a + \frac{a_a}{t_a}(2t_a - t)$.

Therefore, expression (18) may be written:

$$a(t) = a_a\left(1 - \frac{t}{t_a}\right) \tag{19}$$

It is observed that the optimum law of variation of acceleration both during acceleration as well as during deceleration is limited, imposing a parabola variation of speed during these periods.

Integrating equation (19), speed, space and energy dissipated during transitional starting and breaking periods, laws of variations are obtained:

$$\begin{aligned} v(t) &= \int_0^t a(t)\,dt = \\ &= \int_0^t \left[a_a\left(1 - \frac{t}{t_a}\right)\right]dt = a_a t\left(1 - \frac{t}{t_a}\right) \end{aligned} \tag{20}$$

$$\begin{aligned} h(t) &= \int_0^t v(t)\,dt = \\ &= \int_0^t \left[a_a t_a\left(1 - \frac{t}{t_a}\right)\right]dt = a_a\frac{t_2}{2}\left(1 - \frac{t}{3t_a}\right) \end{aligned} \tag{21}$$

$$\begin{aligned} W(t) &= \int_0^t a^2(t)\,dt = \\ &= \int_0^t \left[a_a\left(1 - \frac{t}{t_a}\right)\right]^2 dt = a_a^2 t_a\left(1 - \frac{t}{t_a} + \frac{t^2}{3t_a}\right) \end{aligned} \tag{22}$$

The t_a and a_a constancies are determined from the limit conditions:

$$v(T) = a_a T\left(1 - \frac{T}{t_a}\right) = 0 \tag{23}$$

$$h(T) = a_a\frac{T^2}{2}\left(1 - \frac{T}{3t_a}\right) = H \tag{24}$$

From the above equation it results:

$$t_a = \frac{T}{2};\quad a_a = 6\frac{H}{T^2} \tag{25}$$

The above case presents the starting and breaking transitional processes considering a two phase tachogram. Introducing the speed limit imposed by the operational norms of vertical transport installations, $v(t) \le v_{\max\,adm}$ then the tachogram transforms into a three phase one where $t_a' < t_a$ (figure 3).

It is observed that the duration of the transitional periods t_1 (acceleration) and t_3 (deceleration) depend on the level of the maximum adopted speed, namely on the ordinate intersected by the optimum variation curve of the speed (parabola) with a horizontal line corresponding to the maximum speed.

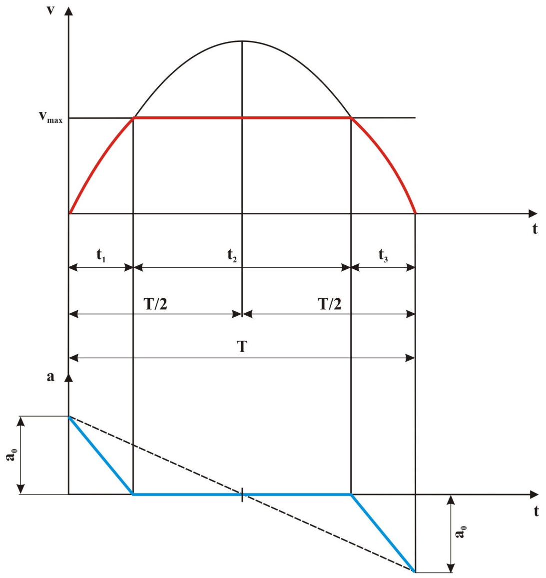

Fig.3 Speed limit tachogram

In the same time, it results that the acceleration needn't be kept at a constant level, imposing a smooth linear variation.

2 Dynamic parameters optimisation

A method for the optimisation of the electric operation is constituted by adopting a trapezoidal tachogram and considering a constant static torque according to the criteria of equivalent power. The objective was the development of an optimum trapezoidal tachogram for the minimisation of the equivalent power (figure4). The mathematical model used is based on relative coordinates with the purpose of generalising the results.

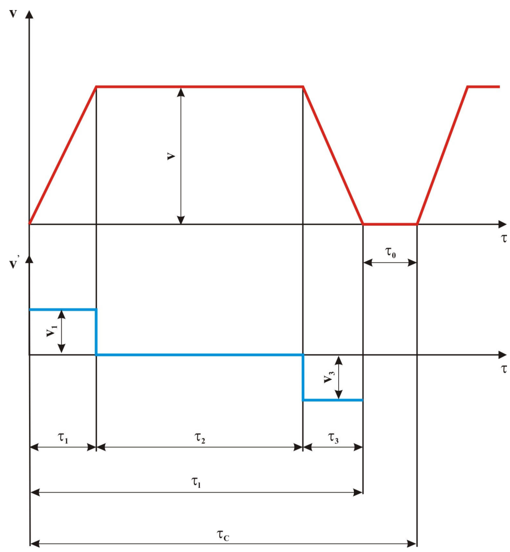

Fig.4 Analysed tachogram

The following have been considered for the time reference:

$$\tau = \frac{t}{T} \tag{26}$$

where:

T represents the mechanical time constancy:

$$T = \frac{I \cdot \omega_N}{M_N} = \frac{m \cdot v_N}{F_N} \tag{27}$$

where:

- I is the inertia moment of moving elements;
- m the weight of the moving elements;
- M_N and F_N the peripheral momentum and force;
- ω_N and v_N the angular and peripheral speed of the operating mechanism.

For the speed, torque and power, their nominal values have been considered as reference values:

$$v = \frac{\omega}{\omega_N} = \frac{v}{v_N}; \quad \mu = \frac{M}{M_N} = \frac{F}{F_N}; \quad \rho = \frac{P}{P_N} \tag{28}$$

The following relations result for the movement and acceleration:

$$t = \frac{\theta}{T\omega_N} = \frac{H}{Tv_N}; \quad v' = \frac{\omega' T}{\omega_N} = \frac{v' T}{v_N} \tag{29}$$

Therefore the movement equation in absolute measures $M = M_s + \left|\frac{d\omega}{dt}\right|$ may be written in relative measures as:

$$\mu = \mu_s + \frac{dv}{d\tau} \tag{30}$$

The expressions of speed and space are:

$$v = \int v' dt; \quad h = \int v dt \tag{31}$$

The power in relative measures is:

$$\rho = \mu\, v = \left(\mu\, s + v'\right) v \tag{32}$$

The total movement of a trapezoidal tachogram, after making the integrals (31) is:

$$x_0 = \frac{1}{2} v\tau_1 + v\tau_2 + \frac{1}{2} v\tau_3 \tag{33}$$

Introducing a dimensional variables:

$$\alpha = \frac{\tau_1}{\tau_2}; \quad \beta = \frac{\tau_3}{\tau_1}; \quad 0 < \alpha; \quad \beta < 1 \tag{34}$$

The periods of the tachogram become:

$$\tau_1 = \alpha\tau_2; \quad \tau_3 = \beta\tau_1; \quad \tau_2 = \left[1 - (\alpha - \beta)\right]\tau_1 \tag{35}$$

And the regime movement and speed will be:

$$x_0 = \left[1 - \frac{1}{2}(\alpha + \beta)\right] v\, \tau_1 \tag{36}$$

$$v = \frac{x_0}{\tau_1} \cdot \frac{1}{1 - \frac{1}{2}(\alpha + \beta)} \tag{37}$$

ISBN: 978-1-61804-162-3

The corresponding accelerations for the two ends of the tachogram are:

$$v_1^{'} = \frac{v}{\tau_1} = \frac{1}{\alpha\left[1 - \frac{1}{2}(\alpha + \beta)\right]} \cdot \frac{x_0}{\tau_1^2}$$
$$v_3^{'} = \frac{v}{\tau_3} = \frac{1}{\beta\left[1 - \frac{1}{2}(\alpha + \beta)\right]} \cdot \frac{x_0}{\tau_1^2} \qquad (38)$$

Equivalent torque:

$$\mu_{ech}^2 = \frac{1}{\tau_c}\int_0^{\tau c} \mu^2 d\tau = \frac{\varepsilon}{\tau_1}\int_0^{\tau 1} (\mu_s + v^{'})^2 d\tau \qquad (39)$$

Where ε is the connection period:

$$\varepsilon = \frac{\tau_1}{\tau_c} \qquad (40)$$

For a trapezoidal tachogram, it results the following equivalent torque:

$$\mu_{ech}^2 = \varepsilon\left[\mu_0^2 \frac{\frac{1}{\alpha} + \frac{1}{\beta}}{\left[1 - \frac{1}{2}(\alpha+\beta)\right]^2} \cdot \frac{x_0^2}{\tau_1^4}\right] \qquad (41)$$

Equivalent power depending on the torque and speed: $p_N = v\sqrt{\frac{1}{\tau_c}\int_0^{\tau c} \mu^2 d\tau}$.

$$p_N^2 = \varepsilon \cdot x_0^4 \cdot \left[\frac{\mu_0^2 \cdot \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)}{\tau_1^4 \cdot \tau_0^2 \cdot \left[1 - \frac{1}{2}(\alpha+\beta)\right]^4}\right] \qquad (42)$$

The extreme of the equivalent power is obtained for $\alpha = \beta$:

$$p_N^2 = \varepsilon\left[\mu_0^2 \frac{1}{(1-\alpha)^2} + \frac{2}{\alpha(1-\alpha)^4} \cdot \frac{x_0^2}{\tau_1^4}\right]\frac{x_0^2}{\tau_1^2} \qquad (43)$$

The minimum condition of the equivalent power results from cancelling the derivative of the power with the restrictions: $v \le 1$; $\mu \le \mu_{max}$; $\mu_{ech} \le 1$:

$$\frac{\partial\, p_N^2}{\partial\, \alpha} = \frac{\mu_0^2}{x_0^2}\tau_1^4\alpha^2(1-\alpha)^2 - (1-5\alpha) = 0 \qquad (44)$$

For the particular case of no-load operation ($\mu_0 = 0$), it results the optimum value of the power:

$$\alpha = \beta = 0{,}2 \qquad (45)$$

The regime speed:

$$v = 1{,}25\frac{x_0}{\tau_1} \qquad (46)$$

Minimum power:

$$P_{N\,min} = 4{,}94\sqrt{\varepsilon}\frac{x_0^2}{\tau_1^3} \qquad (47)$$

In case of load operation ($\mu_0 \ne 0$) it results:

$$0 < \alpha_{opt} \le 0{,}2$$

Analysing the above presented method, the main conclusion is that it is a practical, operative method but it is valid only for a linear variation of speed. There is no certainty that this type of variation is optimum for ensuring the minimum value of speed. Moreover, choosing the trapezoidal tachogram is not scientifically justified, being made only empirically based on experience. Therefore, there may be another form of the operational diagram to ensure the minimum value of the actuating power.

3 Conclusions

Vertical transport installations are a machinery, equipment and mechanism complex ensuring the connection between different levels of materials and personnel imposing therefore special operational requirements which eliminate break-downs causing increased material and human damages. From the analysis of this study, the following important conclusions are emphasised:

- Due to increased actuating powers, the vertical transport installations represent one of the most important energy consumer as the power requirements increase together with the height of transport;
- The two phase tachograms are used for small heights;
- Three phase tachograms with constant acceleration and linear speed variation during transitional periods are used for installations powered by asynchronous motors, while for the same installations but powered by a continuous current, tachograms with linear acceleration variation during start-up (parabola speed variation) and with constant deceleration (linear speed variation) during breaking, are used;
- Five phase tachograms are also used for installations powered asynchronously, asymmetric, with constant acceleration, while for the installations powered by continuous current, second phase linear acceleration variation tachograms are generally used, leading to a parabola variation of speed in the same phase;
- Three phase parabola tachograms, with speed limit, although advantageous, are not widely spread due to the imperfections of the speed regulator;

- In order to avoid shock in the mechanical system during transportation, the elimination of sudden acceleration variations is imposed;
- The momentum equation of constant ray reeling installations coincides in form to the equation of the peripheral forces;
- In order to avoid large limit variations of peripheral forces and powers, dynamic balanced installations use is recommended;
- The transitional periods (acceleration and deceleration) highly influence the actuating power and energy consumption;
- During the transitional periods, the variation in time of the current absorbed by the actuating motor of the installation is equivalent to the variation of the acceleration during the same periods. The high decrease, therefore, of the acceleration leads to the increase of the period of the movement imposing the adoption of a compromising solution;
- The actuating power and consequently the energy consumption are mainly influenced by the speed variation and the acceleration of extraction containers law. In order to minimise the energy consumption during transitional periods the variation determination is used, proving that the optimum acceleration variation law is the linear one, while of the speed is the parabola one, both during acceleration as well as during deceleration.

References:

[1] Fransua Alexandru, *Maşini şi sisteme de acţionări electice – Probleme fundamentale*, Editura tehnică, Bucureşti, 1978;

[2] Magyari Andrei, *Instalaţii mecanice miniere*, Editura Didactică şi Pedagogică Bucureşti, 1990

[3] Nan Marin Silviu, *Capacitatea sistemelor de transport*, Editura Universitas Petroşani, 2000

[4] Florin Dumitru Popescu, Ilie Mitran, *The analysis of vertical transport installations considering the theory automatic systems*, Proceedings of the 8th WSEAS International Conference on System Science and Simulation in Engineering (ICOSSSE '09), University of Genova, Italy October 17-19, 2009, pp.125-130

[5] Florin Dumitru Popescu, Ilie Mitran, *Vertical Transport Installations Balance Survey*, WSEAS TRANSACTIONS on SYSTEMS and CONTROL, Issue 10, Volume 4, October 2009, ISSN: 1991-8763, pp.509-518

[6] Florin Dumitru Popescu, *Aplicaţii industriale ale tehnicii de calcul*, Editura AGIR, Bucureşti, 2009, ISBN 978-973-720-234-5

[7] Florin Dumitru Popescu, *Instalaţii de transport pe verticală*, Editura Focus, Petroşani, 2010, ISBN 978-973-677-182-8

[8] Florin Dumitru Popescu, Gabriel Dimirache, *Instalaţii de transport pe verticală –Calcule de verificare şi dimensionare*, Editura Focus, Petroşani, 2011, ISBN 978-973-677-182-8

[9] Pop Emil, *Automatizări în industria minieră,* Editura Didactică şi Pedagogică, Bucureşti, 1983

[10] Şabac I.Gh., *Matematici speciale*, Editura Didactică şi Pedagogică, Bucureşti,1981

[11] Tunsoiu Gheorghe, *Acţionări electrice*, Editura Didactică şi Pedagogică, Bucureşti, 1982.