Analyzing the Slabs by Means of the Finite Difference Method

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Abstract: Slabs rank among structures which are used very frequently. The slabs can be analysed by the finite difference method which is described in this paper. Two types of square concrete slabs are used for the analysis. Deflections, specific bending moments and torsion moments were chosen for the analysis. Algorithms for the calculation were developed and calculated for various systems of equations. This paper also aims at evaluating time which each method needs for solution of a set of linear equations with algorithms developed in Matlab. Results of this method are compared with those obtained by the finite element method.

Key-Words: Finite element method, finite difference method, slab, algorithms, speed calculation, deflection

1 Introduction
The paper deals with the analysis of concrete structures. Much attention has been paid to slab analyses in past to experimental [1] and theoretical research [2, 3]. Slab made from concrete are often used.

Designs of concrete structures have been dealt with in recommendations [4]. This task is closely connected with non-linear analyses which have been used by the authors for 2D tasks [5, 6]. Original relations and theories for analysing internal forces of slabs are listed in [7, 8]. The most frequent methods used for the slabs are the Finite Element Method [9, 10] and Finite Difference Method [11, 12]. The Finite Element Method is used in [13, 14, 15]. When analysing the slabs by means of the Finite Difference Method, orthotropic properties can be also taken into account [16]. For some tasks the Finite Difference Method was used also for the non-linear analysis [17, 18].

The goal is to develop an algorithm by means of the finite difference method (which is also referred to as the network method) and compare results with those obtained by the finite element method. Deflections, specific bending moments and torsion moments were chosen for the analysis. The program Matlab [19] was used to develop an algorithm and the program Scia Engineering 2012 [20] was used to the verify results. One of other possible approaches used for development of algorithms for such tasks is described in [21]. Another goal of this paper was to compare the methods used for calculation of the system of linear equations [22, 23, 24] depending on the density of the network.

Two directs methods and two methods integrated in the program Matlab [19] were chosen for purposes of such comparison. More details about algorithm development are described in [25].

2 Finite difference method
Thin slabs can by analysed using the finite difference method. For theory and more details relating to this method see [7]. The algorithm assumes that the structure fulfils conditions specified in Kirchhoff theory [11]. The unknown variable in the slab is \( w \) deflection which can be obtained by solving the slab equation:

\[
\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D}
\]

(1)

where \( w \) is deflection of the slab, \( p \) is magnitude of the load, and \( D \) is slab rigidity. The basic principle of the finite difference method is that the structure is divided into nodes and the differential equation (1) of the slab is modified by means of difference replacement to give a system of linear equations for node values of the deflection. The system of equations was created using a difference scheme which defines environment for each point in the network, creating thus a system of linear equations. The calculation used a reduced deflection which depends on real deflection and the formula is as follows:

\[
W = \frac{D}{a \cdot b} w
\]

(2)

where \( a \) and \( b \) are distances between network nodes (the difference step), \( D \) is slab rigidity and \( w \) is real
vertical dislocation (deflection) of a point in the network. When using the reduced deflection, the right side of the difference equations equals, for each point, to:

\[ P = p \cdot a \cdot b + F \]

where \( F \) is the value of a single load applied in a node point. Because the difference scheme requires 13 points in the surrounding and such points may exceed beyond edges of the slab, boundary conditions should be used to define the outer points.

3 Numerical example

The concrete slab, which is analysed by means of the finite difference method is simply supported along its perimeter and loaded with the even load \( p = 10 \text{ kNm}^{-2} \). Dimensions of the slab are \( b = 5 \text{ m} \) and \( l = 5 \text{ m} \). The thickness is \( h = 180 \text{ mm} \). The static modulus of elasticity is \( E = 30 \text{ GPa} \) and Poisson coefficient is \( 0.2 \). The calculation is carried out for 25 internal points.

The maximum calculated values were as follows: deflection \( w = 1.6715 \text{ mm} \), bending moments \( m_x = m_y = 11.038 \text{ kNm} \) and torsion moments \( m_{xy} = \pm 9.177 \text{ kNm} \). Results obtained in the program Matlab [19] were checked by calculations made in the program Scia Engineer [20]. The program Scia Engineer [20] is based on the finite element method [9]. The calculated values were slightly different: deflection \( w = 1.792 \text{ mm} \), bending moments \( m_x = m_y = 54.51 \text{ kNm} \) and torsion moments \( m_{xy} = \pm 43.63 \text{ kNm} \).

Fig. 1 shows 3D results obtained for the finite difference method. Fig. 2 shows results obtained in Scia Engineer [20].

4 Comparing the methods used for calculation of the system of linear equations

The system of linear equations is solved using matrix calculation where the system can be briefly described as follows:

![Fig. 1. Internal forces and deflections calculated using the finite difference method](image_url)
Fig. 2. Deflections and internal forces calculated in Scia Engineer [20]

Fig. 3. Internal forces and deflections calculated using the finite difference method
\[ A \cdot x = b \]  
(4)

where: \( A \) is the matrix of the system (the left side of the equations), \( b \) is the vector of the right sides and \( x \) is the vector of system roots which should be determined. Four methods were used for purposes of comparison:

- direct methods: Gauss-Jordan method and LU decomposition method,
- Matlab functions: division \((x = b/A)\) and inversion \((x = b^{-1} \cdot A)\).

Variability of the network size and, in turn, the number of linear equations were also taken into account. The modified algorithm changes gradually the network size and measures the time needed for calculations.

The basis for comparison was a slab which is fixed at all sides and loaded with the even load of 15 kNm\(^2\). Dimensions of the slab are \( b = l = 6 \) m. The thickness is \( h = 200 \) mm. The static modulus of elasticity is \( E = 30 \) GPa and Poisson coefficient is 0.2. The final deflection in this case (for 59 internal points where the distance between the points in the network is 100 mm) is \( w = 1.1835 \) mm. The final bending moments are \( m_x = m_y = 11.423 \) kNm, \( m_x- = m_y- = -27.680 \) kNm and the torsion moment is \( m_{xy} = \pm 5.269 \) kNm. Fig. 3 shows 2D results for the finite difference method, while Fig. 4 shows results obtained in the program Scia Engineer [9] (\( w = 1.181 \) mm, \( m_x = m_y = 11.416 \) kNm, \( m_x- = m_y- = -27.698 \) kNm and \( m_{xy} = \pm 5.284 \) kNm). The graphic comparison in Fig. 6 was based on 5 calculation cycles where the density of network varied from 5 to 50 internal points. An average was made for the values from each cycle, and the average values were plotted in the chart. The maximum time was set at 60 seconds. In order to compare functions in the program Matlab [19] in detail, the function were compared for the network density from 50 to 100 internal points in Fig. 7. After the calculation is completed, the dump print in the command window in Fig. 5 is as follows (the first 16 cycles are shown there):

Fig. 4. Deflections and internal forces calculated in Scia Engineer [20]
Fig. 6. Comparing the duration of each method versus the size of the matrix (the dashed connecting line belongs to the right vertical axis)

Fig. 7. Comparing duration of the methods depending on the size of the matrix (density of the network: 50 – 100 points)
If the maximum preset time is exceeded in the calculation (60 minutes, in this case), the method is not measured anymore. Average times from each cycle are plotted in the chart.

4 Conclusion

Thin slabs can be analysed in an algorithm created in the program Matlab [19] which is based on the finite difference method. The variables which should be identified might be the deflection of the slab or internal forces. Correctness of the calculation was verified by comparing these results with those obtained in the program Scia Engineer [20]. The comparison of times measured for the methods used for the systems of linear equations and evaluation of the charts with the values suggest that the LU decomposition method is better, from among the direct methods, than the Gauss-Jordan method. In case of rather small matrices it is, however, recommended to use the Gauss-Jordan matrix. The fastest method is the direct division of matrices in the program Matlab [19]. Direct methods can be used for rather small systems of equations – for the matrices which are less than 1600. Those methods are not optimised enough for rather big matrices and do not provide necessary results in a real time.

Because the matrix of the system is sparse, each row in the matrix contains not more than 13 values. It is advisable then to use this feature when optimising the calculation. Another objective of the research is the using of the Finite Difference Method for analyses of fibre reinforced concrete structures [26].

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