Choosing heating units using the Electre function

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Abstract: In this paper, we will discuss how to choose heating units using the Electre function. Chapter 2 presents the computation methodology. A case study underlines the way in which one can apply the mathematical model into practice. This paper presents outline and results of these calculations.

Key-Words: boiler, multi-criteria methods, electre function, central heating, installations for constructions, optimization, performance.

1 Introduction

In this article, we have analyzed the possibility of choosing the heating units using the Electre method.

One of the most significant methods for optimizing multidimensional decisions under certainty, whose construction is centered on the utility theory, is the ELECTRE method, which is a product of the French management school [16].

The ELECTRE method was proposed in 1965 by Bernard Roy, a professor at Universitatea Paris-Dauphine University. The ELECTRE acronym comes from the initials of the method’s name: **EL**imination **E**t **C**hoix **T**raduisent la **RE**alité (Elimination and Choice Expressing Reality). The method was developed in time: ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS, ELECTRE TRI [15].

In Romania, multi-criteria methods are well known [12], however there are few studies about their use in the field of installations for constructions. Starting with 1996, studies about how to choose boilers and heating units using multi-criteria methods started to appear in Romania too [1], [2], [4], [7], [8], [9], [17].

2 The work method

2.1 Stages

The choice of the optimal decision-making version using the Electre method is based on a ten-step computation algorithm (Fig. no. 1), namely:

- determining the decisional versions;
- determining the decisional criteria;
- determining the importance coefficients corresponding to decisional criteria;
- filling in the consequence matrix;
- filling in the utility matrix;
- forming the concordance coefficients matrix;
- forming the discordance coefficients matrix;
- forming the outranking matrix;
- forming the preference matrix;
- choosing the optimal version.

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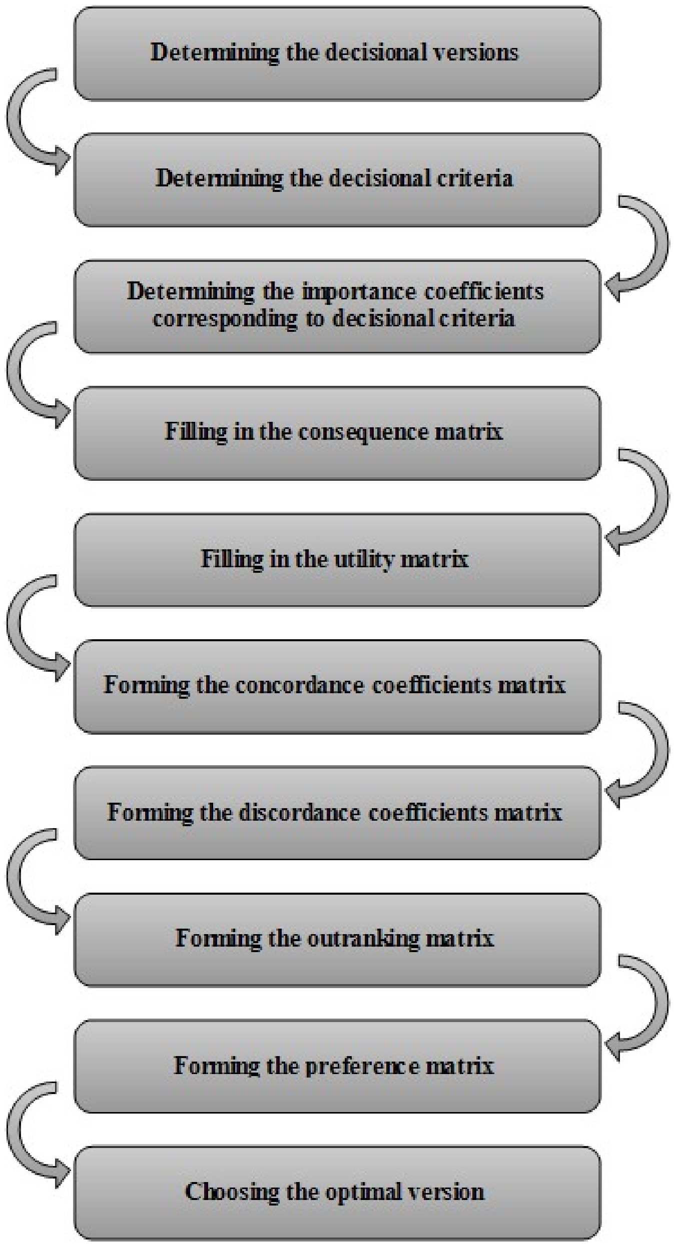

Fig. 1 - Stages of global Electre function

2.2 Calculation algorithm

The difference between the “ELECTRE method” and the “global utility method” consists in the comparison of pairs of versions and in the determination of preference relations (outranking) based on two indices, the concordance coefficient and the discordance coefficient [14].

In papers [7] and [8] the author presented the “global utility method”, and in this paper he is going to detail the particularities of the “ELECTRE method”.

2.2.1 Computation of concordance coefficients

The concordance coefficients (cim) for the two pairs of versions (Vi and Vm) are computed according to the formula:

$$cim = \frac{1}{k1 + k2 + ...kn} \cdot \sum kj \text{ , } j = 1 \ldots n \tag{1}$$

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For those j's for which uij > umj.

where:

kj are the importance coefficients of criteria [5], [6].

Concordance coefficients are sub-unitary numbers and they show that a Vi version outranks a Vm version [14].

These coefficients are transposed into a square matrix (table no. 1), where both the lines and the columns are versions (Vi stands for the lines, while Vm stands for the columns) [10].

Table 1 Concordance coefficients matrix [11]

cim	V1	…	Vi	…	Vm
V1		…	c1i	…	c1m
…	…		…	…	…
Vi	ci1	…		…	cim
…	…	…	…		…
Vm	cm1	…	cmi	…	

The concordance coefficients of the appreciation criteria show in which degree a decision-making version Vi outranks another decision-making version Vm, according to all „m" appreciation criteria [11].

2.2.2 Computation of discordance coefficients

The discordance coefficients (dim) for the two pairs of versions (Vi and Vm) are computed according to the formula:

dim = 0, if umj < uij

$$\text{dim} = \frac{1}{d} \cdot \max(umj - uij) \text{, maximum for those j which umj > uij} \qquad (2)$$

where:

de is the maximum difference that may occur between the values of the states; d = 1, if minimum uij = 0 and maximum uij = 1.

Discordance coefficients are sub-unitary numbers and they show how much a random alternative "Vm" outranks another alternative "Vi" [14].

These coefficients are transposed into a square matrix (table no. 2), where both the lines and the columns are versions (Vi stands for the lines, while Vm stands for the columns) [10].

Table 2 Discordance coefficients matrix [11]

dim	V1	…	Vi	…	Vm
V1		…	d1i	…	d1m
…	…		…	…	…
Vi	di1	…		…	dim
…	…	…	…		…
Vm	dm1	…	dmi	…	

The discordance coefficients are the ones that show the unacceptability of an option, when an unacceptably weak performance is recorded [12].

2.2.3 Forming the outranking matrix

Hereinafter, we have computed the differences between the corresponding concordance and discordance coefficients:

$$Dim = cim - dim \qquad (3)$$

and the data are centralized in the outranking matrix, according to table no. 3.

Table 3 Outranking matrix

Dim	V1	…	Vi	…	Vm
V1					
…					
Vi					
…					
Vm					

2.2.4 Forming the preference matrix

In the preference matrix (table no. 4), we compared the pairs of values marked “Dim” and Dmi”, and according to this relation, we granted pim scores to the two versions (Vi and Vm).

There are three possible situations:

- Dim < Dmi: Vi scores 0 points (pim = 0), while Vm scores 1 point (pmi = 1) (generally speaking, by directly comparing the two versions, Vi proved to be weaker than Vm);
- Dim = Dmi: Vi and Vm scores are identical (there are two working methods: each of them scores 1 point or each of them scores 0.5 points);
- Dim > Dmi: Vi scores 1 point (pim = 1), qnd Vh scores 0 points (pmi = 0).

One shall compute the general score for each version, by summing up the points it scored (in table no. 4, the points are summed on line) [10]:

$$Pi = \sum_{i \neq m} pim \qquad (4)$$

Table 4 Preference matrix [13]

pim	V1	…	Vi	…	Vm	Pi	Place
V1							
…							
Vi							
…							
Vm							

The more points a version scores, the better it proved to be, and therefore it has a better rank in the versions’ ranking. Hence, the ranking is made by taking into account the decreasing order of Pi scores [10].

2.2.5 Choosing the optimal version

The optimal version is given by the maximum sum of utilities from the preference matrix [5], [6].

One compares the importance levels of the decision-making versions and chooses the optimal decision-making version (Vo) [11]:

$$Vo = \max_i \{f(Vi)\} \qquad (5)$$

For noting the ranking, the following notations are used:

- P = preferable: if a version obtains a higher score than another one, then the first one is preferable to the latter;

- I = indifferent: if two versions obtain the same score, then one is indifferent to another [10].

So, if the score obtained by version Vi outranks the score obtained by version Vm, the relation between the versions is Vi P Vm (namely Vi is preferred to Vm), and if the scores are identical, the relation between the versions is Vi I Vm (namely there is an indifference relation between the two versions) [6].

3 Case study

We present bellow a case study related to how to choose heating units using the Electre function.

3.1 Set of decisional versions

We take into account 4 mini-heating units marked P1, P2, P3 and P4 [9].

In table no. 5 we presented the set of versions [Vi].

Table 5 Set of versions [Vi]

V_i	Name
V_1	P_1
V_2	P_2
V_3	P_3
V_4	P_4

3.2 Set of decisional criteria

Out of the set of characteristics of one mini-heating unit, we have chosen as analysis characteristics: lifespan, nominal thermal power, nominal output, automation degree, accessories, template, electrical power, noise level and price.

In order to make this study, the following classification of the above mentioned features is also useful, namely:

- features directly proportional to the product's quality (the bigger is the value of the quantity associated to the feature, the more product quality increases): lifespan, nominal thermal power, nominal output, automation degree, accessories;

- features inversely proportional to the product quality (the smaller is the value of the quantity associated to the feature, the more product quality increases): template, electrical power, noise level, price [9].

In table no. 6, we presented the set of decisional criteria [Cj].

Table 6 Set of Criteria [Cj]

C_j	Criterion Name	M.U.	Nature
C_1	Lifespan	years	maximizing
C_2	Nominal thermal power	kW	maximizing
C_3	Nominal output	%	maximizing
C_4	Automation degree		maximizing
C_5	Accessories		maximizing
C_6	Template		minimizing
C_7	Electrical power	W	minimizing
C_8	Noise level	dB(A)	minimizing

3.3 Set of assessment criteria consequences

The consequence matrix (table no. 7) contains the values of the quantities characterizing these products (price, nominal thermal power, template, and so). The values necessary for the study are offered directly by the manufacturer in the documentation. For other features (automation degree and accessories), an assessment is made based on the information found in the documentation, using grades from 1 to 3 (where 1 is the lowest grade and 3 is the highest grade) [9].

For the example studied, the data obtained shall be centralized in table no. 7.

Table 7 Consequence matrix [aij]

V_i	C_j							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
V_1	22	42	92,5	3	2	0,823	130	65
V_2	15	40,7	90	2	1	0,513	150	55
V_3	20	47,2	92	1	2	0,533	100	60
V_4	20	47	92	3	3	1,273	130	65
V_4	20	47	92	3	3	1,273	130	65

4 Results and discussion
4.1 The obtained results

By applying the utility method, we obtained the utility matrix (table no. 8).

Table 8 Utility matrix [uij]

Version	Importance coefficients								Utility sum
	K1	K2	K3	K4	K5	K6	K7	K8	
	0.2105	0.1842	0.1579	0.0921	0.0789	0.0526	0.1053	0.1184	
	Criteria								
	C1	C2	C3	C4	C5	C6	C7	C8	
V_1	1.00	0.20	1.00	0.00	0.50	0.41	0.40	0.00	3.51
V_2	0.00	0.00	0.00	0.50	1.00	0.00	0.00	1.00	2.50
V_3	0.71	1.00	0.80	1.00	0.50	0.03	1.00	0.50	5.54
V_4	0.71	0.97	0.80	0.00	0.00	1.00	0.40	0.00	3.88

Hereinafter, based on the utility matrix and on the relation no. 1, we determined the concordance coefficients, and the data were transcribed in the concordance coefficients' matrix (table no. 9).

Table 9 Concordance coefficients' matrix [cih]

c_{ih}	V_1	V_2	V_3	V_4
V_1		0.71	0.50	0.76
V_2	0.29		0.20	0.29
V_3	0.50	0.80		0.95
V_4	0.24	0.71	0.05	

Based on the utility matrix and by using relation no. 2, we also determined the discordance coefficients, and the data were transcribed in the discordance coefficients' matrix (table no. 10).

Table 10 Discordance coefficients' matrix [dih]

d_{ih}	V_1	V_2	V_3	V_4
V_1		1.00	1.00	0.77
V_2	0.38		1.00	1.00
V_3	0.38	0.50		0.97
V_4	0.50	1.00	1.00	

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Based on the data from the concordance coefficients' matrix, on the data form the discordance coefficients' matrix and on relation no. 3, we built the differences' matrix (table no. 11).

Table 11 Differences matrix [Dih]

D_{ih}	V_1	V_2	V_3	V_4
V_1		-0.29	-0.50	-0.01
V_2	-0.09		-0.80	-0.71
V_3	0.12	0.30		-0.03
V_4	-0.26	-0.29	-0.95	

Based on the differences' matrix, we built the points' matrix (table no. 12).

Tabel 12 Points' matrix [pih] and computation of scores [Pi]

p_{ih}	V_1	V_2	V_3	V_4	P_i	Place
V_1		0	0	1	1	2
V_2	1		0	0	1	2
V_3	1	1		1	3	1
V_4	0	1	0		1	2

Based on the data presented in table no. 12, the following versions' ranking resulted: V3 P V1 I V2 I V4.

The versions' ranking shall be interpreted as follows:

- version V3 is preferable to version V1;
- versions V1, V2 and V4 obtained the same score, therefore they are in an indifference relation.

4.2 Discussions

By comparing the ranking obtained in this paper using the Electre method to the ranking established by the author using the utility method, we notice that the product noted V3 is on the first place in both cases. For the rest of the products, the ranking is different, namely product V4 takes the 2nd place, product V1 the 3rd place and product V2 the 4th place respectively.

The results obtained were also compared to the results obtained by other authors [9], and the conclusion is that the multi-criteria method may influence the final ranking of the technical solutions.

5 Conclusion

The main conclusions of this article are the following:

a) When choosing technical solutions based on multi-criteria methods, the final ranking may be influenced by the multi-criteria method used.

b) The Electre method is relatively easy to apply if there are relatively few criteria. When the number of criteria increases, in order to solve the decision-making problem more rapidly, we recommend the use of some calculation software.

c) The French school recommends that one should not use more than 49 criteria [3].

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