Alternative Numerical Techniques to Approximate the Electric Field of the Electrostatic Precipitators

IOANA-GABRIELA SIRBU
Department of Electrical, Power Systems and Aerospace Engineering
University of Craiova
107 Decebal Boulevard, 200440 Craiova
ROMANIA
osirbu@elth.ucv.ro http://www.ie.ucv.ro

Abstract: - This paper proposes to show how different numerical methods can be used to estimate the electric field distribution of an electrostatic precipitator. The principle of each method and the equations underlying each approach, with concrete implementation for this application, are briefly presented. The results obtained through their implementation in computer programs are then explained and compared. Finally some conclusions are drawn, which highlights the advantages of certain methods.

Key-Words: - Electrostatic precipitator, electric field, domain methods, boundary element method

1 Introduction
The development of the computing equipment and of their performance has allowed, over time, that much of the design and optimization stages of the equipment or of the installation to go from laboratory and test bench in the virtual environment. In order to facilitate certain operations it became appropriate to develop some software packages, specialized for the study of some particular or more complex aspects of these structures. In this context some programs for the electromagnetic and thermal fields modeling of the equipment have been developed, and also for the mechanical stress analysis of some components, for the simulation of the electrical and electronic circuits or of the electrical networks, and so on, programs that can operate independently or even correlated each other (the so-called ‘coupled problems’) [1]-[4].

Over the time for the electromagnetic field modeling several numerical techniques for solving the problems were developed, which, considering the approach, can be divided into [5]:

● domain methods, that analyze the field in the entire computational domain (which is discretized into cells or finite elements) based on the imposed boundary conditions and source terms [6], and

● boundary methods, that discretize completely the boundary of the domain, using approximation functions that satisfy the problem equations inside the domain. The first category includes many well-known methods such as finite difference method (FDM), finite element method (FEM), finite volume method (FVM), etc.. In the second category the boundary element method (BEM) is included. To note that each of these methods meets a series of versions to make them to be applied more easily in specific problems. Also they might be coupled in so-called hybrid methods [7].

This paper aims to analyze comparatively both domain methods and boundary methods. The study is applied for the determination of the electric field inside an electrostatic precipitator [8]-[10]. The modeling results, obtained using both commercial softwares (e.g. QuickField or FEMM) and our own programs created in MATLAB, highlight the facilities of each technique and the differences between the results obtained using each method separately. We investigated both data accuracy and computation time required for solving the problem.

2 Electric Field Modeling in the Electrostatic Precipitators
The electrostatic precipitator is an electrical installation serving an industrial plant that uses positive or negative corona phenomenon for charging electrically the dust particles (resulted from combustion) that cross it. The collecting chamber of the electrostatic precipitator includes a series of equidistant emissive electrodes placed in the middle plane between two parallel plates (collecting electrodes).

Due to its symmetry only a quarter of the electrostatic precipitator channel needs to be modeled (Fig.1).
By discretizing the equation (2) using finite differences around an interior node, \((i, j)\), for a uniform mesh both on the \(x\)-axis direction (of step \(h\)) and on the \(y\)-axis direction (of step \(k\)) [6], and considering a constant permittivity all over the domain, we obtain:

\[
\frac{1}{h} \left( \varepsilon \frac{V_{i+1,j} - V_{i,j}}{h} - \varepsilon \frac{V_{i,j} - V_{i-1,j}}{h} \right) + \\
\frac{1}{k} \left( \varepsilon \frac{V_{i,j+1} - V_{i,j}}{k} - \varepsilon \frac{V_{i,j} - V_{i,j-1}}{k} \right) = \rho_{i,j}.
\]  

\[
\text{(3)}
\]

The equations of type (3), written for each node inside the domain (with some features for neighboring border nodes) and arranged to highlight the coefficients of each unknown \(V_{i,j}\), lead to obtain a system of algebraic equations that can be solved numerically.

### 2.2 Finite Volume Method

The finite volume method uses an integral formulation of the field problem in terms of an appropriate potential [6], [12]. The equations of the problem are expressed as:

\[
\oint \int \gamma \cdot \left( \varepsilon \cdot \text{grad} V \right) \cdot dS = \oint \gamma \cdot \rho_{Dz}.
\]  

\[
\text{(4)}
\]

where \(\Sigma\) is the boundary surface of the domain and \(qDz\) is the electric charge of the domain.

Neglecting the permanent polarization, for a constant \(\varepsilon\) and using as intermediate unknown the electric potential \(V\), relation (4) written for a finite volume is [6]:

\[
\oint \int \gamma \cdot \left( \varepsilon \cdot \text{grad} V \right) \cdot dS = \oint \gamma \cdot \rho_{V} \cdot dx \cdot dy.
\]  

\[
\text{(5)}
\]

Using as intermediate unknown the electric potential \(V\), \(\overrightarrow{E} = -\text{grad} V\), the system (1) is reduced to the equation:

\[
\text{div}(\varepsilon \cdot \text{grad} V) = -\rho_{V}.
\]  

\[
\text{(2)}
\]
\( \gamma \) is the boundary contour of the finite volume, and \( S_p \) is its surface (Fig.2).

By discretizing the equation (5) on a finite volume surrounding an interior node, \((i,j)\), for a uniform mesh both on the \(x\)-axis direction (of step \(h\)) and on the \(y\)-axis direction (of step \(k\)) \[6\], \[11\], one obtains (Fig.2):

\[
\begin{aligned}
\varepsilon \cdot k \cdot \frac{V_{i+1,j} - V_{i,j}}{h} - \varepsilon \cdot k \cdot \frac{V_{i,j} - V_{i-1,j}}{h} + \\
+ \varepsilon \cdot h \cdot \frac{V_{i,j+1} - V_{i,j}}{k} - \varepsilon \cdot h \cdot \frac{V_{i,j} - V_{i,j-1}}{k} = h \cdot k \cdot \rho_{i,j}.
\end{aligned}
\]

(6)

By comparing the relation (6) with (3) one notes that they are similar. Therefore, for this application, the two methods can be analyzed simultaneously, using the same computer program.

The relation (6), adapted for each node of the mesh, has been introduced into a program created in MATLAB as:

\[
\begin{aligned}
a_{i,j} V_{i,j} + a_{i+1,j} V_{i+1,j} + a_{i-1,j} V_{i-1,j} + \\
+ a_{i,j+1} V_{i,j+1} + a_{i,j-1} V_{i,j-1} = b_{i,j},
\end{aligned}
\]

(7)

where: \(a_{i,j} = -2 \cdot (\varepsilon \cdot k / h + \varepsilon \cdot h / k)\); \(a_{i+1,j} = a_{i-1,j}\)

\(= \varepsilon \cdot k / h\); \(a_{i,j+1} = a_{i,j-1} = \varepsilon \cdot h / k\); \(b_{i,j} = h \cdot k \cdot \rho_{i,j}\).

To solve the system of algebraic equations we used the Thomas algorithm \[12\].

In Fig.3 the distribution of the electric potential \(V\) in the analyzed domain is presented, obtained using the program implemented in MATLAB.

Fig.3 Distribution of the electric potential \(V\) in the computational domain, obtained using finite volume method

### 2.3 Finite Element Method

The finite element method uses an integro-differential formulation of the field problem \[13\], \[14\]. Here the unknown functions are approximated polynomials on parts of the domain (on finite elements). The initial equations of the problem take the same form (1). In Galerkin’s version the equations of the finite element method are \[6\]:

\[
\sum_l V_l \cdot \int_{D_L} \varepsilon \cdot \nabla \phi_l \cdot \varepsilon \cdot \nabla \phi_j \cdot d\Omega = \\
= \int_{\Omega} \phi_j \cdot \rho \cdot d\Omega + \int_{S_D} \phi_j \cdot \varepsilon \cdot g \cdot dS,
\]

(8)

where \(\phi_j\), \(\phi_j\) are the shape functions associated with the problem domain, and \(g(r) = \frac{\partial V}{\partial n}|_{S_D}\) corresponds to the Neumann boundary condition. The result is a system of algebraic equations for unknown nodal variables (electric potentials).

This method is well developed and optimized in many software programs specific to the electromagnetic field modeling, available on the market (QuickField, ANSYS, FEMM, etc.), so the creation of a new program becomes useless. In the present study we used as softwares QuickField Professional and FEMM. Thus we could get both the electric potential distribution in the entire domain of analysis, and the distribution of the electric field strength (Fig.4). It is easily observed the electric field concentration near the emissive electrode.

Fig.4 Distribution of the electric field strength \(E\) in the analyzed domain, obtained using FEMM

### 2.4 Boundary Element Method

In this case one starts also from an equation of type (2) \[15\], \[16\] which, for a permittivity constant all over the domain, takes the form of a Poisson type equation:

\[
\Delta V = -\rho_f / \varepsilon.
\]

(9)

It can be shown that for a boundary node, using the weighted residual method \[5\],
\[
\int -\frac{\rho V}{\varepsilon} V^* d\Omega + \int V^* g^* d\Gamma + \int V g^* d\Gamma + \int g V^* d\Gamma + k \cdot V_j = \int g V^* d\Gamma + \int g V^* d\Gamma,
\]

where \( V = \overline{V} \) on the portion \( \Gamma_1 \) of the boundary \( \Gamma \) corresponds to the Dirichlet condition, and \( g = \frac{\partial V}{\partial n} = \frac{\partial V}{\partial n} \) on the portion \( \Gamma_2 \) of the boundary \( \Gamma \) corresponds to the Neumann condition. \( V^* \) corresponds to the weighting function and \( g^* = \frac{\partial V^*}{\partial n} \). The constant \( k \) takes the value 1 for the internal nodes and 0.5 for the boundary nodes [5]. Through discretization, for a boundary node \( i \), the relation (10) takes the form:

\[
\int -\frac{\rho V}{\varepsilon} V^* d\Omega + \sum_{j=1}^{n} V_j \int g^* d\Gamma + \frac{1}{2} V_i = \sum_{j=1}^{n} g_{ij} \int V^* d\Gamma,
\]

that can be brought to the form

\[
B_i + \sum_{j=1}^{n} H_{ij} V_j = \sum_{j=1}^{n} G_{ij} g_{ij}.
\]

The integral terms \( H_{ij} \) and \( G_{ij} \) can be calculated by numerical integration (e.g. using Gauss quadrature rule) [5],[17].

Based on these relations a program in MATLAB was developed that calculates the electric field produced inside the electrostatic precipitator using the boundary element method. Some results are shown in Fig.5 and Fig.6.

By comparing the results obtained using each of these methods one observed that similar results both qualitatively and quantitatively were achieved. Moreover they are in accordance with the numerical and experimental results obtained by other experts [1], [7], [10], [13].

To more thoroughly examine the deviations of the results, a comparative study of them is necessary.

3 Comparative Analysis of the Results

In order to emphasize the differences between the results obtained using each method described above a comparative study was carried out. It aimed to establish the order of magnitude of the recorded deviations and also to estimate the time required to perform the calculations and display the results.
Some of the obtained results were presented graphically in Figs.7-9. They were focused mainly on the analysis of electric potential variations, given that this is the main variable used in the electric field modeling and it is used afterwards to calculate other field quantities ($\vec{E}$, $\vec{D}$).

For a suggestive analysis it was aimed as the number of points corresponding to the boundary to be, in each case (each method), the same (400 nodes, 100 nodes on each side). However, at the finite element method (FEM) this number was higher to obtain a high precision of the results (13343 nodes in the entire domain).

Fig.7 shows the variation of the electric potential on the emissive electrodes direction. It was found that the values obtained using FEM are average values between those obtained by the other two methods (FVM-FDM and BEM). In this case the highest recorded error (reported to FEM) was about 10% (Table 1).

![Fig.8 Variation of the electric potential between the emissive electrode and the collecting electrode](image1)

![Fig.9 Variation of the electric potential along the line $L_x = 0$, $y$ direction (Fig.1)](image2)

Table 1

<table>
<thead>
<tr>
<th>Potential [V]</th>
<th>Absolute value [V]</th>
<th>Relative value [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>on direction $y = 0$</td>
<td>10180</td>
<td>10.81</td>
</tr>
<tr>
<td>on direction $x = 0$</td>
<td>10072</td>
<td>10.77</td>
</tr>
<tr>
<td>on direction $x = L$</td>
<td>2321</td>
<td>4.89</td>
</tr>
</tbody>
</table>

Fig.8 shows the variation of electric potential between the emissive electrode and the collecting electrode. In this case it is found also that the values of the FEM are averages between those obtained by the other two methods. The maximum relative error that was recorded in this case was about 10%.

Fig.9 depicts the electric potential variation along the line $x = L$. In this case the maximum relative error was below 5% (Table 1).

Reduced differences between results could be achieved by increasing the number of nodes (in the mesh or on the boundary respectively) and by imposing an even smaller error in the numerical solving.

In Table 2 the working time that is necessary to obtain the results is specified.

Table 2

<table>
<thead>
<tr>
<th>Method</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FVM, FDM</td>
<td>279.9</td>
</tr>
<tr>
<td>BEM</td>
<td>8.484</td>
</tr>
<tr>
<td>FEM</td>
<td>3.57</td>
</tr>
</tbody>
</table>

The relatively high value of time for FVM-FDM is obtained due to the use of a program in MATLAB that was not optimized. This program (that is based on the Thomas algorithm) uses an iterative process and requires the achievement of the convergence; the smaller the imposed error, the greater the computing time. Accuracy is, however, satisfactory.

The boundary element method, using a small number of points, requires less data and a substantially reduced working time. This is the reason why this method has gained ground in the last years, finding applicability in numerous technical problems, including the coupling with other numerical techniques.

The lower time required to solve the problem using FEM is due to the development and optimization of the commercial softwares that use this method, both in terms of domain discretization and solving the systems of algebraic equations.

In conclusion, each numerical method can be used successfully in solving the problem of the electric field inside the electrostatic precipitator.
The results are comparable. The differences in choosing the method consist in the facilities in the obtaining and the implementation of the discretized equations (at FVM and FDM), in the time and computational effort decreasing (at BEM) or in the easy use of some already developed programs (at FEM).

4 Conclusion

This work aimed to compare some numerical methods known for the modeling of the electromagnetic field, with application to the electric field calculation of an electrostatic precipitator. To simplify the analysis some simplifying hypotheses were considered (constant permittivity, constant charge density, airflow neglecting, etc.).

The study highlighted the possibility of using alternative methods for modeling the electromagnetic field, each method having its advantages. Any of them can be implemented in own computer programs that can be viable alternatives to commercial software, especially for some specific applications.

References:


