Numerical problems in 3D magnetostatic FEM analysis

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Abstract: - In this paper some numerical problems that occur in finite element analysis of the 3D magnetostatic fields are discussed. The impact of the Coulomb gauge condition on the accuracy of magnetostatic field computed in homogenous and non homogenous media is evaluated, using iterative and direct solvers.
The influence of the Coulomb gauge condition on the iterations number is evaluated. The influence of the boundary conditions on the magnetic flux density and MVP distribution is described. The behavior of the magnetic field at the interface of the media is evaluated.

Key-Words: - **Coulomb gauge, finite element method, magnetic vector potential, vector elements**

1 Introduction

The magnetic vector potential(MVP) is not uniquely defined by the Maxwell equations.For 3D magnetostatic problems the divergence of A should be specified in order to make the solution unique. Even if the MVP is not unique the magnetic flux density vector B is unique.
The topic has certain approaches in the literature. Imposing this gauge with the nodal elements causes some problems
The finite element formulation can obtain the solution even for the ungauged equations.
But this approach implies a high number of iterations of the solver used, because of the system matrix is ill conditioned [1].One solution is to use penalty functions.
In [2] this approach is investigated for Coulomb gauge using FE model simulations.
In [3],[4],[5] different finite element solutions using nodal and vector elements, with gauged and ungauged formulations are presented.
In [6], [7], [8] and [9],[10] there are comprehensive presentations of potential formulations used in finite element methods applied in electromagnetic field.
The influence of the Coulomb gauge condition on the solving process is evaluated, considering homogenous and non homogenous media, with vector quadratic elements.
Direct and iterative solvers are used and also the accuracy of the results are discussed.

2 Theoretical considerations

2.1 Magnetostatic field equations

The Maxwell equations describing the magnetostatic field are:

$$\nabla \cdot \overline{B} = 0 \qquad (1)$$
$$\nabla \times \overline{H} = \overline{J} \qquad (2)$$

The curl of the magnetic vector potential (MVP) is defined :

$$\overline{B} = \nabla \times \overline{A} \qquad (3)$$

Relation (3) satisfies automatically (1).
By replacing (3) in (2) we obtain :

$$\nabla \times \nu \nabla \times \overline{A} = \overline{J} \qquad (4)$$

where ν is the magnetic reluctivity.
The Coulomb gauge condition is described by:

$$\nabla \cdot \overline{A} = 0 \qquad (5)$$

In order to have a unique solution for the MVP the divergence of it should also be defined.
For a constant ν a vector Poisson equation is obtained:

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$$\Delta \overline{A} = -\mu \overline{J} \qquad (6)$$

where μ is the magnetic permeability.

2.2 3D finite element formulations for linear magnetostatic analysis

In 3D problems equation (4) has to be solved. Any function:

$$\overline{A} = \overline{A'} + \nabla f \qquad (7)$$

is a solution of (4).Hence, a gauge condition should be used in order to obtain a unique solution.
Depending on the gauge condition there are three variational functionals used in the finite element method formulations [8]:
- The first functional doesn't include a gauge condition :

$$F_1(A) = \frac{1}{2}\int_V [\frac{1}{\mu}(\nabla \times \overline{A})^2 - 2 \cdot \overline{J}\overline{A}]dV \qquad (8)$$

-The second functional adds the divergence term as a penalty term :

$$F_2(A) = \frac{1}{2}\int_V [\frac{1}{\mu}(\nabla \times \overline{A})^2 - 2 \cdot \overline{J}\overline{A} + \lambda(\nabla \cdot \overline{A})]dV \quad (9)$$

- The third functional imposes a gauge condition explicitly :

$$F_3(A) = \frac{1}{2}\int_V [\frac{1}{\mu}(\nabla \cdot \overline{A})^2 - 2 \cdot \overline{J}\overline{A}]dV \quad (10)$$

2.3 Nodal 3D elements vs. vector elements

At the interface of certain two media, 1 and 2, certain conditions for the state quantities of the magnetic field have to be satisfied. The normal component of the magnetic flux density and the tangential component of the magnetic field intensity vectors should be continous [8]:

$$B_{n1} = B_{n2} \qquad (11)$$

$$H_{t1} = H_{t2} \qquad (12)$$

Using the MVP conditions (11) and (12) are expressed as follow:

$$\frac{\partial A_{t2,1}}{\partial t_{1,1}} - \frac{\partial A_{t2,1}}{\partial t_{1,1}} = \frac{\partial A_{t2,2}}{\partial t_{1,2}} - \frac{\partial A_{t2,2}}{\partial t_{1,2}} \qquad (13)$$

$$\frac{1}{\mu_1}(\frac{\partial A_{n,1}}{\partial t_{2,1}} - \frac{\partial A_{t_{2,1}}}{\partial_{n,1}}) = \frac{1}{\mu_2}(\frac{\partial A_{n,2}}{\partial_{t2,2}} - \frac{\partial A_{t2,2}}{\partial_{n,2}}) \quad (14)$$

where t_1 and t_2 are two orthogonal tangential directions at the interface.
The conventional nodal elements give a overall magnetic field solution but it fails to obtain an accurate field solution at the material interfaces.
The nodal element formulation presumes that not only the tangential but also the normal component of the MVP is continuous at any element interface.
The derivative of every component of MVP,along the tangential direction, is always continuous.
The first requirement (13) is automatically satisfied with the nodal MVP formulation. In the second requirement, two derivative terms $\frac{\partial A_n}{\partial t_2}$ and $\frac{\partial A_{t2}}{\partial n}$ need to represent the jump caused by the large difference between the magnetic permeabilities of the media.The first term, $\frac{\partial A_n}{\partial t_2}$, is continuous, because is the derivative in the tangential direction.
Only the second term, $\frac{\partial A_{t2}}{\partial n}$, should represent the necessary jump.In some cases the first continous derivative term is much larger than the second discontinous derivative term.
As a consequence the nodal MVP formulation fails to satisfy condition (14).
The shape function is in a vector form and represents the direction of the MVP.this different type of approximation allows the discontinuity of the derivative term $\frac{\partial A_n}{\partial t_2}$ and condition (14) is satisfied.

2.4 Representation of vector functions using vector finite elements

Inside each vector finite element a vector function is approximated by a linear combination of shape functions associated with edges. Within an element, a vector function ***T*** is approximated as [10]:

$$\overline{T} = \sum_{i=1}^{e} T_{ti}\overline{N_{ei}} \qquad (14)$$

where the coefficient T_{ti} is the degree of freedom at edge I and $\overline{N_{ei}}$ is the edge shape function corresponding to edge i.
The index e is the number of edges in the element and is equal to 6 for the tetrahedral element.
The line integral of $\overline{N_{ei}}$ along edge i equals unity, yielding that the line integral of $\overline{T}$ along edge i can be written as:

$$\int_i \overline{T} \cdot d\overline{l} = \int_i T_{ti} \overline{N_{ei}} d\overline{l} = T_{ti} \qquad (15)$$

So T_{ti} is the line integral of $\boldsymbol{T}$ along edge i. The degrees of freedom, instead of being components of the vector function at element nodes, must be understood as the line integrals of the approximated vector function along element edges.
When two elements share an edge i, the degrees of freedom *Tti* at edge i are set to be equal. Applying this procedure throughout a mesh makes the vector function $\overline{T}$ tangentially continuous across all element interfaces. The vector function thus constructed is not normally continuous (Fig.1)

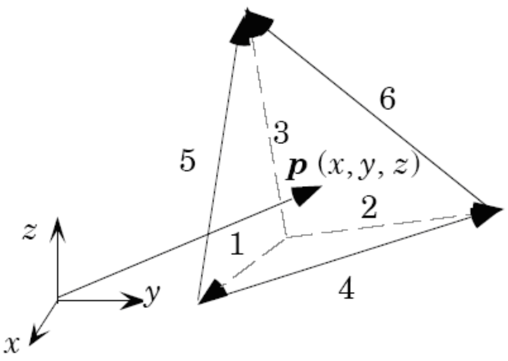

Fig.1 Low order tetrahedral vector finite element

3 Finite element simulations

3.1 Non-ferromagnetic sphere in homogenous magnetostatic field

Two cases were considered: gauge on and gauge off, for the MVP formulation.

3.1.1 Gauge off

The geometry is presented in Fig.2.There is an interior sphere is subjected to an external homogenous magnetic field $\overline{B} = B_z \overline{k}$, of 0.1 T, in a spherical domain (Fig.2).

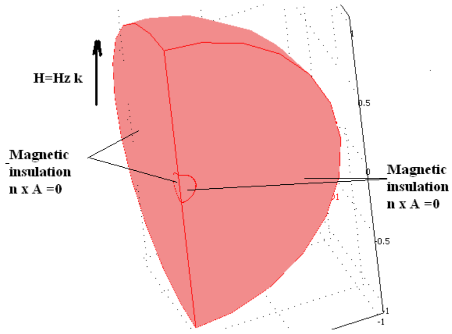

Fig.2 Model and boundary conditions

For reasons of symmetry only a quarter of the geometry is modeled. The radius of the spherical domain is $r_e = 1000$ mm and of the internal sphere is $r_i = 100$ mm.The finite element method Comsol Multiphysics software was used, the AC/DC, Magnetostatics submodule, that uses MVP and allows to activate or not the gauge condition [10].
A 53000 tetrahedral quadratic vector mesh was used (Fig.2).In this figure only elements that satisfy the condition $0.05 < z < 0.09$ m are presented.

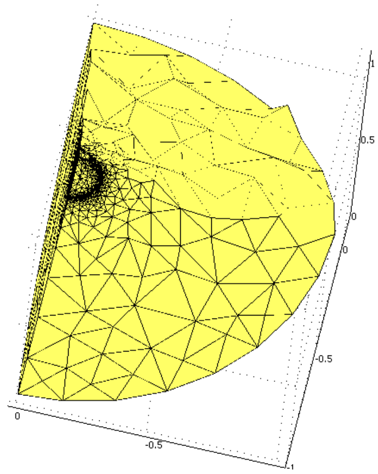

Fig.3 Selected mesh elements

A zoomed detail that emphasizes the mesh distribution is described in Fig.3

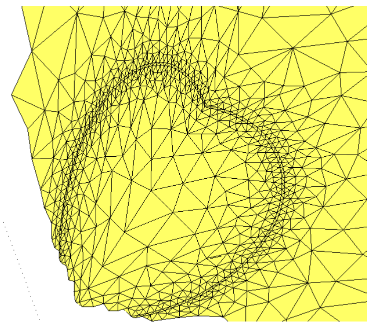

Fig.4 Detailed mesh

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The source of the field was introduced as a boundary condition: a magnetic field value or a MVP value.
For the MVP formulation the number of DOFs is 63771 and the execution time is 48.7 s, using a direct solver Spooles and for magnetic field BC the execution time is 52.9 s.
The Spooles solver uses the multifrontal method and direct LU factorization of the matrix A.
The distribution of the magnetic flux density B is slightly different for different types of source boundary conditions. In Fig.5 is the MVP source description and Fig.6 is the magnetic field strength H.

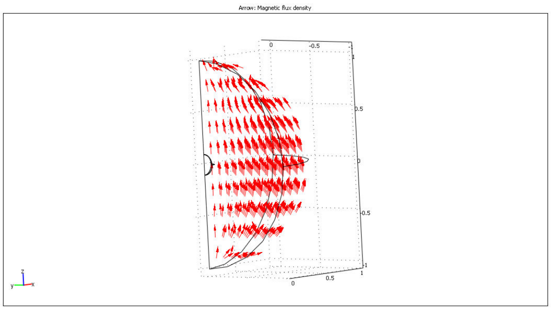

Fig.5 Distribution of the magnetic flux density for MVP magnetic field BC

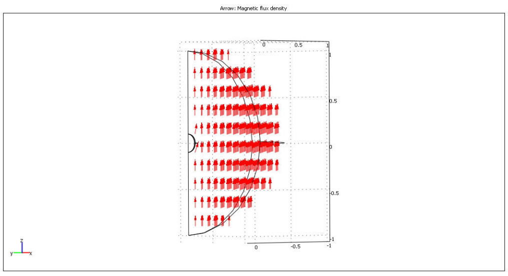

Fig.6 Distribution of the magnetic flux density for the magnetic field strength BC

The difference is rather natural because the MVP is not unique in this case. The MVP distribution is almost random, in both cases, as in Fig.7 and 8.

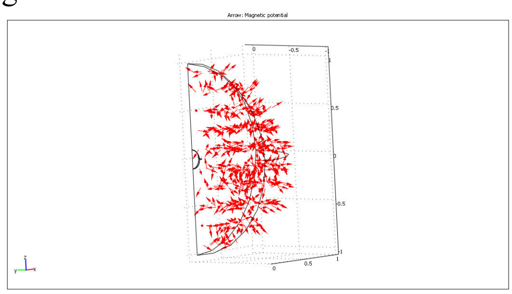

Fig.7 Distribution of the MVP

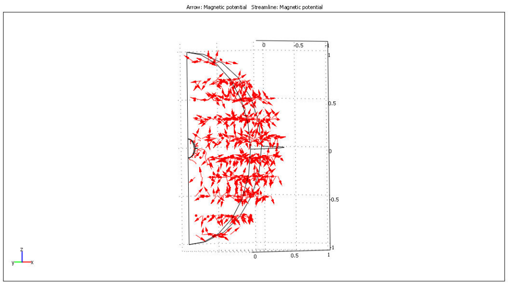

Fig.8 Distribution of the MVP

The values in the origin for the magnetic field case is Bi(0,0) = 0.103 T and Bmax =0.426 T.
It indicates a good accuracy.
For the MVP formulation Bi(0,0) = 0.15 T with a Bmax = 2.42 T. This indicates that this approach is not a proper one.
The iterative solver doesn't work for the H source describing but works for the A formulation.
The solver used was FMGRES with the preconditioner Vanka.
In order to use an iterative solver a preconditioner should be selected. The convergence and the number of iterations are influenced by the selection.
If a linear system of algebraic equations is considered:

$$A \cdot x = b \qquad (16)$$

The preconditioner is a matrix M that approximates A (sometimes in a very rough sense).The preconditioned system becomes :

$$M^{-1} \cdot A \cdot x = M^{-1} \cdot b \qquad (17)$$

Because the $M^{-1} \cdot A$ matrix is close to the identity matrix, in some sense, the iterative solver converges faster when applied on the preconditioned system.
The convergence is presented in Fig.8.The value of magnetic flux density, Bi(0,0) = 0.102 T.

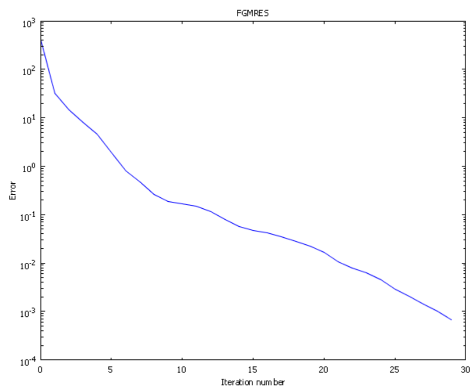

Fig.9 Error vs.number of iterations for gauge off

3.1.2 Gauge on

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For a direct solver Spooles, Bi = 0.1 T, and Bmax=0.1T.The execution time is 127.7 s for 73000 DOFs.
When the gauge is on the distribution of the magnetic flux density and of the magnetic vector potential is normal, ie orthogonal.The Oz orientation of the B magnetic flux density is presented in Fig.10 and the oy direction of the MVP in Fig.11.

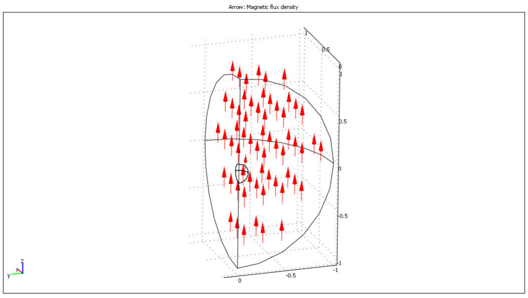

Fig.10 Distribution of the magnetic flux density

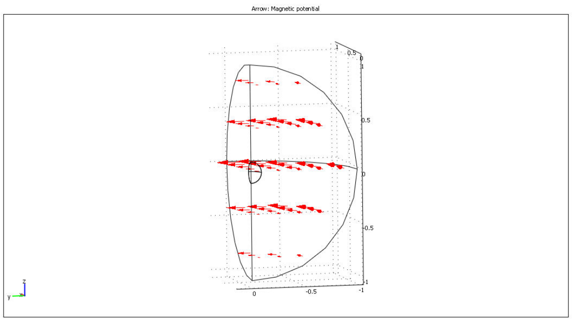

Fig.11 Distribution of the MVP

The iterative solver used for homogenous media was FMGRES with a Vanka preconditioner. The Bi = 0.1 T, in 43.6 s having 65971 DOFs, with vector quadratic elements.
From Fig.11 it can be noticed that the number of iterations is smaller, for gauge on approach than for the gauge off.

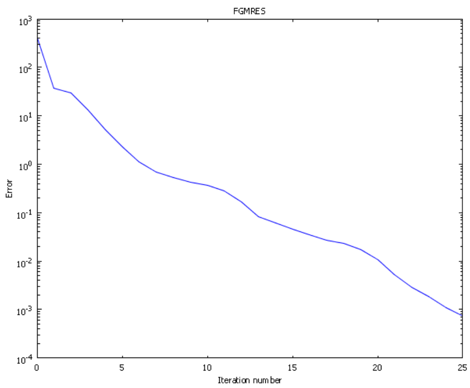

Fig.12 Error vs.number of iterations for gauge on

3.2Ferromagnetic sphere in homogenous magnetostatic field

The same two cases were considered.

3.2.1 Gauge off

The Bi=0.299 T obtained with direct Spooles solver for 54076 DOFs, 8188 elements vector quadratic, needed an execution time t=15.53 s on a 2GHz dual core laptop. This result is in good agreement with the theoretical value 0.3 T. In Fig.13 the random distribution of magnetic potential is presented.

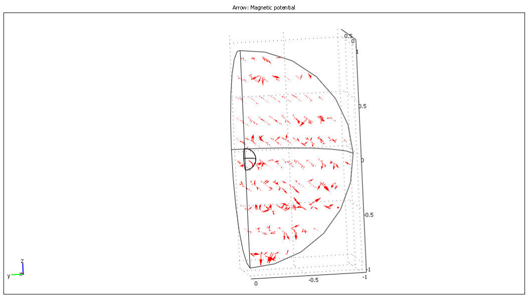

Fig.13 Distribution of the MVP

A ferromagnetic sphere with the relative magnetic permeability $\mu_r = 1000$ is considered.
In Table 1 the comparison of the results for the gauge off and on cases, solved using the Spools solver.

Table 1 The magnetic flux density values

Gauge	No. of elements	DOFs	Execution time [s]	Bi
on	21616	65071	45.6	0.299
off	21616	139588	12.64	0.466

It can be noticed that the gauge off state decrease significantly the accuracy of the magnetic flux density. Iterative solvers didn't converged.

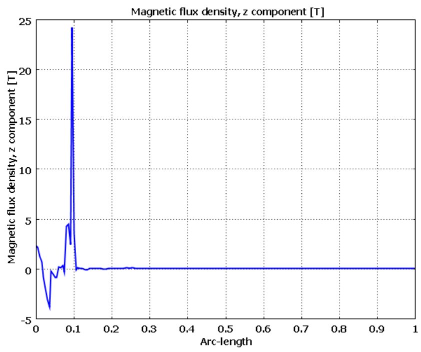

Fig.14 Variation of the Oz component of the magnetic flux density at the media interface for gauge off

In Fig.14 the variation of the z component of the magnetic flux density, along an arc between the origin of the spheres and the external boundary, is presented. It can be noticed the strange variation of the Bz inside the ferromagnetic sphere and at the peak at the interface.

3.2.2 Gauge on

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The iterative solver was FMGRES with a Vanka preconditioner. The Bi = 0.3 T, in 37.8 s having 65971 DOFs, with vector quadratic elements.
In Fig.15 the variation of the z component of the magnetic flux density, along an arc between the origin of the spheres and the external boundary ,is presented.
The jump from 0.3 T to 0.1 T, caused by the difference of the magnetic permeabilities, has to be noticed.

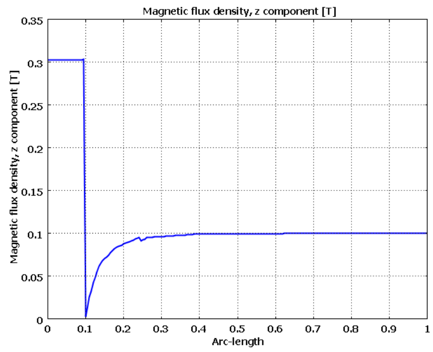

Fig.15 Variation of the Oz component of the magnetic flux density at the media interface for gauge on

4 Conclusion

The Coulomb gauge condition has a significant impact on the values and distribution of the magnetic flux density and MVP. The magnetic flux density preserves its unique value and its distribution despite of the state of gauge condition. When the gauge is off the distribution of the MVP is affected. The BC has a certain influence on MVP distribution, when using a direct solver. For the iterative solver the impact of the gauge is higher, especially for the non homogenous media.

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