Afterburning Control System Based on Fuel Injection Dosage System with Respect to the Turbine’s Pressure Ratio

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Abstract: - This paper deals with a control system for an aircraft single-spool single-jet engine with afterburning, which fuel injection system operates with respect to the basic engine’s turbine pressures. The author has established the mathematical model for the fuel dosage system and has also determined its time behavior (quality). Based on other mathematical models, determined for similar controllers, the author has studied an embedded control system, for the basic engine and its afterburner, during their common operating. Using this new model combination, system’s quality (time behavior) was studied, concerning the embedded system’s total thrust, as well as some other important output parameters (such as engine speed, exhaust nozzle area, fuel flow rates, afterburner’s temperature). System’s model and its time behavior can be useful for further studies concerning aircraft engines with afterburning.

Key-Words: - fuel, injection, afterburning, control, pressure, hydraulic, jet engine, fuel pump.

1 Introduction
Aircraft gas turbine engines, especially jet engines with or without thrust augmentation facilities, considered as controlled objects, have as main input parameter the fuel flow rate. Aircraft jet engines with afterburning systems (EAS) have at least two input parameters, which are the combustor fuel flow rate (basic engine’s input parameter) and the afterburner fuel flow rate.

While the basic engines have as main output parameter (controlled parameter) the engine speed \( n \) (which is an indirect, but a very effective, performance indicator), an engine with afterburning system has as output parameters the afterburner’s temperature and/or the combined engine thrust (which is a direct performance indicator).

Afterburning fuel injection control is achieved, in most cases, by follower systems, which output is the afterburner’s fuel flow rate. As input parameter for such a control system can be used: the throttle positioning, as direct method, as well as some basic engine’s output parameters (e.g. turbine’s pressure ratio), as indirect method.

The gas-dynamic principles and the motion equations, for both of the EAS components (basic jet engine, as well as the afterburning system), are presented in [4], [5] and [9]. Meanwhile, the basic single-spool single jet engine and the afterburning system as controlled objects are depicted in [7], [9] and [10]; a possibility for the afterburner’s fuel pump automatic control was presented by the author in [11]; an embedded system (engine-afterburner) was also presented by the author in [14].

This paper deals with an embedded EAS control system, which consists of an engine speed controller and an exhaust nozzle controller, both operating...
with respect to the throttle’s position, as well as of an afterburning fuel injection controller, which operates with respect to the engine’s turbine’s pressure ratio. Such a system is depicted in fig. 1.

2. System description and operation

As fig.1 shows, EAS main parts are: a) air inlet; b) axial compressor; c) engine’s combustor; d) gas-turbine (its rotor is connected to the compressor’s rotor through the shaft, resulting the engine’s spool); e) afterburner with fuel injectors and flame stabilizers; f) adjustable exhaust nozzle with actuator.

EAS consists of two interconnected jet propulsion systems (engines): a single-spool turbo-engine and an afterburner, each one of them having its own control and controlled parameters.

As control parameters (inputs) for a single-jet single-spool engine, only two inputs can be identified: the fuel flow rate \(Q_c\) (which is the most important control parameter) and the exhaust nozzle area \(A_5\); meanwhile, the combustor’s temperature \(T^*_3\) is a limited controlled parameter, its limitation being realized through the same fuel flow-rate control.

EAS has, eventually, a single input parameter, which is the throttle’s position \(\theta\), as fig. 2 shows. The throttle is the unique command that the pilot can use, but it generates, by a complex input mechanism, the two input signals (presetting the reference signals for \(n\) and for \(A_5\)). Engine’s shaft, through a gearbox, drives engine’s fuel pump rotor. Meanwhile, the exhaust nozzle’s opening is controlled by a follower system, similar to the one studied in [1] and [12].

The afterburner is supplied by a fuel pump (driven by the same engine shaft) and controlled by an actuator, which correlates the fuel flow rate \(Qf\) to the engine’s gas-turbine pressures \(p^*_3\) (or \(p^*_2\)) and \(p^*_4\) (or to the turbine pressure ratio). This fuel dosage system scheme is shown in fig. 3.

Consequently, the embedded control system has a single input, throttle’s position, but three control parameters (combustor fuel flow rate \(Q_c\), exhaust nozzle area \(A_5\) and afterburner fuel flow rate \(Qf\)); as main output parameter (controlled parameter) one has chosen the combined thrust \(F_p\), but, in order to build the mathematical model and to perform some simulations, a few other relevant secondary output...
parameters (speed \( n \), total pressures \( p_2^* \) and \( p_4^* \), afterburner’s temperature \( T_{4p}^* \), etc) are selected.

3. Afterburning fuel dosage system
As fig. 3 shows, afterburner’s fuel supply system consists of: a) fuel pump; b) injection pressure regulator; c) fuel dosage valve; d) fuel valve’s actuator; e) turbine pressure transducer.

3.1. Fuel dosage system mathematical model
System’s mathematical model consists of each above-mentioned part motion equations, as follows:

a) fuel pump equation
\[ Q_p = Q_p(n); \]

b) fuel dosage valve equations
\[ Q_s = \mu_s \frac{n d_i^2}{4} \sqrt{\frac{2}{\rho}} \sqrt{p_p - p_i}, \]
\[ d_s = d_s(y); \]

c) injection pressure regulator equations
\[ Q_p - Q_s - Q_g = \beta V \frac{dp}{dt} - S \frac{du}{dt}, \]
\[ Q_s - Q_f = \beta V_o \frac{dp}{dt} + S \frac{du}{dt}, \]
\[ Q_g = \mu_g b_g u \left( \frac{2}{\rho} \right) \sqrt{p_p}, \]
\[ Q_f = \mu_f \frac{n d_i^2}{4} \sqrt{\frac{2}{\rho}} \sqrt{p_i}, \]
\[ S_g(p_i - p_p) = m_g \frac{d^2 u}{dt^2} + \xi \frac{du}{dt} + k_{cf}. \]

where \( Q_p \) is the fuel flow rate supplied by the pump, \( Q_s \) – flow rate through the dosage valve, \( \mu_s, \mu_g, \mu_f \) – flow rate coefficients, \( p_p - p_r \) – pressure behind the pump, \( p_i - p_r \) – fuel injection pressure, \( \rho \) – fuel density, \( d_s \) – dosage valve’s diameter, depending on actuator’s rod position \( y \), \( Q_g \) – fuel flow rate through regulator’s slot, \( b_g \) – regulator’s slot width, \( u \) – regulator’s piston displacement, \( S_g \) – regulator’s piston area, \( \beta \) – compressibility coefficient (assumed as null for the liquid fuel), \( V_g \) – regulator’s chamber volume, \( V_0 \) – fuel injection ramp volume, \( Q_f \) – injected fuel flow rate, \( d_i \) – fuel injector’s diameter, \( r \) – number of injectors, \( m_g \) – regulator’s piston mass, \( \xi \) – viscous friction coefficient, \( k_{cf} \) – regulator’s spring elastic constant.

Besides these equations should be considered both actuator’s and transducer’s equations, depicted and studied in [10], [12] and [13].

3.2. Linearized mathematical model
The above determined non-linear equation system is difficult to be used for further studies, so it can be linearized, using the small perturbation method, considering formally any variable or parameter \( X \) as \( X = X_0 + \Delta X \) and \( \bar{X} = \frac{\Delta X}{X_0} \), where \( \Delta X \) – parameter’s deviation, \( X_0 \) – steady state regime’s value and \( \bar{X} \) – non-dimensional deviation.

Introducing the new form of each parameter into the above-mentioned equation system, separating the steady state terms and introducing the flow rate expressions (1), (2), (6) and (7) into (4) and (5), one obtains a new form of the equation system, a linear one. Assuming that the used fuel is incompressible, so \( \beta = 0 \), all terms multiplied by \( \beta \) become null.

Using some appropriate chosen amplifying terms, the new determined mathematical model can be transformed in a non-dimensional one. After applying the Laplace transformer, one obtains the non-dimensional mathematical model, as follows
\[ \bar{p}_p = k_{p_0} \bar{n} - k_{p_0} \bar{n} - k_{p_0} \bar{p}_i, \]
\[ \bar{p}_i = k_{p_0} \bar{p}_p + k_{0} \bar{y} - \tau_u \bar{s} \bar{n}, \]
\[ (s^2 + 2\omega_2 T_1 s + 1) \bar{x} = k_{u_1} \bar{p}_i - k_{u_0} \bar{p}_p, \]
\[ \bar{Q}_f = k_{f_0} \bar{p}_i. \]

One has to add to the above-presented equations the actuator’s simplified model (see [10])
\[ \bar{y} = \frac{1}{\tau_u s + p_a}, \]

as well as the pressure transducer model, determined and studied in [13]
\[ (s^2 + 1) \bar{p}_R = k_{2R_2} \bar{p}_2 \bar{s} - k_{2R_2} \bar{s} + 1 \bar{x}, \]
\[ k_{p_0} (\bar{p}_4 - \bar{p}_R) = (s^2 + 2\omega_1 T_1 s + 1) \bar{x}. \]

The above-used annotations are
\[ k_{g_0} = \mu_g b_g \frac{2}{\rho} \frac{p_{p_0}}{\sqrt{2 P_{p_0}}, k_{g_p} = \frac{2}{\sqrt{2 P_{p_0}}, k_{p_0} = \frac{\partial Q_p}{\partial n}}, \]
\[ k_{s_p} = \frac{\mu_s n d_i^2}{4 \sqrt{2 P_{p_0}}}, k_{y_2} = 2 \mu_s d_i^0 \frac{2}{\sqrt{2 P_{p_0}}}, k_{p_0} = \frac{k_{g_0}}{k_{g_p}}, \]
\[ k_{f_0} = \frac{\mu_f n d_i^2}{4 \sqrt{2 P_{p_0}}}, k_{p_0} = \frac{k_{g_0}}{k_{g_p}}, k_{p_0} = \frac{k_{g_0}}{k_{g_p}}. \]


3.3. Fuel dosage system quality

Based on equations (9) to (15), the block diagram with transfer functions for the fuel dosage system may be built and a simulation may be performed, in order to study system’s step response, which gives system’s quality.

System’s inputs are: turbine’s pressures $p^*_1$ and $p^*_2$, as well as engine’s speed $n$; system output is the fuel flow rate $Q_f$ injected in the afterburner. For each input parameter a step variation was considered, while the other two parameters were kept constant. Simulations results are shown in fig. 4, where three curves are presented. As a conclusion, the dosage system is an asymptotic stable one, no matter the input were, but with small static errors (between 1.2% and 2.4%). Response times are various, from 1 s for speed step input, 2.5 for $\overline{p}_2^*$ step input, to 4.5 s for $\overline{p}_1^*$ step input.

4. Embedded system mathematical model

This study is realized for a hypothetic EAS, consisting of an engine similar to VK-1F, which is equipped with brand new controllers. As fig. 2 shows, one can identify five operational blocks, as follows:

a) jet engine with afterburning (EAS);

b) speed controller;

c) exhaust nozzle controller;

d) afterburning fuel injection controller (fuel injection dosage system);

e) input signal formatting block (which can be very simple, as here is, or more complicate, as presented in [10]).

4.1. EAS model

As presented in [14], EAS model has a matrix description, which gives, eventually (using a Cramer solving method), the output parameters’ equations. For the particular case of VK-1F propulsion system, these equations are, as follows:

a) basic engine’s speed equation

$$\bar{n} = \frac{3.402 \Lambda_5 + 2.3837 \bar{Q}_c + 3.402 \bar{Q}_f}{2.9413s + 13.486},$$  \hspace{1cm} (18)

b) engine’s turbine pressures’ equations

$$\overline{p}_2^* = \frac{4.836 \Lambda_5 + (0.5187s + 5.763)\bar{Q}_c + 6.3182\bar{Q}_f}{2.9413s + 13.486},$$  \hspace{1cm} (19)

c) afterburner’s temperature equation

$$\bar{T}_{i,\beta}^* = \frac{1}{2.9413s + 13.486} \left[ \left( 2.3531s + 0.9942 \right) \Lambda_5 + (0.1452s + 7.5263)\bar{Q}_c - (2.3531s + 0.9942)\bar{Q}_f \right],$$  \hspace{1cm} (20)

d) total thrust equation

$$\bar{F}_p = \frac{1}{2.9413s + 13.486} \left[ \left( 4.4883s + 1.3816 \right) \Lambda_5 + (2.1196s + 24.743)\bar{Q}_c + (6.0591s + 6.325)\bar{Q}_f \right],$$  \hspace{1cm} (22)

which are EAS mathematical model equations and will be used for further studies.

![Fig. 4. Fuel dosage system (flow rate) time behavior](image-url)
4.2. Throttle’s model
Embedded control system, as fig. 2 shows, has a single input, which is throttle’s position \( \theta \). Throttle’s positioning has two operation intervals:
- from “idle” to “maximal” (or “full”), when it controls the basic engine’s speed, \( \theta \) being proportional to the speed reference \( n_{\text{ref}} \);
- beyond “maximal”, into afterburning domain, when \( \theta \) is conceived to be proportional to \( F_p \) total thrust. In fact, it can be assumed as proportional to \( A_5 \), which gives, indirectly, the level of \( Q_f \) (and \( T^*_4 \)).

Such a throttle assisting system (input signal formatting block) is presented in [3], for R11-F300 jet engine with afterburning; a similar system is described in [12], operating after a similar command law, \( A_5 = A_5(\theta) \), as follows.

4.3. Exhaust nozzle’s controller
Exhaust nozzle’s area control is realized by a system as presented in [12], by a hydro-mechanical control system. Its simplified mathematical model consists of a single equation
\[
A_5 = \frac{0.0712 \times (0.0422s + 0.1025)}{(0.1474s + 1)(0.6376s + 1)} \theta. \quad (23)
\]
Comparing to the form in [12], the hydraulic supplying pressure influence was neglected, assuming that the hydraulic system uses constant pressure supply valves.

4.4. Basic engine speed controller
For this EAS, one has considered a simple speed controller, as described and studied in [11]. It establishes the fuel flow rate for the basic engine, with respect to the throttle’s position; fuel pump is driven by the engine’s shaft, so pump’s speed is equal or proportional to the engine’s speed. Its equation is
\[
Q_5 = \frac{0.683 \theta}{0.078s^2 + 1.813s + 5.3068 + (0.9065s + 2.4795) + 1.6183s + 6.308}, \quad (24)
\]
so, the fuel flow rate supplied by the engine’s main pump depends on the throttle’s position, as well as on the effective engine’s speed.

5. System’s quality
As fig. 2 shows, embedded control system has a single input, the throttle’s position. For the

Fig. 5. Fuel dosage system time behavior (flow rate) simulation.
affected by static errors; the smallest values, around 0.5%, are those for the afterburner’s flow rate (fig.5.b) and for the exhaust nozzle’s area (fig.5.a).

Basic engine’s output parameters $\bar{Q}$ and $\bar{n}$ are shown in fig. 5.a; the pressures before and behind the turbine (outputs for the engine, but inputs for the fuel dosage system), as well as the afterburning fuel flow rate (output for the fuel dosage system), are presented in fig. 5.b.

Main output parameter, the total thrust $\bar{F}_p$, as fig. 5.c shows, has also a small static error (4.3%) and an acceptable time response (around 2.5 s); afterburner’s total temperature $\bar{T}_{i,p}$ has a similar behavior, but with a bigger static error (5.4%) and the same response time.

6. Conclusion

This paper has studied an aircraft jet engine with afterburning as controlled object. Jet engine VK-1F was considered as basic engine and three controllers were theoretically adapted to it. As fuel system for afterburning, one has studied a dosage system working with respect to the total pressures before and behind basic engine’s turbine, which uses a micro-jet type pressure transducer and a fuel valve commanded by an actuator. Control laws were established in order to keep the embedded system (engine and afterburning) stable running, no matter its regime was. Embedded system has a single input parameter (throttle’s position), which generates (directly or indirectly) EAS input parameters’ level. Most important output parameters are afterburner’s temperature and total thrust. Some simulations were performed, using the mathematical model(s), for each system part, as well as for the whole embedded system; one has established system’s quality as system step response.

As the step responses in fig. 5 demonstrates, the studied embedded control system makes a stable system from the EAS, with small static errors and acceptable response time(s).

As fig.5.c shows, comparing to a similar system (described in [14], dash line in fig. 5.c), the hereby studied one has better performances (continuous line in fig. 5.c); although its static errors are bigger, the stabilization is asymptotic for both the total thrust and temperature, the initial small override(s) of the system described in [14] being eliminated.

The paper subject and used method can be extended for multi-spool jet engines with afterburning, as well as for further improved studies, concerning other engines, with different coefficient values.

References: