Krill herd (KH) algorithm for portfolio optimization

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Abstract: This paper presents novel krill herd (KH) nature-inspired metaheuristics for solving portfolio optimization task. Krill herd algorithm mimics the herding behavior of krill individuals. The objective function for the krill movement is defined by the minimum distances of each individual krill from food and from higher density of the herd. Constrained portfolio optimization problem extends the classical mean-variance portfolio problem by adding constraints to the basic problem definition. For optimizing constraint portfolio problem, traditional optimization techniques do not obtain satisfying results, and the usage of metaheuristics approach is necessary. Experimental results show that the krill herd algorithm is a promising technique for tackling portfolio optimization problems.

Key–Words: portfolio optimization problem, metaheuristic optimization, swarm intelligence, krill herd (KH) algorithm, nature-inspired algorithms

1 Introduction

Portfolio optimization problem is also known as portfolio selection problem and such represents one of the most studied topics in economics and finance. Financial portfolios represent the collection of financial instruments (investments), all owned by the same organization or by an individual. They usually include bonds (investments in debts), stocks (investments in individual businesses), and mutual funds (pools of money from many professional investors).Portfolio structure is generally designed according to the investor's risk sensitivity, objectives of an investment and a time frame.

In its basic definition, portfolio optimization problem refers to dealing with the selection of portfolio's assets (or securities) that minimizes the risk subject to the constraint that guarantees a given level of returns. Individual and institutional investors prefer to invest in portfolios rather than in a single asset because by doing this, the risk is mitigated with no negative impact on the expected returns [1]. In other words, portfolio optimization problem seeks for an optimal way to distribute a given budget on a set of available assets [2].

Portfolio optimization's aim is to select portfolio with minimum risk at defined minimal expected returns. In other words, the goal is the reduction of non-systematic risks to zero. Also, portfolio optimization problem can be defined as multi-criteria optimization in which risks have to be minimized, while, on the other hand, return has to be maximized. Unfortunately, this approach to the problem has several drawbacks [3]. Firstly, it might be difficult to gather enough data for accurate estimation of the risk and returns. Secondly, when estimating return and risks using covariance, errors can frequently occur [4]. Third, and finally, this model is considered to be too simplistic for practical purposes because it does not capture essential properties of the real-world trading, such as maximum size of portfolio, transaction costs, preferences over assets, cost management, etc. These properties can be modeled by adding additional constraints to the basic problem definition which transform unconstrained portfolio optimization problem into the constrained one. Constrained problem is more complex, and belong to the class of NP-Complete problems [5].

With the addition of real-world requirements to the basic portfolio optimization problem formulation, the problem is being transformed into constrained, and such it becomes intractable in a reasonable amount of computational time. In those cases, exact methods can not obtain results, and the use of approximate algorithms, and in particular metaheuristics in necessary. Modern metaheuristics algorithms are typically highlevel strategies which guide an underlying subordinate heuristic to the desired objec-

This research was supported by Ministry of education and science of Republic of Serbia, Grant III-44006.

tive. Metaheuristics methods can find satisfying feasible solution in a reasonable amount of computational time.

Swarm intelligence employs principles of the collective behavior of social insect colonies and other animal groups in the search process. Swarm intelligence can be classified in the group of population based metaheuristics. Those metaheuristics start with initial (usually random) population of candidate problem solutions and iteratively improve them. The key concept of swarm intelligence lies in the effect of emergent behavior of many individuals which exhibit extraordinary collective intelligence without any centralized supervision mechanism. Entire swarm intelligence system is fully adaptive to internal and external changes, and it is established on four basic properties on which self-organization rely: positive feedback, negative feedback, multiple interactions and fluctuations. Swarm intelligence approach has obvious advantages over other optimization methods and techniques: scalability, adaptation, fault tolerance, parallelism and speed.

Ant colony optimization (ACO) showed satisfying performance in solving many hard optimization problems [6], [7], [8], [9]. This metaheuristics was inspired by the foraging behavior of ants who deposit pheromone trails which help them in finding the shortest path between food sources and their nests. The basic philosophy of ACO algorithm involves the movement of an ant colony which is directed by two local decision policies: pheromone trails and its attractiveness. Particle swarm optimization (PSO) is a swarm intelligence algorithm that imitates social behavior of fish schooling or bird flocking. PSO was tested on portfolio optimization problem [10]. Artificial bee colony (ABC) metaheristics is one of the latest simulations of the honey bee swarm. The simulated bee colony consists of employed, onlooker and scout bees. ABC showed outstanding results in global optimization problems [11], [12], [13].

In this paper, we present the krill herd (KH) algorithm for portfolio optimization problem. KH was recently proposed by Gandomi and Alavi [14]. The implementation of the KH for portfolio optimization problem was not found in the literature.

After Introduction, in Section 2, this paper illustrates mathematical formulation of the portfolio optimization problem. In this Section, we present different problem formulations that can be found in the literature. Section 3 introduces KH metaheuristics. Experimental data, problem setup and experimental results are presented in Section 4, while Section 5 concludes the paper.

2 Portfolio optimization problem

The fundamental guideline in making financial investments decisions is diversification where investors invest into different types of assets. Portfolio diversification minimizes investors' exposure to the risks while maximizing returns on portfolios.

Many methods can be applied to solving multi-objective optimization problems such is portfolio optimization. One essential method is to transform the multi-objective optimization problem into a single-objective optimization problem. This method can be further divided into two sub-types. In the first approach, one important objective function is selected for optimization, while the rest of objective functions are defined as constrained conditions. Alternatively, only one evaluation function is created by weighting the multiple objective functions [15].

The first method is defined by Markowitz and is called the standard mean-variance model [16]. It was first introduced more than 50 years ago and its basic assumptions are a rational investor with either multivariate normally distributed asset returns, or, in the case of arbitrary returns, a quadratic utility function [2]. If those assumptions hold, then the optimal portfolio for the investor lies on the mean-variance efficient frontier.

In this model, the selection of risky portfolio is considered as one objective function and the mean return on an asset is considered to be one of the constraints [10]. It can be formulated as follows:

$$min\ \sigma^2_{R_p} = \sigma^2_p = \sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\omega_j Cov(\bar{R}_i\bar{R}_j) \quad (1)$$

Subject to

$$\bar{R}_p = E(R_p) = \sum_{i=1}^{N}\omega_i\bar{R}_i \geq R \quad (2)$$

$$\sum_{i=1}^{N}\omega_i = 1 \quad (3)$$

$$\omega_i \geq 0,\ \forall i \in (1,2,...N) \quad (4)$$

where N is the number of available assets, $\bar{R}_i$ is the mean return on an asset i and $Cov(\bar{R}_i\bar{R}_j)$ is covariance of returns of assets i and j respectively. Weight variable ω_i controls the proportion of the capital that is invested in asset i, and constraint in Eq. 3 ensures that the whole available capital is invested. In this model, the goal is to minimize the portfolio risk σ^2_p, for a given value of portfolio expected return $\bar{R}_p$.

ISBN: 978-960-474-332-2

In the presented standard mean-variance model, variables are real and they range between zero and one, as they represent the fraction of available money to invest in an asset. This choice is quite straightforward, and has the advantage of being independent of the actual budget.

The second method refers to the construction of only one evaluation function that models portfolio selection problem. This method comprises two distinct models: efficient frontier and sharpe ratio model [15].

The models we talked about so far refer only to the basic problem definitions. Those definitions do not seem realistic because they do not consider several aspects, such as [17]:

- the existence of frictional aspects like the transaction costs, sectors with high capitalization and taxation;
- the existence of specific impositions arising from the legal,economic, etc. environment;
- the finite divisibility of the assets to select.

Taking into account all above mentioned additional portfolio optimization constraints, new portfolio optimization problem can be established [10]. This model is called extended mean-variance model and it is classified as a quadratic mixed-integer programming model necessitating the use of efficient heuristics to find the solution. It can be formulated as follows:

$$min\ \sigma^2_{R_p} = \sigma^2_p = \sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\omega_j Cov(\bar{R}_i\bar{R}_j) \quad (5)$$

where

$$\omega_i = \frac{x_i c_i z_i}{\sum_{j=1}^{N} x_j c_j z_j},\ i = 1, ..., N \quad (6)$$

$$\sum_{i=1}^{N} z_i = M \leq N,\ M, N \in N,\ \forall i = 1, ...N,\ z_i \in \{0, 1\} \quad (7)$$

Subject to

$$\sum_{i=1}^{N} x_i c_i z_i \bar{R}_i \geq BR \quad (8)$$

$$\sum_{i=1}^{N} x_i c_i z_i \leq B \quad (9)$$

$$0 \leq B_{low_i} \leq x_i c_i \leq B_{up_i} \leq B,\ i = 1, ...N \quad (10)$$

$$\sum_{i_s} W_{i_s} \geq \sum_{i_{s'}} W_{i_{s'}}$$
$$\forall y_s y_{s'} \neq 0,\ \ s, s' \in \{1, ...S\},\ s < s' \quad (11)$$

where

$$y_s = \begin{cases} 1 & \text{if } \sum_{i_s} z_i > 0 \\ 0 & \text{if } \sum_{i_s} z_i = 0 \end{cases} \quad (12)$$

where M represents the number of selected assets among possible N assets. B is the total available budget, while B_{low_i} and B_{up_i} are lower and upper limits of the budget that can be invested in asset i respectively. S is the total number of sectors. c_i represents the minimum transaction lot for asset i, and x_i denotes the number of c_i' that is purchased. According to this, $x_i c_i$ are integer values that show the units of asset i in the portfolio.

Decision variable z_i is defined for modeling cardinality constraint. z_i is equal to 1 if an asset i is present in the portfolio. Otherwise, it is equal to 0. Eq. (7) represents the cardinality constraint and inequality (8) is the same as (2). In order to make the search process easier, budget constraint (Eq.9) is converted to inequality. Eq. (10) shows lower and upper bounds of budget constraint.

3 KH Implementation

Zooplankton aggregation occurs as the result of biological and physical processes [18]. In the ocean environment, the density distribution of plankton depends on the circulation patterns, such as mesoscale vertices and fronts [18]. The behavior of individuals responding to their environment also plays a significant role in the generation of dense, quasi horizontal patches of zooplankton commonly called swarms. With pysical and chemical cues, collective movements and the formation of groups in the population can be triggered, and this is referred to as social behavior.

Antarctic krill is one of the best-studied species of marine animal. Krill herds exist on a space scales of 10 to 100 meters. In the last three decades, many studies have been conducted for the sake of understanding the ecology and distribution of the krill.

Although there are yet notable uncertainties about the forces determining the distribution of the krill herd, conceptual models have been proposed to explain the observed formation of the krill herds.

When predators attack krill, they remove only individual krill,and the krill density is reduced. The formation of the krill after the attack depends on several parameters. The herding of the krill individuals is a multi-objective process including two main goals: (1) increasing krill density, and (2) reaching food.

The position of an individual krill that is time-dependent in 2D surface is govern by the following factors: movement induced by other krill individuals, foraging activity and random diffusion.

The Lagrangian model is generalized to an n dimensional decision space:

$$\frac{dX_i}{d_t} = N_i + F_i + D_i, \tag{13}$$

where N_i is the motion induced by other krill individuals, F_i is foraging motion, and D_i is the physical diffusion of the i-th krill.

The movement of krill individual is defined by:

$$N_i^{new} = N^{max}\alpha_i + \omega_n N_i^{old}, \tag{14}$$

where

$$\alpha_i = \alpha_i^{local} + \alpha_i^{target}, \tag{15}$$

where N^{max} is the maximum induce speed, w_n is the inertia weight of the motion in the range $[0, 1]$, N_i^{old} is the last motion induced, α_i^{local} is the local effect provided by the neighbors α_i^{target} is the target direction provided by the best krill individual.

The sensing distance for each krill individual can be determined using different heuristic methods. Here, it is determined using the following formula for each iteration:

$$d_{s,i} = \frac{1}{5N}\sum_{j=1}^{N} ||X_i - X_j|| \tag{16}$$

where $d_{s,i}$ is the sensing distance for the i-th krill in the population, and N is the number of the krill individuals. The factor 5 in the denominator is empirically calculated [14]. In 16, if the distance of two krill individuals is less than the defined sensing distance, they are neighbors.

The known target vector of each krill individual is the lowest fitness of an individual krill. The global optimum is defined as followed:

$$\alpha_i^{target} = C^{best} K_{i,best} X_{i,best} \tag{17}$$

where C^{best} is the effective coefficient of the krill individual with the best fitness of the i-th krill individual. The value of C^{best} can be defined as follows:

$$C^{best} = 2(rand + \frac{l}{l_{max}}) \tag{18}$$

where $rand$ is a random number between 0 and 1, l is the current iteration numbers, and l_{max} is the maximum number of iterations.

As mentioned above, the krill motion consists of foraging motion, motion influenced by other individuals, and the physical diffusion. The foraging motion formulation is based on two main effective parameters: the food location, and the previous experience about the food location. Foraging motion of the i-th krill individual is formulated as follows:

$$F_i = V_f \beta_i + \omega_f F_i^{old} \tag{19}$$

where

$$\beta_i = \beta_i^{food} + \beta_i^{best} \tag{20}$$

where V_f is the foraging speed, w_f is the inertia weight of the foraging motion, and it is defined in range [0,1], β_i^{food} is the food attractiveness, and β_i^{best} is the effect of the best krill found in the population so far. According to empirical test, the best value for the V_f is 0.02 ms_{-1}.

The effect of the food depends on its location. The center of the food is discovered first and it is used for formulation of food attraction. This can only be estimated. In [14], the virtual center of food concentration is estimated according to the fitness distribution of the krill individuals, which is inspired from the "center of mass". This center of food in each iteration is defined as:

$$X^{food} = \frac{\sum_{i=1}^{N} \frac{1}{k_i} X_i}{\sum_{i=1}^{N} \frac{1}{K_i}} \tag{21}$$

The food attraction of the i-th krill individual is defined as:

$$\beta_i^{food} = C^{food} K_{i,food} X_{i,food} \tag{22}$$

where C^{food} is the food coefficient, and it is defined as follows:

$$C^{food} = 2(1 - \frac{l}{l_{max}}) \tag{23}$$

Physical diffusion of the krill individuals is a random process, and it is used for exploration of the search space. It is formulated using maximum diffusion speed and a random directional vector:

$$D_i = D^{max}\delta \tag{24}$$

where D^{max} is the maximum diffusion speed, and δ is a random directional vector. Empirically calculated maximum diffusion speed is in the range [0.002,0.010] ms^{-1} [14].

ISBN: 978-960-474-332-2

Above defined motions frequently change the position of a krill individual towards the best fitness. Motions contain two global and two local strategies, which make KH very powerful algorithm [14]. The position of a krill individual in the time interval $[t, t + \Delta t]$ is given below:

$$X_i(t + \Delta t) = X_i(t) + \Delta t \frac{dX_i}{dt} \tag{25}$$

Pseudo-code of the KH algorithm is given below [14].

I Definition: defining algorithm parameters, bounds of the problem, etc.

II Initialization: creation of the initial population of solutions

III Fitness evaluation: evaluate all krills based on its current position

IV Motion calculation:

based on the position of other individuals

foraging motion

physical diffusion

V Updating: update the krill individual position in the population

VI Repeating: go to step III until termination criteria is met

VII End

We should note that in implementation presented in [14], genetic operators were used. In our demonstration, no genetic operators were employed.

4 Practical application and results

In this section, we present portfolio optimization problem formulation used in testing KH approach, data used in the experiments and experimental results. We used the same problem formulation and data set like in [19]. Data set is given in Table 1.

Table 1: Data set for the experiments

Year	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5
2007	-0.15	0.29	0.38	0.18	-0.10
2008	0.05	0.18	0.63	-0.12	0.15
2009	-0.43	0.24	0.46	0.42	0.15
2010	0.79	0.25	0.36	0.24	0.10
2011	0.32	0.17	-0.57	0.30	0.25

For testing purposes, we used simple historical data set like in [19].

Tests were performed on Intel Core 2 Duo T8500 processor @4GHz with 4GB of RAM memory, Windows 7 x64 Ultimate 64 operating system and Visual Studio 2012 with .NET 4.5 Framework. Krill number (KN) was set to 40, while maximum iteration number IN was set to 6000, yielding totally 240.000 objective function evaluations (40*6000). The same number of objective function evaluations was used in [12].

We set the foraging speed V_f to 0.02 like in [14], and diffusion speed (D^{max}) to 0.006 (the aritmetic average of the range of recommended paramters, see Section 3). The algorithm was tested on 30 independent runs each starting with a different random number seed.

Experimental results are show in the tables below.

Table 2: Experimental results

	Best	**Worst**	**Mean**
Objective function	4.582	4.702	4.703
Variance	0.039	0.069	0.075
Return	0.209	0.183	0.195

In Table 3, we show portfolio weights for the best and worst results.

Table 3: Portfolio weights for best and worst results

	ω_1	ω_2	ω_3	ω_4	ω_5
Best	0.072	0.415	0.287	0.226	0.079
Worst	0.035	0.188	0.259	0.232	0.286

According to the experiment results presented in Tables 2 and 3, KH for portfolio optimization performs similar like GA approach in [19]. In [19], three variants of GA were shown: single-point, two-point and arithmetic. Arithmetic variant performed significantly better than other two variants, and also better than the KH presented in this paper. But, at the other hand, KH showed similar performance like single-point and two-point variants of the GA presented in [19].

5 Conclusion

In this paper, KH for portfolio optimization problem was presented. The algorithm was tested on a set of five assets, like GA in [19]. The results of the investigation reported in this paper show that the KH metaheuristics has potential for solving this problem.

KH was applied only to the basic portfolio optimization problem definition. Generally, the application of swarm intelligence metaheuristics on the variants of portfolio optimization problem was not enough investigated according to the sources from the literature. In our subsequent work, original, as well as the modified version of the KH algorithm will be applied to the extended-mean variance, and other portfolio optimization formulations. Also, other swarm intelligence metaheuristics will be implemented for solving this family of problems.

Acknowledgements: This research is supported by Ministry of Education and Science of Republic of Serbia, Grant No. III-44006.

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