Reducing harmonic pollution using the method of instantaneous reactive power theory for control active power filters

MARIUS MARCU, FLORIN-GABRIEL POPESCU, LEON PANA
Department of Automation, Computer, Electrical, and Energy Engineering
University of Petrosani
Str. Universitatii nr. 20, 332006, Petrosani, jud. Hunedoara
ROMANIA
mariusmarcu66@yahoo.com, floringabriel82@yahoo.com, leon_pana@yahoo.com, http://www.upet.ro

Abstract: - The paper presents the structure of an active parallel filter for reducing harmonic pollution and reactive power. The active power filter control is based on instantaneous reactive power theory. The authors present the modelling of parallel active filter based on this theory and the simulation results in MATLAB-SIMULINK. The method is based on instantaneous reactive power theory (TPRI_P).

Key-Words: - Power active filter, instantaneous reactive power, harmonic pollution, distorting regime, reactive power, simulation.

1 Introduction

More than 60% of power system is feed by power static converters. There are well known advantages of power static converters, but there are some disadvantages such as distortion of the waveforms of voltages and currents of the power supply, which is equivalent with presence of fundamental harmonic and superior harmonics in electrical networks [11], [18].

In consequence of that, there is necessary development of some solutions to compensate the voltage and current harmonics. Those solutions are represented by passive and active filters.

Active power filters (APF) represent for the network a changeable inductance with a determined value needed for harmonics discharge. These filters are didn’t based on a rigid structure and are appropriate for complex waveforms, which have a complex mathematic model.

In the latest years, there were a large variety of APF topologies [3], [15], [22], due to the development in power converters and power static elements with a better performance in control, current and voltage rated values and reduced switching times [5].

These semiconductor system based on IGBT transistors are extremely robust and include in addition to the circuit "driver" and other circuitry, such as: thermal protection circuits, protection against surges and direct short circuits [11], [17].

All of these associated with a development of digital techniques and a Digital Signal Processors (DSP) leaded to the APF methods improvement.

APF can be classified in [13], [14]:

- Parallel active filters;
- Serial active filters;
- Serial-parallel active filters;
- Hybrid active filters.

The parallel active filters represents the well-known structure and improve the distorting regime, power factor, balanced the network currents and made the null phase current to zero value.

The APF is connected in parallel with the non-linear load. The basic function consists in injection of currents which are load currents in opposite-phase, so that the load harmonic currents are canceled [9], [10].

The APF consists in a DC static converter and an element to stock the energy. The static converter is controlled with a high frequency and with an algorithm which measures the load current and generates switching signals for the semiconductors elements to injects an appropriate currents in the network [19].

The most used structure of parallel active filter (PAF) is based on voltage converter controlled in voltage where the energy is stocked in a capacitor installed on dc part of the converter (Fig.1).

There are other methods to control the parallel active filter, but this control method will be presented in this paper due to their advantages.
2 Modelling of Power Active Filters Based on Instantaneous Reactive Power Theory

As mentioned at the beginnings of the last century have been developed theories of power which considers that there would be non-sinusoidal waveform of voltage and current. [8]. But in 1983 Akagi, Kanazawa and Nabae [1] have introduced instantaneous reactive power theory (TPRI) for three-phase, based on the concept of instantaneous value, for the waveforms of voltage and current arbitrary, inclusive and transitory statuses.

This is one of the most popular theories that have been used, not only in theory but and in practice can be considered as the first successfully implemented through means of methods based on using the static converters constituting what is known under the name Active Power Filters (APF).

2.1 Instantaneous reactive power theory development (TPRI)

This theory was initially developed for the three-phase systems without neutral making use of instantaneous spatial vectors to define the three phase system in a system of two orthogonal axes called α-β, which completely define the three-phase system.

The conventional system three-phase of voltage and current is related to a system of three axes a, b and c, fixed in level and the phase shift with each other by 2π/3.

This three-phase system is transformed into two -phase α-β axes through the following transformations:

\[
\begin{bmatrix}
  v_a \\
v_\beta
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
  v_a \\
v_b \\
v_c
\end{bmatrix}
\]  

(1)

\[
\begin{bmatrix}
  i_a \\
i_\beta
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
  i_a \\
i_b \\
i_c
\end{bmatrix}
\]  

(2)

The instantaneous power is defined as the sum between the product the instantaneous values of voltage and current for each phase, namely:

\[
p = \sum_i v_i(t) \cdot i_i(t)
\]  

(3)

New system of α-β axes which defines the three-phase system must guarantee equality instantaneous power value, independent of the chosen reference axis system, and therefore, need to ensure that:

\[
p = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c = v_a \cdot i_a + v_\beta \cdot i_\beta
\]  

(4)

As a new concept, Akagi, Kanazawa and Nabae [1] defined a vector space called Instantaneous Imaginary Power, as the vector product of voltage and current in the following manner:

\[
q = v_a \times i_\beta + v_\beta \times i_a
\]  

(5)

Fig.2 Spatial representation of Instantaneous Imaginary Power

This vector is located on an axis perpendicular to the α-β plan and consists of two summations, which is the vector product of the voltage according to axis, respective the current, according to the other
axis. In a graphical representation this products are vectors for which direction is perpendicular to the plane formed by the axes opposite α-β as shown in Fig. 2.

The power p is expressed as [W] and the product between the voltages and currents belonging to the same axis, while the power q can not be expressed even in [W], or in [VA], whereas the product between the voltages and currents situated on different perpendicular axes. Therefore this latter power is called Instantaneous Imaginary Power.

The result of the two previous equations can be expressed in the following way:

\[
\begin{bmatrix}
  p \\
  q
\end{bmatrix} =
\begin{bmatrix}
  v_α & v_β \\
  -v_β & v_α
\end{bmatrix}
\begin{bmatrix}
  i_α \\
  i_β
\end{bmatrix}
\]  

(6)

Current values according to the axes will be:

\[
\begin{bmatrix}
  i_α \\
  i_β
\end{bmatrix} =
\begin{bmatrix}
  v_α & v_β \\
  -v_β & v_α
\end{bmatrix}^{-1}
\begin{bmatrix}
  p \\
  q
\end{bmatrix}
\]

(7)

These instantaneous currents according to α-β axes can be expressed by splitting them into two components:

\[
\begin{bmatrix}
  i_α \\
  i_β
\end{bmatrix} =
\begin{bmatrix}
  v_α & v_β \\
  -v_β & v_α
\end{bmatrix}^{-1}
\begin{bmatrix}
  p \\
  q
\end{bmatrix} +
\begin{bmatrix}
  v_α & v_β \\
  -v_β & v_α
\end{bmatrix}^{-1}
\begin{bmatrix}
  0 \\
  0
\end{bmatrix}
\]

(8)

Where the meaning of each component is as follows:

The instantaneous active current according to the axis α:

\[
i_α = \frac{v_α}{v_α^2 + v_β^2} p
\]

(9)

The instantaneous reactive current according to the axis α:

\[
i_α = \frac{-v_α}{v_α^2 + v_β^2} q
\]

(10)

The instantaneous active current according to the axis β:

\[
i_β = \frac{v_β}{v_α^2 + v_β^2} p
\]

(11)

The instantaneous reactive current according to the axis β:

\[
i_β = \frac{v_α}{v_α^2 + v_β^2} q
\]

(12)

Instantaneous power according to the α-β axes, as was previously expressed is the sum of power according of each axle and has the following relationship:

\[
p = p_α + p_β = v_α \cdot i_α + v_β \cdot i_β = v_α \cdot (i_α + i_α) + v_β \cdot (i_β + i_β)
\]

(13)

or

\[
\begin{bmatrix}
p_α \\
p_β
\end{bmatrix} =
\begin{bmatrix}
v_α i_α \\
v_β i_β
\end{bmatrix} =
\begin{bmatrix}
v_β i_β \\
v_α i_α
\end{bmatrix}
\]

(14)

From ones two relationships results that:

\[
p = p_α + p_β = \frac{v_α^2}{v_α^2 + v_β^2} p + \frac{-v_α v_β}{v_α^2 + v_β^2} q + \frac{-v_α v_β}{v_α^2 + v_β^2} q + \frac{v_β^2}{v_α^2 + v_β^2} q
\]

(15)

It results that:

The instantaneous active power according to the axis α:

\[
p_α = \frac{v_α^2}{v_α^2 + v_β^2} p
\]

(16)

The instantaneous reactive power according to the axis α:

\[
p_α = \frac{-v_α v_β}{v_α^2 + v_β^2} q
\]

(17)

The instantaneous active power according to the axis β:

\[
p_β = \frac{v_β^2}{v_α^2 + v_β^2} p
\]

(18)

The instantaneous reactive power according to the axis β:

\[
p_α = \frac{v_α v_β}{v_α^2 + v_β^2} q
\]

(19)
Mathematical Methods and Optimization Techniques in Engineering

That is:
\[ p = p_\alpha + p_\beta = p_a + p_{aq} + p_{\beta p} + p_{\beta q} \] (20)

We obtain:
\[ p_\alpha + p_{\beta q} - 0 \] (21)

The instantaneous real power consists from \( p_a \) and \( p_{\beta q} \) and coincides with active power of the three-phase system and is called instantaneous active power.

Terms \( p_{aq} \) and \( p_{\beta q} \) have the same value and opposite sign cancel each other and do not contribute to the instantaneous power transfer between source and load. So these forms instantaneous reactive power and can be considered it is a power that circulating between phases and not between source and load, as would seem logical. Also to compensation is not necessary to have any power storage system.

The concept of instantaneous reactive power, not to be confused with the instantaneous imaginary power, is in really the most important approach obtained of the theory introduced by Akagi and others.

2.2 Extension theory for three-phase system with neutral

Initially TPRI was conceived for networks three-phase without neutral, but soon after, the same authors, [2], raised the question of the extension theorem at systems with neutral and with the presence of homopolar components of voltage and current.

This is done by transforming a system of axes a-b-c into another system of orthogonal axes, called \( \alpha-\beta \)-0, through next transformation:

\[
\begin{bmatrix}
  v_a \\
  v_\beta \\
  v_0
\end{bmatrix}
= \frac{\sqrt{2}}{3} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2 \\
  1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix}
\begin{bmatrix}
  v_a \\
  v_\beta \\
  v_0
\end{bmatrix}
\] (22)

\[
\begin{bmatrix}
  i_a \\
  i_\beta \\
  i_0
\end{bmatrix}
= \frac{\sqrt{2}}{3} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2 \\
  1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_\beta \\
  i_0
\end{bmatrix}
\] (23)

Besides components of power \( p-q \), according \( \alpha-\beta \) axes is inserted a new component of power associated axis 0 and values the homopolar components of voltage and current so that:

\[ i_0 = \frac{1}{v_0} \cdot p_0 \] (28)

\[ i_\alpha = i_{ap} + i_{aq} = \frac{1}{v_\alpha^2} \cdot v_\alpha \cdot p + \frac{1}{v_\alpha^2} \cdot (-v_\beta \cdot q) \] (29)

\[ i_\beta = i_{\beta p} + i_{\beta q} = \frac{1}{v_\beta^2} \cdot v_\beta \cdot p + \frac{1}{v_\beta^2} \cdot (v_\alpha \cdot q) \] (30)

So the meaning of each component is:

The homopolar current:

\[ i_0 = \frac{1}{v_0} \cdot p_0 \] (31)

The instantaneous active current according to the axis \( \alpha \):

\[ i_{ap} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} \cdot p \] (32)

The instantaneous reactive current according to the axis \( \alpha \):

\[ i_{aq} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \cdot q \] (33)

This power is defined as the instantaneous power sequence zero or homopolar power.

So that, the initial expression of the power applied to a three-phase system with neutral is:

\[
\begin{bmatrix}
  p_a \\
  q_a \\
  p_0
\end{bmatrix}
= \begin{bmatrix}
  v_\alpha & v_\beta & 0 \\
  -v_\beta & v_\alpha & 0 \\
  0 & 0 & v_0
\end{bmatrix}
\begin{bmatrix}
  i_\alpha \\
  i_\beta \\
  i_0
\end{bmatrix}
\] (25)

From previous expression we get the current values:

\[
\begin{bmatrix}
  i_\alpha \\
  i_\beta \\
  i_0
\end{bmatrix}
= \begin{bmatrix}
  v_\alpha & v_\beta & 0 \\
  -v_\beta & v_\alpha & 0 \\
  0 & 0 & v_0
\end{bmatrix}^{-1}
\begin{bmatrix}
  p \\
  q \\
  p_0
\end{bmatrix}
\] (26)

\[ v_{\alpha q}^2 = v_\alpha^2 + v_\beta^2 \] (27)

Can be obtained:

\[ v_{\alpha q}^2 = v_\alpha^2 + v_\beta^2 \] (27)

\[ \begin{bmatrix}
  i_\alpha \\
  i_\beta \\
  i_0
\end{bmatrix}
= \frac{1}{v_0} \cdot p_0 \] (28)

\[ i_\alpha = i_{ap} + i_{aq} = \frac{1}{v_\alpha^2} \cdot v_\alpha \cdot p + \frac{1}{v_\alpha^2} \cdot (-v_\beta \cdot q) \] (29)

\[ i_\beta = i_{\beta p} + i_{\beta q} = \frac{1}{v_\beta^2} \cdot v_\beta \cdot p + \frac{1}{v_\beta^2} \cdot (v_\alpha \cdot q) \] (30)

So the meaning of each component is:

The homopolar current:

\[ i_0 = \frac{1}{v_0} \cdot p_0 \] (31)

The instantaneous active current according to the axis \( \alpha \):

\[ i_{ap} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} \cdot p \] (32)

The instantaneous reactive current according to the axis \( \alpha \):

\[ i_{aq} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \cdot q \] (33)
The instantaneous active current according to the axis $\beta$:

$$i_{\beta q} = \frac{v_{\beta}}{\sqrt{v_{\alpha}^2 + v_{\beta}^2}} q$$  \hspace{1cm} (33)

The instantaneous reactive current according to the axis $\beta$:

$$i_{\beta q} = \frac{v_{\alpha}}{\sqrt{v_{\alpha}^2 + v_{\beta}^2}} q$$  \hspace{1cm} (34)

In the previous expressions it is assumed that we have a voltage homopolar component nonzero, since otherwise it is impossible to calculate the inverse matrix used to determine the current value of the components. However expressions current components according axes $\alpha$ and $\beta$ are independent of the value of $v_0$. Homopolar current component value resulting from the previous equations as a ratio between two values that are zero, resulting in a determination must interpret the result as the current homopolar, in the system of axes $\alpha-\beta-0$, which is the same as the a-b-c axis system, namely:

$$i_0 = \frac{1}{v_0} p_0 \Leftrightarrow i_{0(abc)} = i_0$$  \hspace{1cm} (36)

All these support ones above and mean that homopolar component analysis of voltage and current, leading to a single-phase circuit independently of other components. From the previous expressions can see that:

$$p = v_0 \cdot i_0 + v_{\alpha} \cdot i_{\alpha p} + v_{\beta} \cdot i_{\beta p} = p_0 + p_{\alpha p} + p_{\beta p}$$  \hspace{1cm} (37)

and

$$0 = v_{\alpha} \cdot i_{\alpha q} + v_{\beta} \cdot i_{\beta q} = p_{\alpha q} + p_{\beta q}$$  \hspace{1cm} (38)

Where the meaning of each component is:

- The homopolar power:
  $$p_0 = v_0 \cdot i_0$$  \hspace{1cm} (39)

- The instantaneous active power according to the axis $\alpha$:
  $$p_{\alpha p} = \frac{v_{\alpha}^2}{\sqrt{v_{\alpha}^2 + v_{\beta}^2}} p$$  \hspace{1cm} (40)

- The instantaneous reactive power according to the axis $\alpha$:
  $$p_{\alpha q} = \frac{-v_{\alpha} v_{\beta}}{\sqrt{v_{\alpha}^2 + v_{\beta}^2}} q$$  \hspace{1cm} (41)

- The instantaneous active power according to the axis $\beta$:
  $$p_{\beta p} = \frac{v_{\beta}^2}{\sqrt{v_{\alpha}^2 + v_{\beta}^2}} q$$  \hspace{1cm} (42)

- The instantaneous reactive power according to the axis $\beta$:
  $$p_{\beta q} = \frac{-v_{\alpha} v_{\beta}}{\sqrt{v_{\alpha}^2 + v_{\beta}^2}} q$$  \hspace{1cm} (43)

The same significance attributed to these expressions in the case in which are not present homopolar components, is that the presence of this component is considered as an independent circuit and does not affect the components of the other axes.

### 3 Simulation of the Power Active Filter

We will present the working regime of the compensation method based on instantaneous reactive power theory by simulation results. The instantaneous reactive power theory is developed in time domain and gives the power, voltage and current \([4], [6], [7], [16]\). The simulations are made using Matlab-Simulink \([23]\) software, which permits simulation for control blocks and the power converters in the same time.

In the PAF simulations the network, the load and the converter of the filter are the same, changing only the control circuit of the filter in concordance with the control method.

The network is with symmetric voltage three phase system.

#### 3.1 Load circuit

The load used in the simulations consists in a three phase rectifier (RECT), permanently connected and
a single phase rectifier (RECM) connected to phase R after 0.3 seconds.

- The three-phase load (RECT):
  Consists in a full bridge rectifier with a d.c. R-L load and an R-L circuit on a.c. side with 2mH and Q=30, like in Fig. 3.
  The control angle of the rectifier is 300 and the voltage average value is with THD = 28.78%.

- The single-phase load (RECM):
  Consists in a diode bridge with THD = 32.7%, which is connected after 0.3 s and an R-L circuit on the a.c. side, with 2mH and Q=30.
  Connecting and disconnecting of the single-phase load permit study of the dynamic work of the active filter.

The currents on phase a, b and c, when both loads are connected have an THD with: 17.4%, 23.4% and 23.4%.

The waveforms for currents and instantaneous powers are presented in the following diagrams:

Fig.3 Simulation scheme for load circuit

Fig.4 Diagram of the phase currents

Fig.5 Current harmonics with three-phase load

Fig.6 Current harmonics with both loads
In Fig.8 is shown the power active filter circuit used for simulation of the control method.

The inverter is a PWM one, and the controlled method for the inverter uses a PI controller with an 8 kHz carrier frequency.

The PI controller transfer function is:

\[ K \cdot \left( 1 + \frac{s}{T} \right) \]

where: \( K = 10 \) and \( T = 10 \text{ms} \).

3.2 Compensation methods based on instantaneous reactive power theory

Order to determine the current compensation on which APF need to provide there are two methods of compensation:

- Method TPRI_Q. It is the conventional method, in which is determine the current which APF need to provide, starting from the calculation of the inactive components of power outside the considering homopolar component and the loss in the APF.
- Method TPRI_P. It is the method that we used in simulations and based on direct determination of the active power which energy source must provide and corresponding ac current, while the current in the APF is obtained indirectly:

\[ i_F = i_S - i_L \] (44)

Expressions are the following:

\[
\begin{bmatrix}
L_{\alpha} \\
L_{\beta}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
0 & -v_0 \\
0 & v_0
\end{bmatrix} \begin{bmatrix}
P + \tilde{P}_0 + P_{per} \\
0
\end{bmatrix}
\]

\[ i_{F_0} = -i_0 \] (45)

Power losses are obtained by through the voltage regulator of the dc capacitor.

Homopolar current will be considered as though it were an independent.

Starting from the power values obtained current values, which will come into PWM controller to generate the command signals for the semiconductor elements of the four half bridge.

The simulation blocks for control methods are described in Fig.9 for the TPRI_P method and in Fig. 10 are presentation the simulation scheme for the TPRI_P method.

\[ i_F = i_S - i_L \]

\[ \begin{bmatrix}
L_{\alpha} \\
L_{\beta}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
0 & -v_0 \\
0 & v_0
\end{bmatrix} \begin{bmatrix}
P + \tilde{P}_0 + P_{per} \\
0
\end{bmatrix}
\]

\[ i_{F_0} = -i_0 \] (45)
The results for simulation in both cases are presented in the following diagrams.

Fig.11. The unfiltered and filtered currents on phase “a” (TPRI_P)

Fig.12. The unfiltered current and the current filter on phase a

Fig.13. The control currents of filter on phase a

Fig.14. The active and reactive powers (TGPI_P)

Fig.15. Current harmonics with three-phase load (TPRI_P)

Fig.16. Current harmonics with both loads (TPRI_P)

4 Conclusion

The concept of power in three-phase or single-phase system has been analyzed from different viewpoints both in the time domain and in the frequency domain, establishing definitions of power based on the average value between the product voltages and...
currents and apparent power as the product of the rms values.

Currents obtained in the α-β system are alternating current and refers to the current in the network, resulting in the offset currents based on current knowledge of the load as opposed to the conventional method (TPRI_Q) that obtains the offset currents directly.

It can be seen that there are no large differences between ones two methods quotes, at least for the type of load and an imbalance of 20% on a phase.

It can be seen in fig.5.19 that in calculations involved exclusive active powers, perfectly measurable.

Mathematical calculation is reduced in TPRI_P method proposed.

Also is observed that:
• It produces an increase of the current THD network in the case of the system constant instantaneous power of the energy source.
• Is observed a small fluctuation by active power of network in the system constant instantaneous power of the energy source.
• Is observed a small fluctuation by reactive power of network in the system constant instantaneous power of the energy source.
• Regarding the homopolar power, network evolution is similar in both cases.

References:
[18] Uțu, I., Marcu, M., Voltage and current Harmonic Distortion in Mine Electric


[23] * * * Software Matlab 6.5.